Circuit Theory

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Lecture - 1
Review of Signals and Systems

Welcome to this course E E 202 circuit theory 4 credits 310. The objective of the course is to familiarize you with analysis, characterization and elementary synthesis of networks with l l f p b elements. The meaning of these terms, meaning of the objectives, characterized by a single letter shall be made obvious after a little while.

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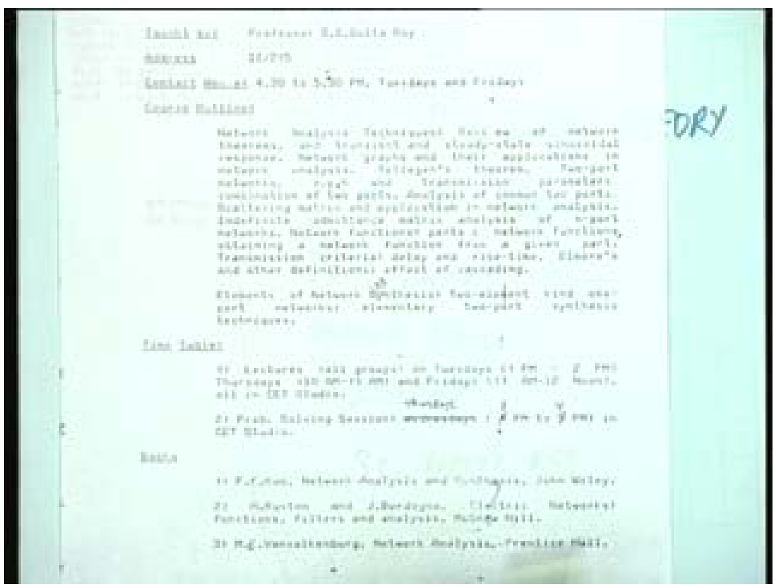

The course is being taught by S.C. Dutta Roy and the contents of the course, the course I have planned includes network analysis techniques review of network theorems and transient and steady state sinusoidal response, network graphs and their applications in network analysis. We will play it rather at a low key, these topics. Tellegen's theorem, we will do these; two port networks, ZYH and transmission parameters shall be done in detail, combination of two ports, then analysis of common two ports. We will discuss scattering matrix characterization and application in network analysis. If time permits we will do a little bit of indefinite admittance matrix analysis, then we shall do parts of network functions and how to obtain a network function from a given part. We shall discuss transmission criteria, definitions of delay and rise time and the various definitions proposed by Elmore and others. We shall also discuss effects of cascading then we shall look at some elementary network synthesis; basically one port networks and elementary two port techniques also. The books that shall be prescribe for this course, number 1, is F.F. Kuo. This is available in cheap edition.

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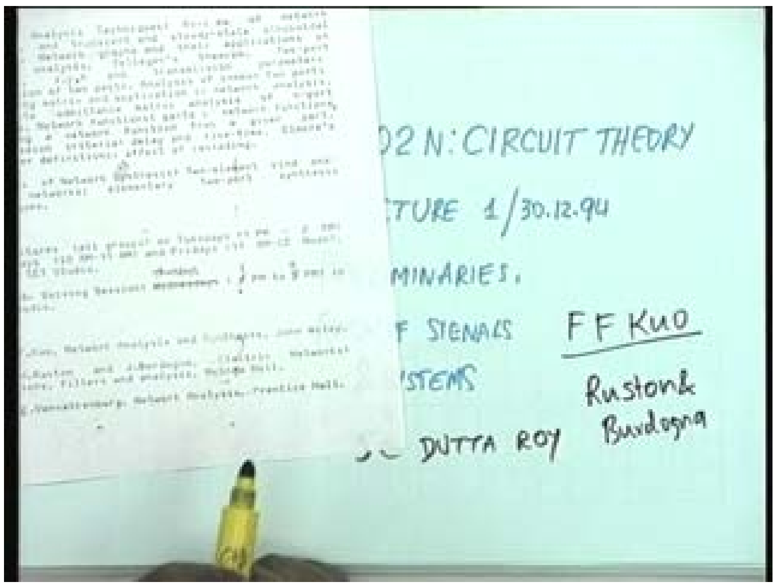

As I have told you F.F. Kuo- Network Analysis and Synthesis, John Wiley. It is a very old book but it is one of the best still available. We will take some material from a book by Ruston and Burdogna. This is available in the library and very small parts will be taken from there. We will we will also have occasions to refer to the third book M. E. Van Valkenburg- Network Analysis, published by Prentice Hall. This is also available in an inexpensive edition and I shall sometimes refer to this particularly, in the problem solving sessions. In the first lecture of this course being held today 30th December, we will discuss some of the preliminaries and we shall review some of the concepts of signals and systems. The first thing that we discuss is the meaning of the word circuit and very often as you see from the name of the books, they talk about networks. So the first

thing we discuss is the difference between a circuit and a network. When is an electrical, a combination of electrical elements is called a circuit and when it is called a network.
A circuit by definition is a closed path. That is, you start from somewhere and come back to the same point. If there exists a path like this, then it is called a circuit. For example, in the Delhi transportation network, bus transportation network, the Mudrika service on the ring road is a circuit because it starts from a point it goes round and comes back to the same point in one trip. On the other hand, the bus that goes from Hauz Khas terminus to Central Secretariat is not a circuit. A single trip is not a circuit, two trips of course is a circuit but it is not a closed path. Therefore a network is a more general term; a circuit is a very special term. A circuit is a closed path and a network may consist of closed as well as open paths. For example, if you have a structure like this, then this is a circuit.

