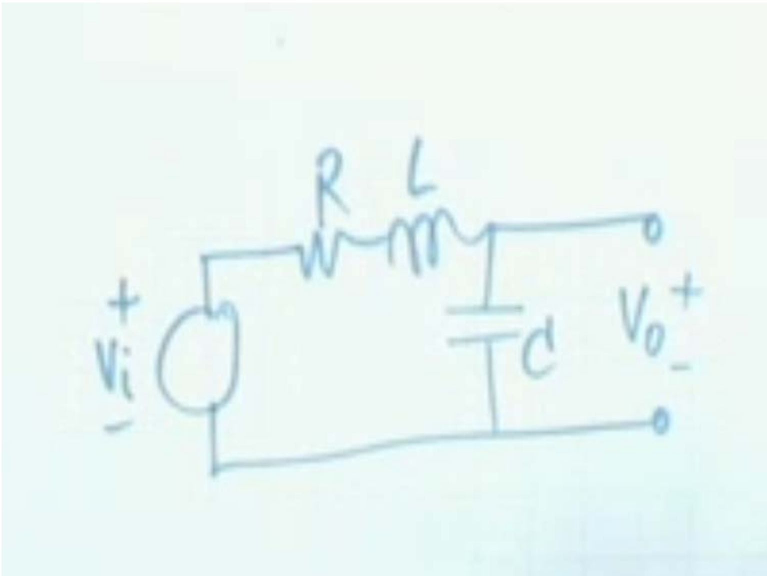
Circuit Theory
Prof. S. C. Dutta Roy
Department of Electrical Engineering
Indian Institute of Technology, Delhi

Lecture - 15
Single Tuned Circuits (Continued)

This is the fifteenth lecture and we continue our discussion on single tuned circuits.

(Refer Slide Time: 1:11)

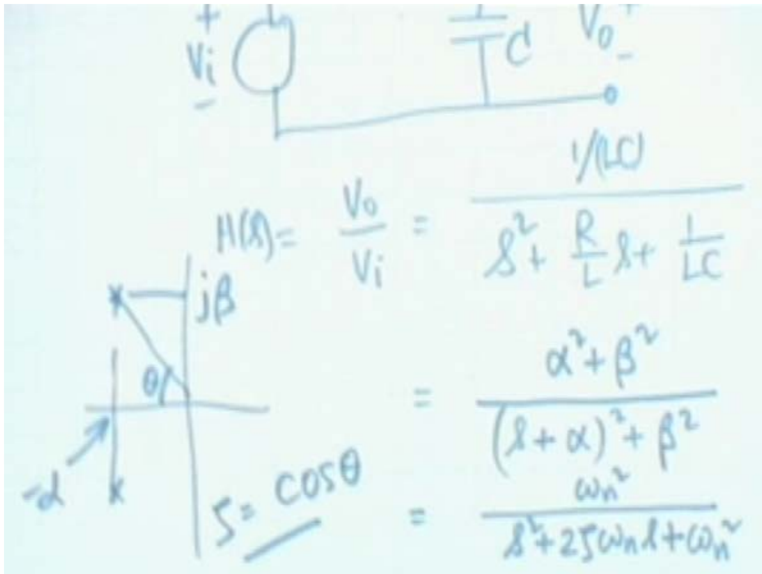


To review briefly what we have done earlier, we have taken a voltage source in series with a resistance and inductance and a capacitance and we chose to take our output voltage across the capacitor.

Student: We can also choose the response

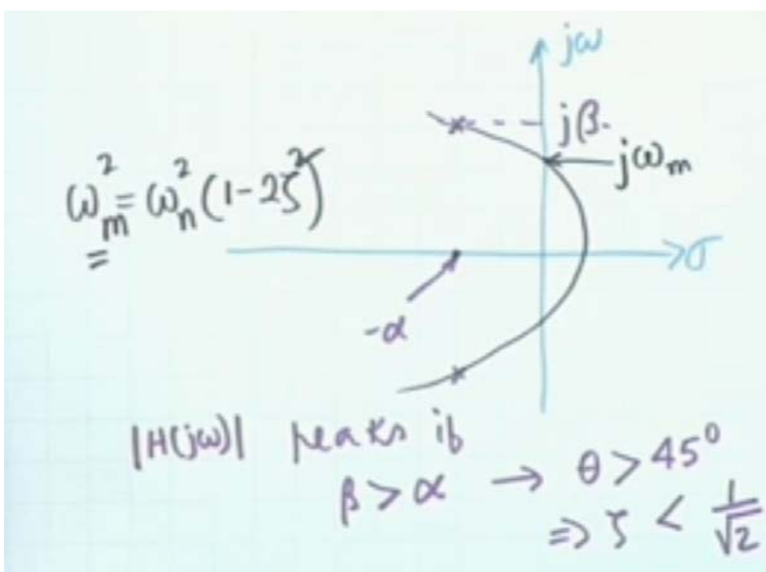
Sir: Sure. It will not be the same response now. Response will be slightly different but our transfer function which is H of s, V 0 by V i, we wrote it in the form 1 over L C divided by s squared plus R by L s plus 1 over L C and we put this in the form, in terms with poles and zeroes as s plus alpha whole squared plus beta squared and alpha squared plus beta squared.

(Refer Slide Time: 01:36)



We also put in a slightly different form, in terms of omega n and zeta and we showed that this is omega n squared divided by s squared plus twice zeta omega n s plus omega n squared, where zeta is defined as the cosine of the angle, theta. This is minus alpha and this is plus j beta. Theta is defined as the cosine of this and zeta is defined as the cosine of this angle theta. These are the three different forms for representation of the same transfer function.