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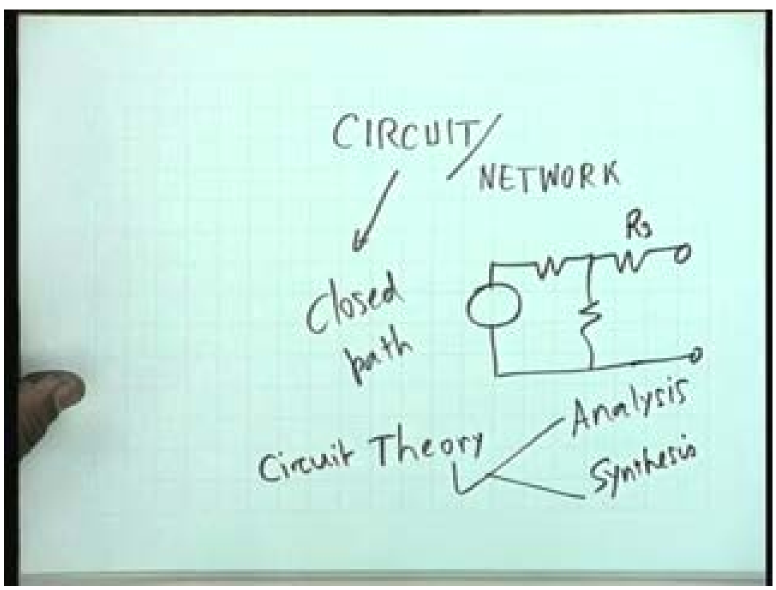

This path is a circuit, it is a closed path on other hand this element let us say R 3 is not part of a circuit because this path is open and therefore the complete structure is not a circuit. The complete structure is a network, a circuit is a part of it and therefore networks are much broader classes of electrical systems than a circuit. All circuits are networks; all networks are not necessarily circuits. Is the distension clear? Our course is concerned with circuit theory; it should have been more properly called network theory because we shall consider networks which are not necessarily circuits and I have already given the example of a transportation network where there are circuits, there are networks and there are non circuit networks also.

A tree, basically, if you considered this as a network, a tree has no circuit except when it has adventitious roots. Have you seen banyan trees? They do have adventitious roots which grow into the ground and may provide a path to the original roots, the genuine roots and then it may be a circuit. So you understand the difference between a circuit and a network, and our course should have been called network theory. But for historical reasons the name circuit theory has been adopted and this is what continues. Now in this course, circuit theory, we shall basically deal with two aspects of circuit theory. Circuit theory contains basically two aspects: one is analysis and the other is synthesis and we shall discuss these two terms in a little more detail now.

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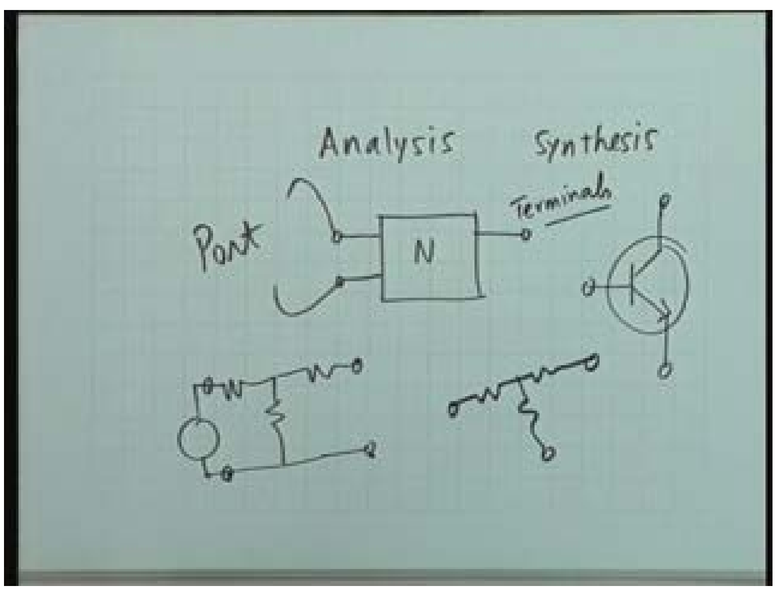

Analysis of circuits and synthesis, both have to do with 3 components that is, in order that a circuit now, I shall use the word circuit and network interchangeably; you should understand that in the context. Let us say we have a network, we represented by a black box; that is, it might contain many other things which we are not showing specifically, and in order to be useful; a network must have what are known as terminals brought out of it. For example, if you have a transistor; it has three terminals. This is a network or a device, it has three terminals. The resistive network that we showed earlier has terminals like this. These two can also be terminals. Terminals are wires of 0 resistance brought out from the black box, so as to be able to connect a source of energy or a load in which power is to be dissipated. So these are terminals and terminals by themselves may not mean anything, because for connecting a load or a source you require at least two terminals. You may require more than two also. For example, the transistor is a three terminal component and therefore you might require three terminals to be able to connect to a network, but minimum is 2.

A single terminal serves no purpose unless it is connected to ground; that is the 0 potential point. So two terminals to which can be docked at a source or a load is called a port and I intestinally use the word docked, it is like a ship docking at a port, and the terminology is basically borrowed from the kind of terminology that is; a port consists of two terminals to which an energy source can be connected or energy can be taken out, energy can be delivered to a load. So port and terminals, the number of terminals sometimes is used to classify a network. For example, this is a four terminal network 1, 2, 3, 4 and if you look at it closely, the potential of this terminal is the same as the potential of this terminal, and therefore it can also looked upon as a 3 terminal network. It is a 3 terminal network. Similarly there can be more than 3 terminals. There can be more than 4 terminals and in general we shall talk of N-terminal networks. N-terminal; the number of terminals can be large it has nothing to do with the other characteristics of the networks.

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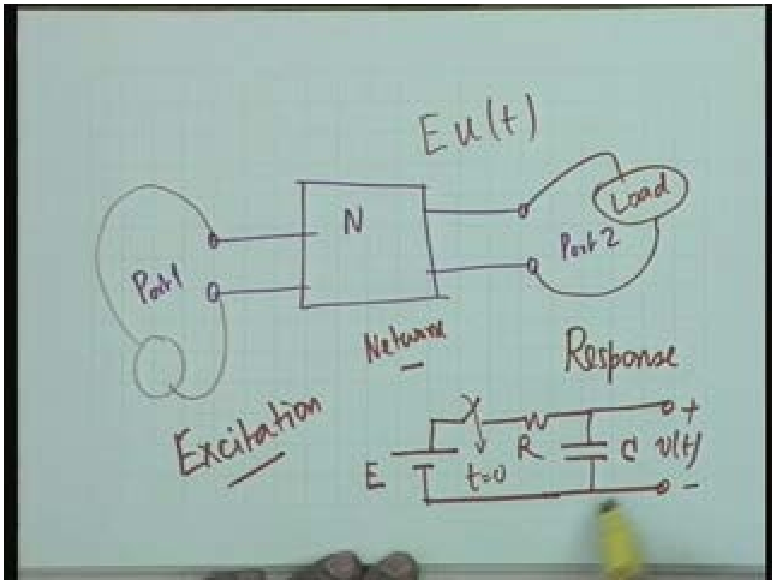

So in order that a network be useful, let us consider the simplest network, which is a 4 terminal and 2 ports. This is port 1 and this is port 2. The simplest possible situation is if a network is to be useful it must have an excitation, it must have an energy source connected. Let us say there is an energy source here, it can be a voltage source or a current source, it does not matter, it can be a voltage source or a current source and there must be at least another port at which you take out the power or it delivers energy, so there can be a load here; the voltage or current at the load, the voltage across the load or the current in the load, this constitutes the response of the network N.

So in order that a network is useful, there are 3 components: excitation, then network and finally response. There are 3 components, which are essential in order that a network or a circuit be useful and the problem of analysis is that if the excitation and the network are given to find out the response. For example, the typical problem could be like this; a battery of voltage E is switched to a resistance capacitance network, RC. The excitation then, this is switched on at t equal to 0, the excitation therefore is E times u of t. u of t is the unit step, and this is the excitation. The network consists of R and C and the response can be the voltage v of t across the output port. So E of u of t is the excitation, RC is the network end and v of t the output, that is the response. This is the problem of analysis; given E and R and C to find out V of t. This is the problem of analysis.

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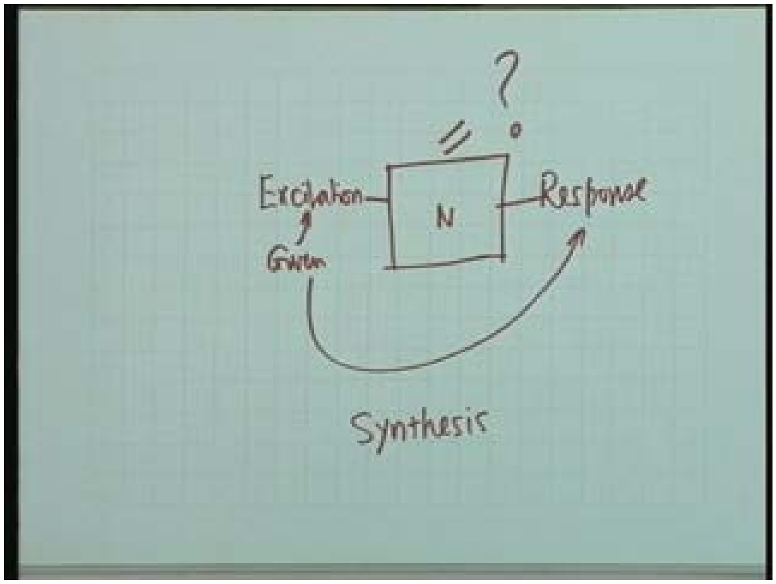