(Refer Slide Time: 02:59)

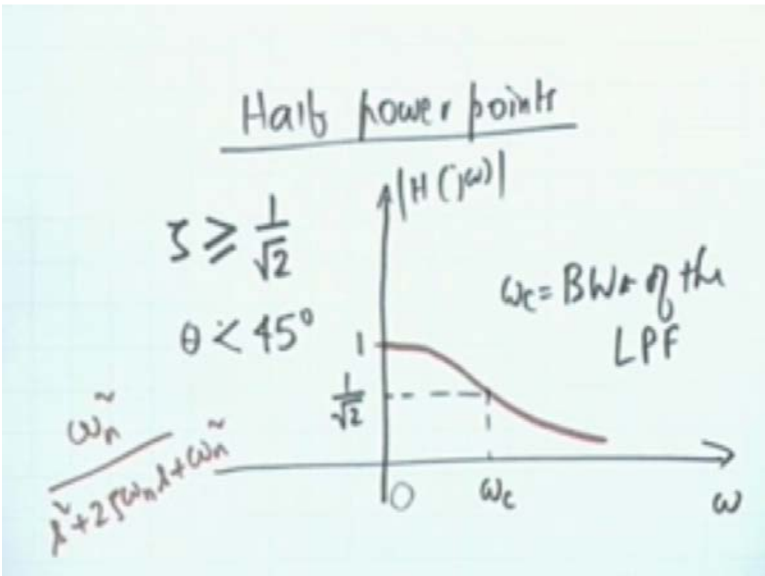


And in terms of alpha and beta, if I write this again j omega sigma and the poles are, let us say, at beta minus beta. This is minus alpha and this is j beta. Then it showed that the response, that is, H of j omega magnitude has peaking if beta, this peaks, if beta is greater than alpha. That is, theta is greater than 45 or zeta, which is equivalent to zeta less than 1 by root 2 and we showed this by means of a peaking circle. That is, what we did was, we drew a circle with this point as center and beta as radius, then this circle cuts the j omega axis at j omega m and this is the frequency at which the peaking occurs. We also showed that omega m square is equal to omega n square multiplied by 1 minus 2 zeta. This is the frequency at which the peaking occurs. This also shows that if zeta is greater than 1 by root 2, I made a mistake here.

Student: zeta square

Sir: Zeta squared, yes. If zeta is greater than 1 by root 2, then this quantity is negative and therefore, there is no maximum. Zeta equal to 1 by root 2 is a critical condition under which the maximum occurs at 0 frequency. Now, these observations will now be useful in finding out the so-called half power points.

(Refer Slide Time: 05:16)

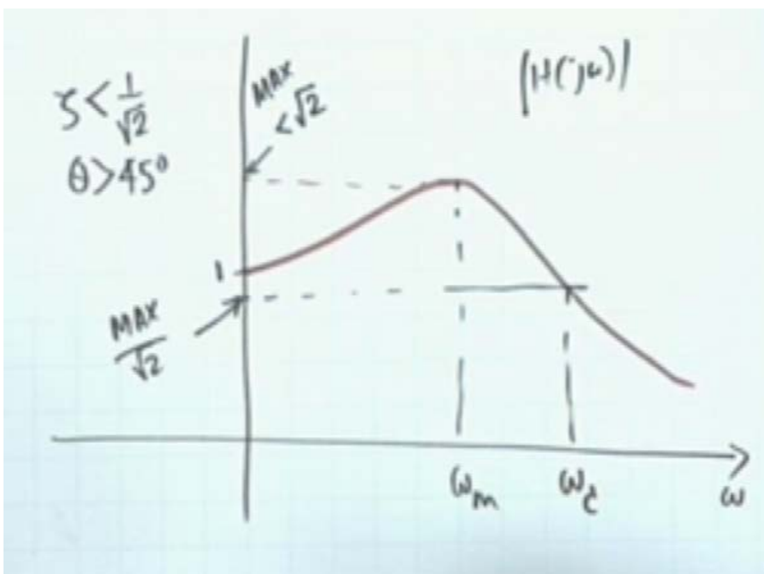


If zeta is greater than or equal to 1 by root 2, that is theta is less than 45, then there is no peaking and the response, the magnitude response versus omega H of j omega magnitude would be something like this. What is the d c response, the response at 0 frequency? It is? You remember the transfer function is s square plus twice zeta omega n s plus omega n square. So it is equal to 0. It is equal to 1. Is not it right?

So it starts from 1 and then if the damping is greater, damping coefficient is greater than or equal to 1 by root 2, no peaking occurs and therefore, it must be a low pass response line. Whether zeta equals to 1 by root 2 or greater than 1 by root 2, it should always be like this and therefore all that one can define here is, if frequency omega sub c at which the response is 1 by root 2 or 70.7 percent down and as you know omega c is called the bandwidth of this low pass filter.

It becomes a low pass filter and this case is hardly of any interest. Nevertheless, it is of interest to find out what omega c is. Omega c is defined as the 3 db cutoff, 3 decibels cutoff. That is, the frequency at which the response is 70.7 percent of the maxim response and the maximum occurs at these peaks.

(Refer Slide Time: 07:19)



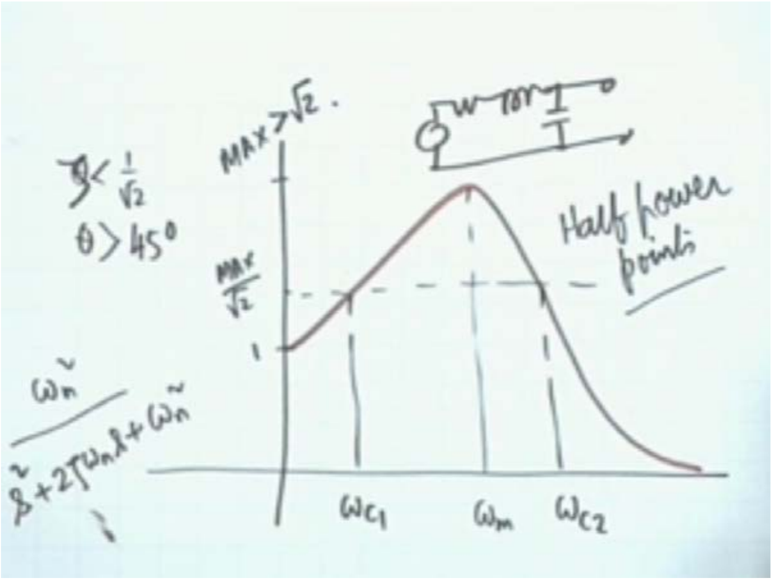
On the other hand, if zeta is less than 1 by root 2, that is, theta is greater than 45, there shall be a peaking and suppose the peaking is like this where the maximum is less than 1 by root 2, this is less than root 2. The maxim occurs at omega m, this is omega but the maxim value is less than root 2. Then obviously, 70.7 percent down from this value shall have, only shall occur only at one frequency.

If this is less than 1 by root 2, then 1 by root 2 times, this will be somewhere here, max. Let us call it max. Max less than 1 by root 2 and this is max by root 2, shall occur only at 1 frequency and this is omega c. So there is still one cutoff frequency. Only one cut off frequency because the d c value is greater than max by root 2. One might argue if I extend on the negative frequencies will it come down?

Student: No

Sir: No, the magnitude response is even and therefore, the magnitude response will be like this. It does not come down. It is even function, magnitude H of j omega. It is the same for omega positive and omega negative.

(Refer Slide Time: 09:19)



Now suppose, theta is less than 1 by root 2, that is, theta, I am sorry, zeta is less than 1 by root 2 and theta is greater than 45 in such a manner that the max is greater than root 2, then my magnitude function shall vary like this. It starts from 1, goes to max at omega equal to omega m and since maximum is greater than root 2, 1 by root 2 of the maximum shall now occur at two frequencies. Let us call them omega c 1 and omega c 2. These are the frequencies at which the magnitude response is 3 decibels down from the maximum value.

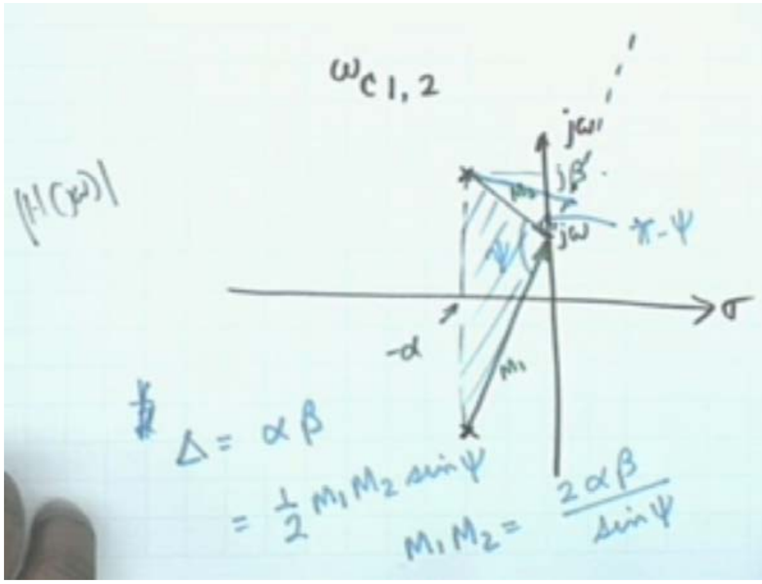
The maximum value is taken as 0 db and these will correspond to minus 3 db or in terms of power, if this voltage appears across a 1 ohm resistor, in terms of power, the power shall be half at these two frequencies as compared to the power at the maximum frequency. So these all are also called half power points, as you know and it is of interest to find out what these half power points are and one of the ways that this can be done is through geometrical construction. Yes?

Student: Excuse me sir, it starts at 1.

Sir: It always starts at 1 because the transfer function is omega n square divided by s square plus twice zeta omega n s plus omega n square and therefore, at s equal to 0, that is, d c, the value is 1 omega n square by omega n square. It should also be obvious from the circuit, if you draw the circuit, we have a resistance and inductance and a capacitance. This is the input, this is the output. So if it d c, the capacitor x is opened, the inductor s is short and therefore, all the voltages appears here. So it always starts with 1.

It would not be so, had we taken the voltage across the inductor. It would have started from origin, at 0, is that clear? If we had taken the voltage across the resistor, then also it would have started from 0. It is because, you are taking across the capacitor, this peculiar situation arises and if the voltage is taken from the inductor or the resistor, there shall occur, whenever there is a peak there, shall occur two cutoff frequencies. But here, two cut off frequencies arise only when the maximum is greater than root 2 not otherwise. If the maximum is less than root 2, there is only one cut off. It is of interest to find out why were these cut off frequencies occurring and this can be done by a graphical construction like this.

(Refer Slide Time: 12:50)



So our aim now is to find the values of omega c 1 and 2. There are two; if there are two cut off frequencies, we want to find both of them. For these, we consider this pole zero diagram again and let us say, these are the poles, somewhere here. These are the poles and this is minus alpha and this is j beta. Let us consider an arbitrary frequency j omega, let us say. We want to find out the magnitude at this frequency, that is, we want to find out H of j omega magnitude. Then by the graphical construction, what you do is, we construct a vector from the pole to j omega and a vector from this pole to j omega. Let us call this as M 1 and M 2.

For reasons we will make clear, let us extend M 1. Use a different color and let us call this angle as psi, the angle between the 2 vectors, as psi, just a magnitude. We do not care about these signs at the moment. Then the area of this triangle, the shaded triangle, obviously, is half. This is 2 beta and the height is alpha and therefore, the area of the triangle is simply alpha beta. Is that clear?

It is a product of the real part and the imaginary part. This is 2 beta, the height is alpha, so half 2 beta multiplied by alpha. It can also be found out in terms of M 1 and M 2 and the angle psi. Now look at this construction. What you do is, we drop a perpendicular on this. We draw a perpendicular on the extension to M 1. This is the perpendicular. Then the area of this triangle is half M 1, the base multiplied by height. Height can be found out in terms of M 2, this angle is 90.

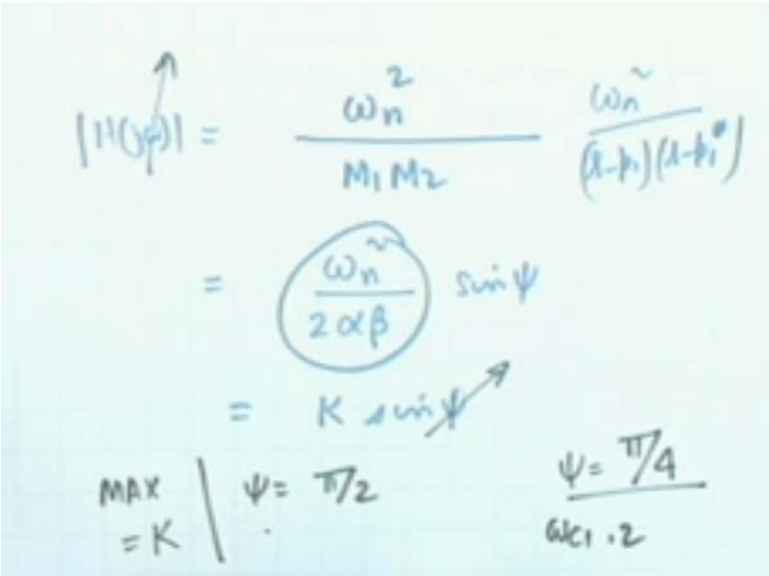
Student: 180 minus psi

Sir: 180 minus psi, pi minus psi and therefore, the height would be M 2. Sign of pi minus psi, this is precisely sine psi. So this would be half M 1 M 2.