On the other hand, the problem of synthesis is that the excitation and the response are given. These are the given quantities, what you have to find out is the network. You have to find out the network. This is the problem of synthesis, and it should be very clear to you that the problem of synthesis is tougher than the problem of analysis. The problem of synthesis can be either in terms of excitation and response or some relationship between them. For example, the ratio of the Laplace transform of the response to the Laplace of the excitation. This is what may be given, and then you will have to find the network. So it is either the excitation and response or some relationship between them which is given, and then you have to find the network; if problem of finding a network or the problem of synthesis is much tougher than the problem of analysis.

It is true, that however difficult the problem of analysis may be, you will always be able to find the solution. For example, if it is a very large network let us say power system, power system network consisting of several grids, sub stations, consumers and so on and so forth, very large network; it may be time consuming for you to find the solution you may have to use a computer. You have to use a computer, Computer aided network analysis. But whatever be the constitution of the network, you shall always be able to find the response. It may take years for you to find the response, it may be a difficult situation, you may have to be careful but a solution exists.

On the other hand for a network, for a synthesis problem, solution may or may not exist. So in the case of the synthesis problem, the first thing to be investigated is; does a solution exist? If a solution does not exist, then there is no point in continuing the investigation. So this is called the realizability test. The synthesis problem starts with test for realizability. Is the network realizable? If it passes the realizability test then you look for a network which satisfies the given relationship between excitation and response, and that problem, the solution to that problem is the solution to a synthesis problem.

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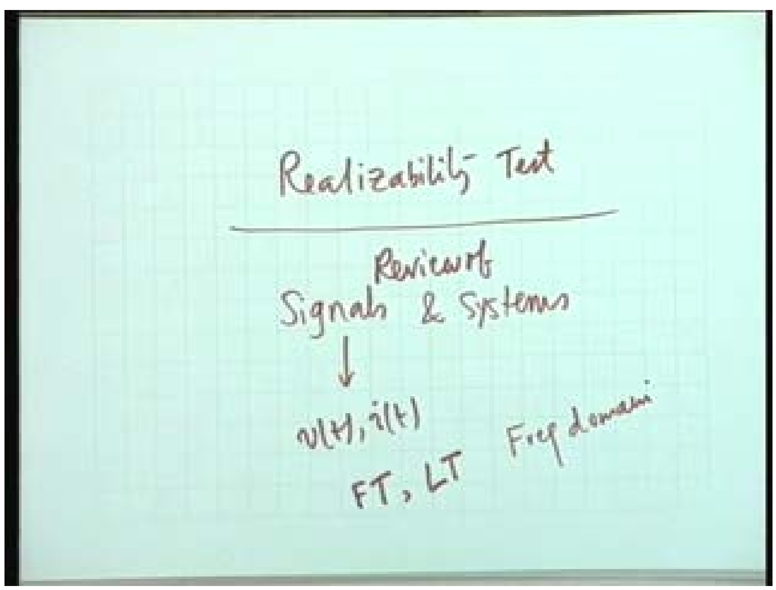

The other distinction between analysis and synthesis is that the solution to an analysis problem is unique. Whether x carries out the analysis or y carries out the analysis, whether you carry out the analysis manually or with a computer, the solution shall be unique, it shall be one. Whereas to a synthesis problem, mind my words now, if one solution exists then there exists an indefinite number of solutions. If one solution exists, then there exists an indefinite number of solutions and therefore what should be the next problem then? Next problem, would be out of the multiplicity of solutions that are available, you will have to find out which one is the most economic. In engineering convenience or the best solution or optimum choice is always determined by money. How much money you have to spend on the particular solution that you adopt, and you will naturally choose the one that costs you least amount of money or one that involves the least amount of complication.

So synthesis problem has 3 phases. Analysis problem is straight forward. All that you need is KCL, KVL, and what else? There are 3 equipments that are needed, KCL, KVL and Ohm’s law. That is all that you need for solving a network analysis problem. On the other hand, for synthesis problem first you require a realizability test, then you require a realization procedure or a synthesis procedure, and finally the question of choice. In this course, in the part that will deal with synthesis, we shall be concerned with the first 2 only. That is, finding a realizability test and then finding a network. The others shall be left to you. That is, finding other possible network and finding which one is the optimum. That, of course, you will learn at the cost of your life that is, in your later life, if you work in an industry or wherever you work, whenever you encounter a synthesis problem not necessarily network, in anywhere; an algorithm or finding a software, finding a product, you will always have to make an optimum choice, and that choice is always determined by economic considerations.