Student: Sir, which triangle we have to find out the area of the psi?

Sir: Area of the shaded triangle, the same triangle, I am first finding out in terms of alpha and beta, next in terms of M 1 and M 2 and I am considering half of the M 1 as the base and this as the height. This height is found in terms of M 2 and this angle is phi minus psi. Therefore, the height is M 2 sign of phi minus psi which is exactly equal to sign psi. So therefore, the reason for finding this out is that M 1 M 2, therefore, is equal to 2 alpha beta divided by sine psi. M 1 M 2 equal to 2 alpha beta divided by sine psi.

(Refer Slide Time: 16:39)



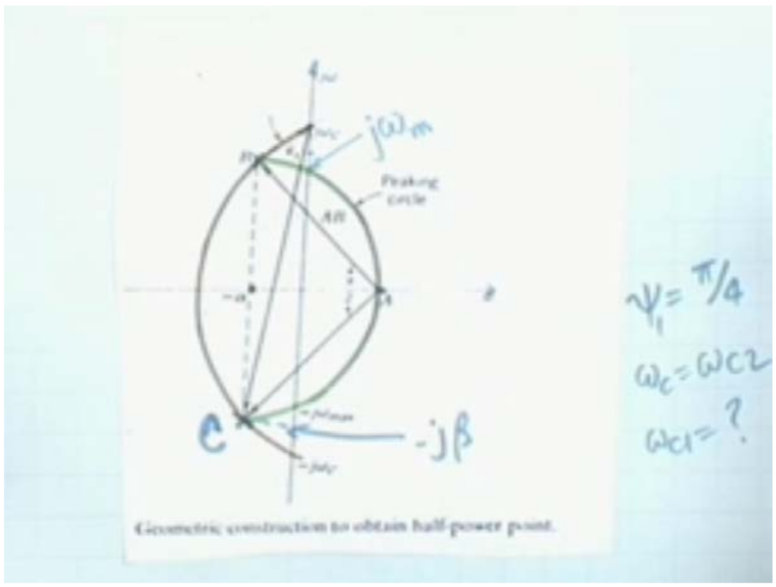
You also know that the magnitude at any point, at any frequency, is given by omega n squared. In the numerator and in the denominator, we shall have the magnitudes of the 2 vectors drawn from the poles to the frequency, that is, M 1 M 2. The magnitude at any frequency is given by, is this clear? What was our function? Omega n squared divided by s minus p 1 s minus p 1 star. If you call this p 1 and this as p 1 star, then the magnitude at any frequency is simply the product of these 0 vectors divided by the product of the pole vectors.

There are no zeros and therefore, it is only the product of the pole vectors j omega minus p 1 j omega minus p 1 star and since M 1 M 2 are found out, we see that this is equal to omega n squared divided by 2 alpha beta multiplied by sine psi. Then you call this k sin psi. This is a constant as frequency varies, as omega varies, k remains a constant; it is only psi which varies. So if the variation of omega, I can find out the values of psi, then I know the variation of magnitude with frequency. But interesting thing is that, instead of 2 pole vectors, both of which are functions of frequency M 1 and M 2, if I increase omega, M 1 will increase, M 2 will decrease and so on.

Instead of two pole vectors, I have reduced the magnitude variation investigation to that of investigation of just one variable, that is the angle psi, which is a very interesting phenomenon because you notice that the max will occur when psi is equal to pi by 2 because the maxim value of sin psi is 1 which occurs at psi is equal to pi by 2 and the maximum value is equal to when psi is equal to phi by 2 and we know where that occurs. We have to draw a peaking circle, with this as the centre and this as the radius, we draw a peaking circle. Wherever it cuts the j omega axis, is the frequency omega m. At omega m, the angle subtended between M 1 and M 2 must be equal to pi by 2.

We can also do this analytically, but we shall lose sight of the physical picture as to what is happening and geometry is much more beautiful than pages of algebra. I find this very intriguing and interesting. So all that we have to find out now is the values of omega at which the angle psi becomes equal to pi by 4. If we want to find out the half power frequencies, all that we have to find out is the frequencies omega c 1 and omega c 2 at which psi is equal to pi by 4, agreed? At this frequency, sine psi will be 1 by root 2, so the magnitude would be precisely k, the maximum value divided by root 2. Maximum value k is 1, that you know, but I have put it for generality sake. So the problem is therefore, to find out omega c 1 2 and for that, all you need to, is to make a geometrical construction to locate the point or points at which psi is equal to pi by 4 and this can be done by means of a a very elegant diagram which I will show now.

(Refer Slide Time: 21:29)



I will explain this carefully so that you can follow this. You see, this is the sigma axis, this is the j omega axis and these are the two poles. One is here and the other is here. This point, we call as A and this point we call as, no I beg your pardon, this point we call as A is already marked. So let us call it as C. Already C is also marked. No, it is not marked. So these are the 2 poles. B and C are the 2 poles, then the distance from the origin is alpha. This point is minus alpha and B has the y co-ordinate or j mega co- ordinate as j beta. This is minus j beta.

Then to draw the peaking circle, what we do is, we take this point as the centre and beta as the radius and draw the peaking circle. The green circle is the peaking circle and naturally, this is equal to j omega m, the frequency of the maximum and you know that if beta is less than alpha, then there will be no peaking circle and no peaking. If beta is equal to alpha, then the peaking circle at the point A shall be at the origin and the j omega axis shall be tangent and all that that you know.

Now what we do is the following. If I draw the peaking circle and I join and this peaking circle intersects the sigma axis at the point A, I join A C and A B, then the angle subtended would be, the angle subtended by a diameter on the circumference and therefore, this will be 90. Is this point clear? This angle would be 90.

What to do now is to take as A as the centre and AB as the radius. AB or AC, which is the same, the distance will be the same and draw this red circle, not red, the orange circle. Let the orange circle intersect j omega axis at j omega c as you can see from the subscript, this is the half power point. The reason is that this chord, the chord BC subtends an angle psi 1 on the circumference and an angle pi by 2 at the centre and therefore, psi 1 must be equal to pi by 4. Is this clear? A very elegant construction, I repeat.

Student: Is it not the same as (..)

Sir: Is not the same as?

Student: The centre (..)

Sir: Oh yes. This is the centre of the orange circle. The chord AB belongs to both the peaking circle. For the peaking circle, it is the diameter, whereas, it is less than the diameter for the orange circle and A is the centre of the orange circle. So the angle subtended at the centre is pi by 2. The angle subtended on the circumference must be pi by 4, half of that, and therefore, j omega c, that this curve that gives you the half power point. Unfortunately, it is only 1.

The other half power point cannot be obtained. At least you do not know how to obtain it geometrically. If you can obtain it, that will be wonderful thing. I will ask professor to put it in his text book with your name mentioned there if you can find out. Unfortunately, this omega c, this is the lower half power point or upper half power point.

Student: Upper half power point

Sir: Upper one, obviously, because it is greater than omega m, is on the right side and therefore, omega c gives you omega c 2. The question now is how to find the lower half power point, omega c 1? Omega c 1, as we know, may or may not exist. Omega c 1 will not exist if the maximum is less than how much?

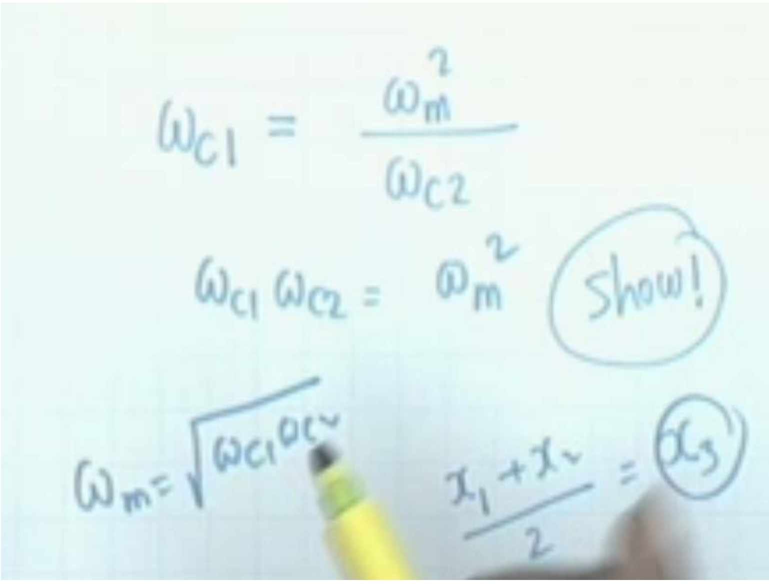
Student: Root 2

Sir: Root 2, if the dc value is 1. If the maximum is less than root 2, now omega c 1 does not exist and that is one of the reasons why a geometrical construction like this does not disclose omega c 1, it only discloses omega c 2. It is a fuzzy logic. It is not very clear as to why we cannot find out and that is why I throw you a challenge.

Student: Excuse me Sir?

Sir: Omega c 1 may or may not exist. If it exits, then it can be shown and that is not too difficult. I also leave that as a challenge.

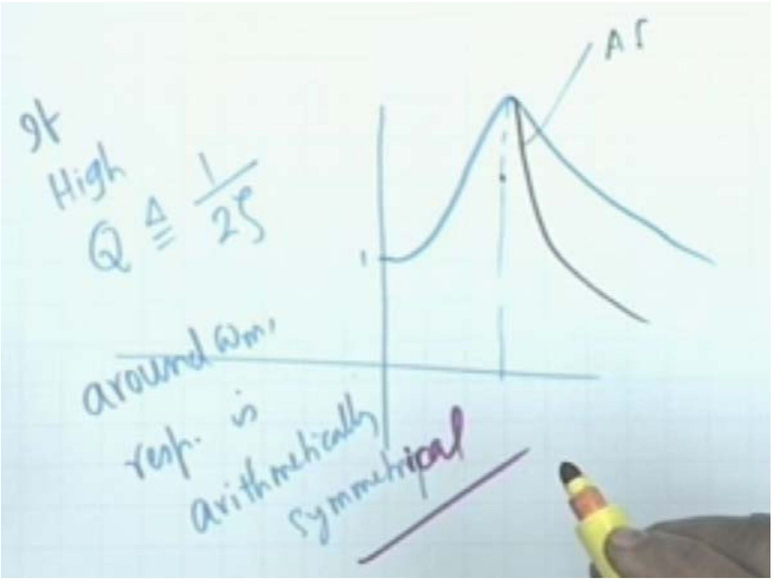
(Refer Slide Time: 26:51)



If it exists, omega c 1 exists then one can show that omega c 1 is equal to omega m squared divided to by omega c 2, which is obviously lower than omega m, agreed? That is, omega c 1, and omega c 2, a product is equal to omega m squared. If one is greater than omega m, obviously, other must be less than omega m. Now let me put this as a problem to you. Now this relationship, that is, the product equal to omega m squared is described in mathematical terms in geometrical terms as in terms of a symmetry.

You see, if I have, let us say, x 1 plus x 2 divided by 2 equals to some x 3, then I say x 3 is the arithmetic mean of x 1 and x 2. On the other hand, if I have omega m as square root of omega c 1 and omega c 2, then I say omega m is the geometric mean. So omega m, the maximum is geometrically symmetrical, the frequency of maxim is geometrically symmetrical about the 2 half power points. In fact, it can be shown that the complete response is also geometrically symmetrical. In other words, what it means is the following.

(Refer Slide Time: 28:29)



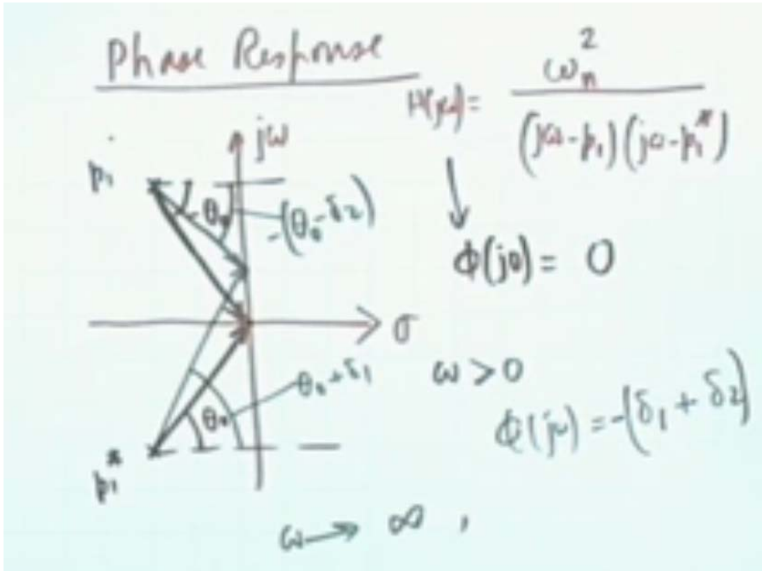
It starts from 1. On the other side, well, it does not have the same kind of symmetry as this. Same kind of symmetry would have meant that the curve falls like this. This is arithmetic symmetry. On the other hand, geometric symmetry will imply that the curve falls like this. It does not fall at the same rate at equal frequencies, deviations from the maxim. Let me make a a prophetic statement that this state, which also you can prove that near omega n, one thing is that the response here does not show asymmetric symmetry, it shows geometric symmetry, this also can be shown, mathematically. Second thing that I make is that, the statement that I make is that, if the Q is high, what does this mean?