Then we review some basic concepts of signals and systems in electrical engineering as you know, our signals are either voltage v of t or current i of t. Our signals are always current or voltage and a current or voltage qualifies as a signal if it is a time varying filament. A constant voltage or a constant current carries no information. It might carry power but no information. For communication through electrical signals; it is always a current, it is always a time varying current or a voltage and we can describe a signal either in the time domain, as the time varying one, or in the frequency domain, and the translation from time domain to frequency domain is done through transforms and these are the usual transforms, the Fourier transform or the Laplace transform.

In the course on circuit theory we shall have we shall depend more on Laplace transform because of its versatility than on Fourier transforms. So, we can have the description of the signal either in the time domain or in the frequency domain and most of the time we shall depend on the frequency domain description of a signal rather than time domain because it is easier to deal with the frequency domain description. This is a great boon to circuit theories that Laplace and Fourier were born and they formulated their transforms. Otherwise, life would have been absolutely miserable if you have to work solely in the time domain.

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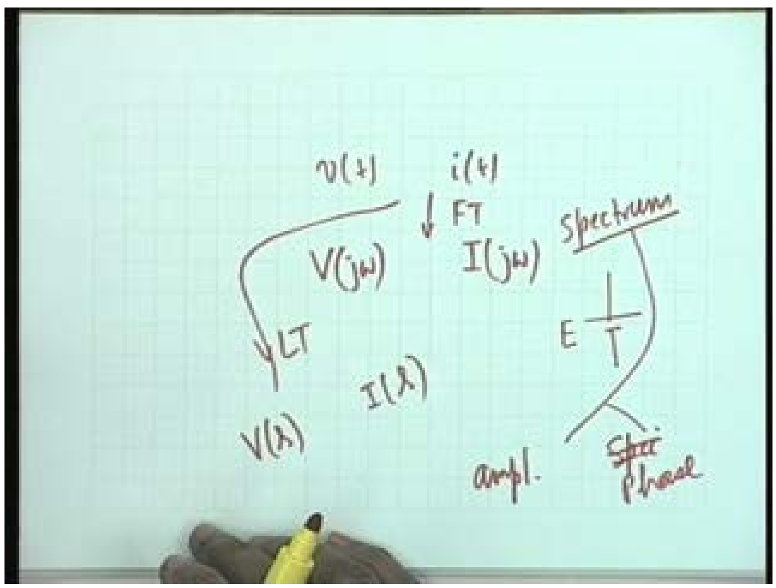

The signal v of (t) or i of (t) as I said it can be transformed to capital V of j omega or capital I of j omega. If we take Fourier transform or if we take Laplace transform then capital V of s and capital I of s; this will be our notations that is, small letter quantities shall always denote signals in the time domain and Capital letters in the frequency domain unless otherwise specified. For example, when you describe, when you call a battery as capital E, obviously it is not it is neither time domain nor frequency domain. It is a constant unless it is specified otherwise. This is what we shall use. That is, if small v of t and capital V of s occurs in the same discussion then one is the Laplace transform of the other and you know that if you take the Fourier transform, then what we are essentially finding out is the spectrum that is the frequency components of the signal. V (j) omega or capital I (j) omega is the spectrum and as you know, this the Fourier transform, similarly Laplace transform also, is a complex quantity and therefore if you wish to specify a signal in terms of its spectrum, you should specify its amplitude spectrum as well as the phase spectrum. So if you go to the frequency domain, it does not matter whether you use Laplace or Fourier, you have 2 components, namely the amplitude and the phase, whereas in the time domain no such complication. Time domain is simply the amplitude and of course the sign, it can be either positive or negative that is, a binary thing. But here, amplitude and phase both have to be paid attention too.

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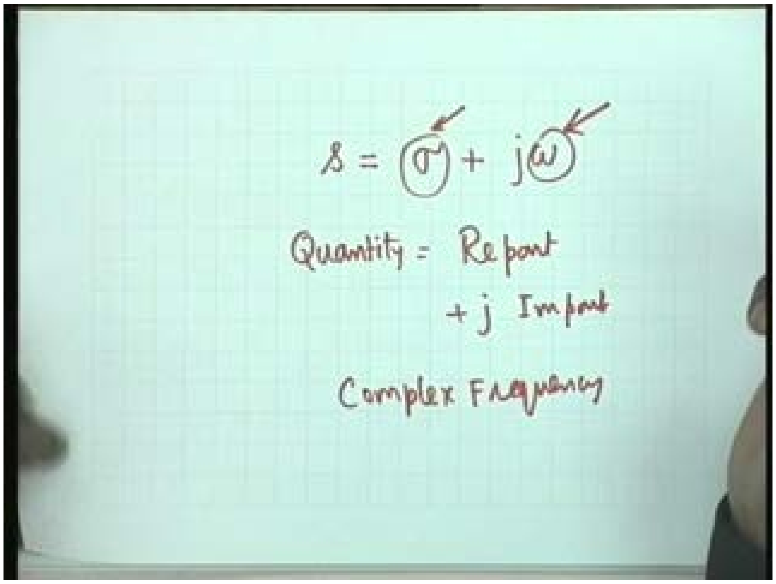

Then we talk about the Laplace variable s it will play a great role in circuit theory and as you know s is a complex quantity. It consists of a real part sigma and an imaginary part j of omega. As a matter of definition, the real part is sigma, the imaginary part of s is not j of omega it is simply omega. So, what we write is quantity is equal to real part plus j times imaginary part. That is, imaginary part itself is a real quantity when we multiply by j; this constitutes a part of the total quantity. It is a matter of definition. So the quantity that multiplies j is the imaginary part and as you know s is called the complex frequency.