Your definition is Q, figure of merit is 1 by 2 zeta. That means, if zeta is small, what does zeta small means that the peaking is sharper and sharper. That is, you have a high Q case, in other words, you have a sharp peak. If this is so, if the Q is high, then one can show that around omega m, that is, for frequencies very close to omega m, the curve shows arithmetic symmetry. Response is arithmetically symmetrical. This request some pages of algebra and it is most convenient for me to leave that to you. So I have made several statements and I have asked you to investigate.

One, is there a possibility to find out omega c 1? Geometrically, can it be done and I promise that if you find a way, I will get it included in the next edition of course book. That responsibility, I take. Number 2, I ask you to show that the product of omega c 1 and omega c 2 is omega m square. Then the third point that I ask you to show is, that in general, the frequency response curve is geometrically symmetrical for a situation like this and the fourth thing that I have shown is, I have told you that, if it is a high Q situation, then around omega m response is arithmetically symmetrical and I can assure you, none of them will be trivial problems.

I did, at one point of time, I thought I might do it in the class, but I decided finally that it is more convenient for me to leave it to you.

(Refer Slide Time: 32:06)



Then, let us look at the phase response. We have only looked at the magnitude response so far. Now, the phase response, if I recall the frequency, the transfer function H of j omega is j omega minus p 1 j omega minus p 1 prime, p 1 star. This is the frequency response for H equal to j omega. Now let me look at the phase response more systematically. I can of course substitute p 1 and p 1 star and do it analytically. I can find an expression for the frequency, for the phase response but if you go from mathematics, from the mathematical expression the chances of making a mistake is much greater in the phase response than in magnitude response because in the phase response there are ambiguities.

0 and 2 pi cannot be distinguished from the tangent of the function. Tangent is the same tangent. Why tangent? All trigonometric functions are the same at 0 theta is equal to 0 or theta is equal 2 pi. A change by plus pi cannot be distinguished from a change through minus pi and therefore, a picture in the case of phase response is worth more than one thousand words, not just one thousand. Let us take the plot. This is p 1 and this is p 1 star in terms of alpha and beta all we have done so far.

Now, if I want to find out the phase at 0, frequency 0, frequency is here. So draw the two vectors. Let me use a different color. Draw the two vectors, no, this is not I wanted to use. Draw the two vectors and find their angles. Let this angle be called theta 0, then obviously, this angle is minus theta 0 and the sum of the 2 angles is equal to 0 and therefore, phi j 0 would be equal to 0, agreed? Is that okay? Minus 0 is same thing as you see. Actually, these are angles of the denominator factors. So when they go to the numerator, to make up the phase of H of j omega you have to take with it a negative sign. Minus 0 is same as the 0, yes.

Student: Sir, here we are considering always the phase to be in the range minus pi to pi or 0 to 2 pi.

Sir: No. I do not understand this question.

Student: Sir, if it were on the other side?

Sir: You can take it to the 2 pi, is that what you mean? This angle?

Student: But there should be a way, a convenient rule that will take from minus pi to 0 to pi

Sir: Oh, whatever the rule is, there is no ambiguity here. Whatever the rule is, we will clarify the rule when we come to a particular situation. But here, you see, one of the possibilities was, take this angle, then the sum of the two angles is 2 pi and therefore the angle would be minus 2 pi, agreed? Now we are not going beyond 2 pi in any case, not beyond 2 pi, between 0 and 2 pi. It could be minus pi to plus pi or 0 to 2 pi; it does not matter. If minus pi comes near, no other way. We cannot convert it to any other phase. So this phase, this phase is 0 or minus 2 pi. As the case may be, if you want to extend beyond 2 pi, you are most welcome. It does not make a difference.

Now, if I take a frequency slightly away from origin, let us say here, then what happens to this angle? This angle increases. This becomes, let us say theta 0 plus delta 1. On the other hand, this angle decreases. Let us say, this angle becomes theta 0 minus delta 2 and the sum of them, that is, as omega is greater than 0, the sum of them, the phase, now becomes delta 1 plus delta 2, the sum of them but with a negative sign minus delta 1 plus delta 2. When it is here, when omega is close to j beta, when omega is close to beta, then this angle becomes 0 and this angle becomes close to pi by 2. Close to pi by 2 not quite pi by 2.

Students: (..)

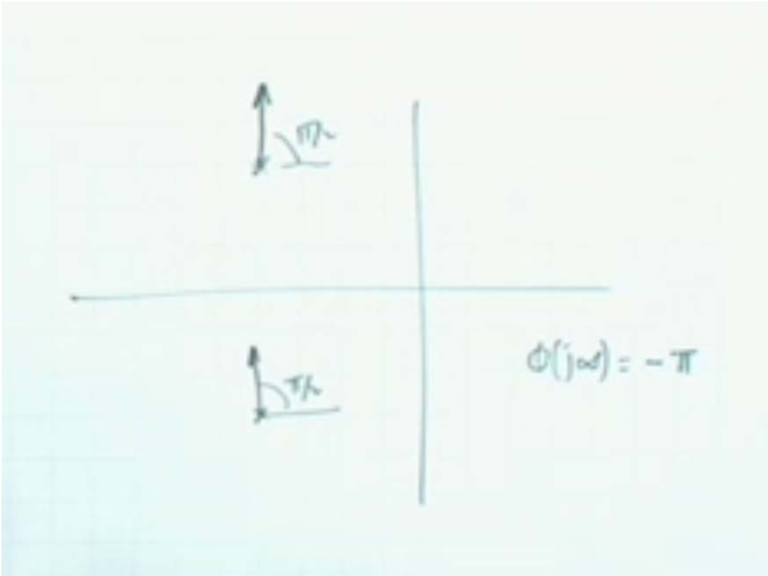
Sir: This is delta 1 minus, no, there is a minus here. So minus and minus makes plus. This plus this, yes, what did you say.

Student: Sir, you are saying that it will be become close to pi by 2. Sir, it will be depending upon alpha and beta.

Sir: Correct. So what I am saying is, if beta is much larger than alpha high Q situation, it will be close to pi by 2 not quite pi by 2. It will be less than pi by 2, minus pi by 2, not plus. The angle is still negative. Why because these angles are in the denominator, you have to take it to the numerator, so the sign is negative. What happens, next point I have to consider is, but at the same time you notice that as soon as we cross the peaking, that is, omega m, we will come to that later. We will come to that later.

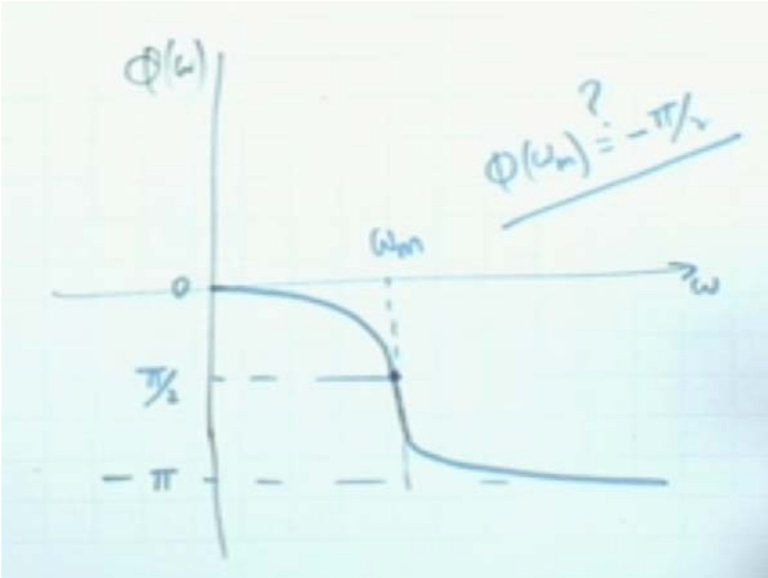
Now, the other frequency that you have to consider is infinity. As omega goes to infinity, the phase of the denominator would be 90. So the total phase will be minus 180. Well, this can also be shown graphically.

(Refer Slide Time: 38:50)



If you have 2 poles like this, when omega goes to infinity, the vector becomes parallel to the j omega axis. This vector also becomes parallel to the j omega axis. So this angle is pi by 2 and this angle is pi by 2 and therefore, phi j infinity shall be equal to minus pi, agreed, minus pi. So it starts from 0, remains negative and goes to minus pi. This is the confusion. If you had taken only the expression, you would have had difficulty interpreting whether it is minus pi or plus pi because the tangent or any other trigonometric function is the same and therefore, the phase frequency response, if I plot phi omega versus omega, the phase response would be like this.

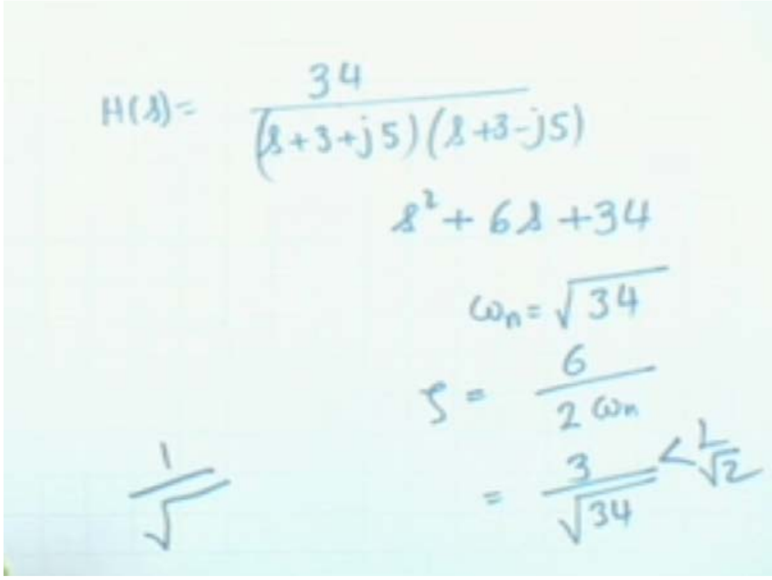
(Refer Slide Time: 39:39)



This is minus pi, this is 0 and pi by 2 is somewhere here and it can be shown that the phase at omega equal to omega m is equal to pi by 2. I leave it a question mark, minus pi by 2, phase at omega equal to omega m. Is that okay but definitely, I am leaving it to the question mark, definitely the rate of change of phase is the highest at omega equal to omega m. That is, if you take slope of this curve, the slope is the highest at this point. The rate of change of slope is the highest at this point. That is as regards the phase response is concerned.

Let us, in the remaining period of time, let us take an example to illustrate all these calculations.

(Refer Slide Time: 41:18)



The example that we take is a single tuned circuit, in which, the transfer function is of the form 34 divided by s plus 3 plus j 5 s plus 3 minus j 5. Actually it is given, as can you tell me, what the polynomial form would be, s square plus 6 s.

Student: No

Sir: Okay, you settle this controversy and let me know the in the next class. That is why I left the question mark. Here, come back here. This is how much? 9 plus 25, is it 34, 25 plus? Yes, it has to be 34. There is no other way because the numerator is 34, omega n square is 34. You can also find out from here, what is omega n. Omega n is square root of 34 and you can find zeta. Zeta is, this factor must be 2 zeta omega m. So zeta would be equal to 6 divided by 2 omega n. So this is equal to 3 divided by square root of 34. Is it less than 1 by root 2 or greater than 1 by root 2?

Student: Less than

Sir: Less than, because it is approximately 1 by square root of 11.

Sir: Pardon me. It is approximately 1 by square root of?

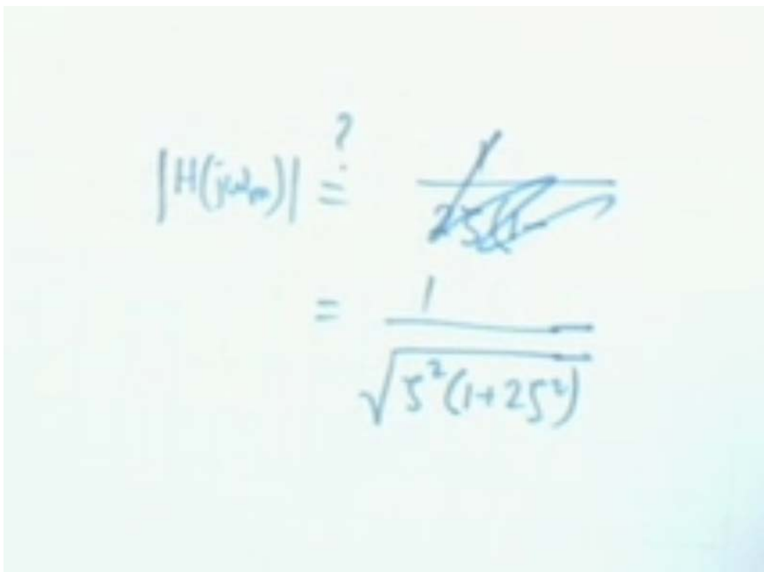
Student: 1 by (..)