This description of the variable in the frequency domain is an extremely useful description and you will see that the most of circuit theory works in terms of variable s only. The real part of s sigma is a characterization of whether the signal is decaying or growing. For example, e to the power s of t. Let us take this signal e to the power s of t. This is e to the sigma t, e to the j omega t and as you know e to the power j omega t by Euler's theorem is cosine omega t plus j sine omega t and if you plot let us say the real part; e to the sigma t cosine omega t then it would be a signal like this, cosine omega t so it starts from here. It would be a decaying sinusoid if sigma is less than 0. It is a cosine omega t but the envelope will decay as e to the sigma t and we have considered sigma less than 0. If sigma is greater than 0 then naturally it would be a growing sinusoid. In a similar manner we can draw the imaginary part. Now, the imaginary part is e to the sigma t sine omega t not j times this. This is the imaginary part and imaginary part is sigma negative shall be like this. It shall start from here sine omega t 0 at t equal to 0 so it goes like this. On the other hand, if sigma is positive then it shall be growing and therefore sigma, the real part of the complex variable s is a characterization of whether that signal is decaying or growing. That is the characterized of signals.

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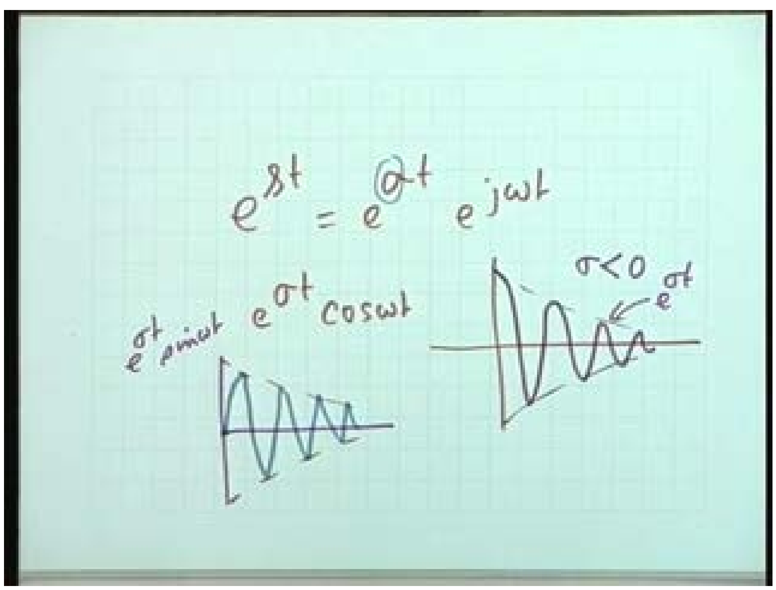

Now how does one characterize a system or the network N? It has an excitation, it has a response. Excitation is described either in the time domain or frequency domain as you have as you have already discussed. How does one describe a network? Well, network can be described in many different ways. You can go into the micro structure of the network that is, what are the components inside and how are they connected to each other. That is the component level; what are the components and how are they connected. You characterize each component by ohms law and then write KVL or KCL to be able to solve for the network. But suppose you want the description of the network or a computer aided analysis description would be that you denote how many nodes are there and from node N to node N plus one what is the connection? How many connections? What is the connection? What is the nature of the element? Is it a resistance, capacitance, or inductance or a diode or something else? Or you can have a gross description that is, a black box description or a terminal description. Like one of the terminal descriptions is the unit impulse response h of t that is, if you apply a ((…)) at the unit impulse of the input, what is the output? I hope the meanings of these are clear; unit impulse and unit step.

So h of t is a good description, good black box description of the system. It is in the time domain. It could be h of t or it could be psi of t, that is the unit step response. That is, instead of unit impulse; if you have a unit step at the input as we usually do when we connect a battery to a network or it could be the unit ramp response. For example, if you apply a ramp that is a voltage which is increasing linearly with time, obviously it cannot go up to infinity. What you do is you break it and then again you make it to rise. Therefore what you apply is a saw tooth, this looks like a saw tooth, tooth of a saw, a saw tooth waveform. You can apply this to a linearly increasing voltage like this which is called a ramp for obvious reasons, it looks like a ramp, the unit ramp response are of it.