Sir: Whatever it is, it is 1 less than 1 by root 2. So there shall be peaking. Will there be 2 peaks?

Student: No

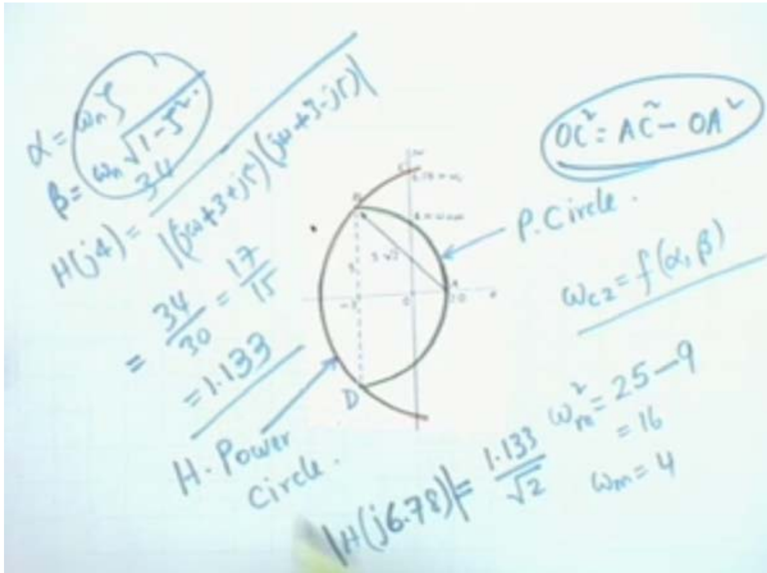
Sir: We do not know yet, how do you know? How do you know so definitely, whether there will be 2 peaks or not? The maxim value, I have not found out yet.

(Refer Slide Time: 43:19)



May I point out once again with a question mark sign the maximum value in terms of zeta would be 2 zeta multiplied by 1 minus. Well, I found this out. Let me give you the result. Maximum value, no, I beg your pardon, is equal to 1 by square root of zeta square 1 plus 2 zeta squared. So unless you substitute here, you would not know whether there are 2 cut off frequencies or 1 cut off frequency. It cannot be said by inspection. So there may be 2 cut off frequencies, there may be 1. Let us see what is true of this particular situation and the easiest thing to do for solving this example is to take help of the two circles.

(Refer Slide Time: 44:23)



One is the peaking circle, this is the peaking circle, the green one and the other one in absence of any other name, can you give it a name?

Students: (..)

Sir: Half power circle we call it, because this circle is used to define the half power points. So we will call this half power circle and you notice the locations of the poles? Minus 3 plus j 5, minus 3 minus j 5, this is the peaking circle and if the peaking circle intersects the j omega axis at j 4, well, this can be found out very easily. Omega m squared would be equal to beta squared which is 25 minus alpha squared, which is 9. So this is equal to 16. Therefore, omega m is equal to 4.

It can also be found out by geometrical construction from here, any manner you like. Then what you do is, you draw the peaking circle, let the peaking circle intersect the sigma axis at A, naturally, the co-ordinate of A, because this distance is 5, this is 3 and therefore, this distance must be 2. Is that clear? This is 3, so OA must be equal to 2. That is, the co-ordinate of A, the point A would be 2 0, 2 j 0. It is on the real axis.

Then if A is the centre and AB as the radius, now what is the distance AB? 5 root 2. This is 5, this is also 5 and this is the hypotenuse of a right angled triangle and therefore, this must be 5 root 2. With 5 root 2 as the radius, you draw the peaking circle. This is BC and we have called this, no, we cannot call this anymore C. Let us call this D. DBC is the half power circle, draw the half power circle and you find geometrically, that this, it intersects this at 6 point 7 8, j 6 point 7 8. Well, if you think you cannot read correctly, this distance, on a diagram, you can also find out analytically.

After all, you have to do is to find out this distance OC and you notice that OC squared is the hypotenuse square, that is, AC squared minus OA squared. You know OA. Do you know AC? AC, of course, you know this is the radius of the half power circle and therefore, this is 5 root 2 and therefore, you can find out OC and the results are as follows. Now the results I have already told you, 6 point 7 8. Then to find out the maximum H of j 4, well, there are various things, you can find out in terms zeta or you can find out directly from the equation, that is, it is 34 divided by magnitude of j omega minus, I am sorry, what is that? s minus 3 plus j 5, so j omega minus 3 plus j 5.

Student: Plus 3

Sir: j omega plus 3 plus j 5 j omega minus, no, plus 3 minus j 5. Magnitude of this and substitute omega equal to 4, because that is the maximum and the final result is 34 by 30, where 17 by 15 is the final result less than

Student: (..)

Sir: This is less than 1 point 1 3 3, less than root 2 and therefore, there shall be only 1 half power frequency and now, if one asks you, quickly if one asks you what is the value at 6 point 7 8,

Sir: What is the magnitude at 6 point 7 8?

Student: 1 by root 2

Sir: So it is simply 1 point 1 3 3 divided by root 2 and the complete problem is solved. There is nothing that is unknown. Now through this example, I also demonstrate that you can obtain an expression for omega c, the upper cut off frequency. You can obtain the expression for omega c in terms if alpha and beta or zeta and omega n, which I have not done in the theory, but you can do that. That problem also, I leave it, express omega c 2 as a function of alpha and beta is very simple. As I did here, all you have to do is utilize these expressions.

You see, OC squared is AC squared. AC squared is obviously, beta times 2 beta square and OA squared is beta minus alpha whole squared and therefore, you can find out in terms of beta and alpha and substitute alpha as equals to omega n times zeta and beta as equal to omega n times square root of 1 minus zeta square to obtain the expression in terms omega n and zeta, it may be helpful to have this expression also ready. We stop here today, we continue on the next Tuesday.