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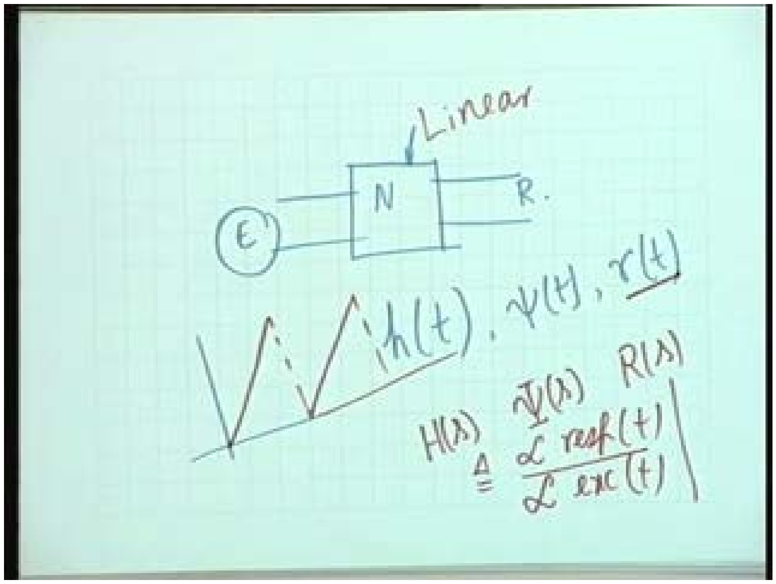

These are time domain descriptions and the time domain descriptions are valid; the unit impulse response is valid only for a very specific type of network and these are the linear network. Provided it is linear, you can describe the network grossly by a unit impulse response, unit step response or unit ramp response and equally valid description of the network would be in the frequency domain. That is, if you take the Laplaces of these, Laplace transform of h of t, psi of t or r of t. That is a function H of s or capital psi of s or capital R of s. This will be an equally good description. But one should remember that in such descriptions, frequency domain descriptions that is, if you want to take the Laplace transform of H of t to find H of s one should remember that the network initially should be relaxed. That is there should be no initial energy storage in the network.

The network the capacitors should all be with 0 charge, the inductor should all be with 0 initial current. It is only then the such gross descriptions suffice and this shall be one of our conditions. For example H of s as you shall see, is the Laplace of the response; Response as a function of t divided by Laplace of excitation as a function of t with the network initially relaxed and initially relaxation means that all capacitors are initially discharged with 0 charge and all inductors are with initially 0 energy. That is, there is no initial current in the inductors. So, that takes care of time domain and frequency domain descriptions of a network. Student: Excuse me sir, in this case do we require that the system be linear? Sir: Yes. The impulse response description or the transfer function description holds only if the network is linear. If it is non linear, you have some kind of description that is not as neat and as elegance as the transfer function for a linear network. We do require that the network be linear and most of the time we shall work with linear networks. If the network is non linear, then we do not go to the frequency domain, we work in the time domain and that is a good way. For example, a demodulator circuit where a diode is concerned, you do not go to transfer function. What we do is, we actually find the input voltage, then we take the square law relational or whatever relation it is and then work it out. So, for non linear we prefer to work in the time domain. Frequency domain description is a kind of a concocted idea.

Student: Sir, Should not the system be time invariant too in this case?

Sir: That restriction, we will not put now. Most of the networks that we shall deal with are time invariant but even if this is not time invariant, even it is time varying, frequency domain description is possible. Although a bit more difficult conceptually, our concern however will be linear time invariant networks. Is there any other question? Okay.

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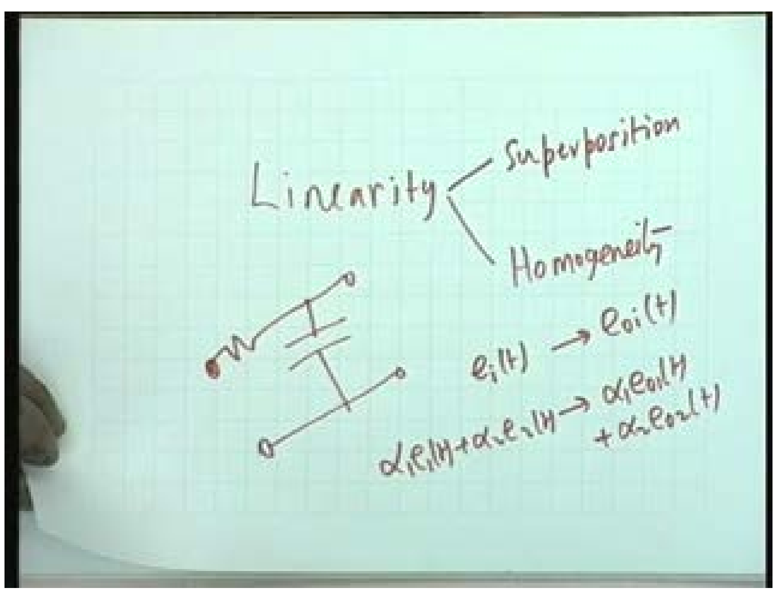

As you know linearity involves two principles, that is super position, these are things which have been dealt with in signal and systems, super position and homogeneity or sometimes called proportionality. Super position, as you know, is that if you have more than one source, then you can find out the response of the network to individual source by suppressing the other sources and then adding them up. Super impose the responses due to the various sources acting alone. On the other hand, homogeneity means that if you have one source and you multiply the source k times, the response should also be multiplied k times and one of the byproducts of homogeneity is that the 0 input should lead to 0 output.

Now, I will ask you a question. Suppose you have a resistance capacitance network in which, the capacitance is initially charged, will these be a linear system? No, because 0 input does not lead to 0 output, there is an output. So initially relaxed networks can be linear, if the network is not initially relaxed, then nine out of ten chances is that the network is non linear. I can of course connect a battery in opposition so that the total voltage across this is 0, I can do that, but these are concocted situations and super position, homogeneity both are included in this type of a description that is if e_i of (t) leads to e 0 i of (t) then alpha 1 e 1 of (t) plus alpha 2 e 2 of (t) should lead to alpha 1 e 01 of (t) plus alpha 2 e 0 2 of (t).

This you know, these signals are system description, which contains both super position homogeneity but as far as circuit theory is concerned, super position individually is going to be useful, homogeneity individually is going to be useful, for example, to judge whether a network is linear or not. Suppose 0 input does not lead to 0 output. Immediately you can say, sorry, the network is non linear. You do not have to go to any other. But suppose the network satisfies homogeneity. Is that sufficient? No. It is a necessary condition but not sufficient. You will have to test for super position.

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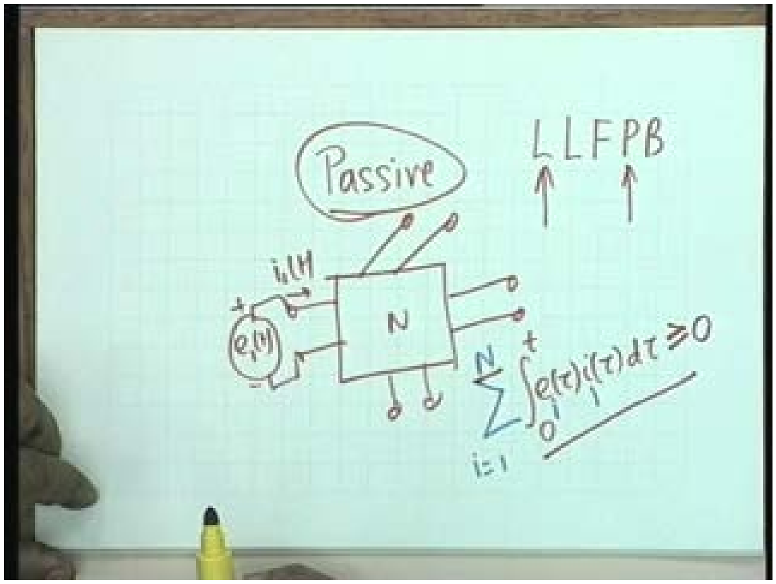

I want to close this class with the definition of what is, while you must have noticed that of the five words LLFPB; we have described the first one that is linear. This L stands for linear. We have said what a linear network is, then we take the fourth letter P this P stands for passive; we shall mostly discuss passive network, a passive network is one in which; suppose you have in general, a multiport, many ports, and you connect voltage sources, let us say e_1 of (t) and find the current i_1 of (t) or you can do it in other way round, you can put a current source and determine the voltage across the port.

Now if you connect sources all sources, sources all around to every pair of terminals or to every port and take this integral 0 to t of e (tau) of i(tau) of d (tau). If you take this integral and find that this integral is greater than or equal to 0, what does this indicate? It indicates that the total energy input to the network, you see the e (tau) i(tau) is power; voltage multiplied by the current, multiplied by t (tau) is the element of energy and if you integrate from 0 to t, it is the total energy supplied to the network by external sources in time small t. If this is greater than equal to 0, with the network initially relaxed because if it is energy sources inside, then obviously the network can give out energy, that is, it can be negative then. This integral can be negative. So the condition is that with the network initially relaxed, if this energy integral is non negative, it can be 0, if it is non negative then the network is called a passive network. Student: Sir should it be the further sum of all these ports, all these terminals or. Sir: Okay, good point I was waiting for this.

This is for one, so it should be summation you call this e i of (t) Ii (t); I equal to one to capital N that is, sum of all the energy supplied by all the sources. You have to sum this up over all the ports and if this qualifies to be non negative, mind you the condition is initially the network must be relaxed, only then this condition can be applied, then it is called a passive network. You know resistance, ordinary resistance, capacitance, inductance, they are all passive. On the other hand a negative resistance, a tunnel diode, for example, is an active element because it can deliver energy. An initially charged capacitor is not a passive element because it can give out energy. So initially relaxed network, if the energy integral summed up over all the ports is non negative, then it is called passive network.

We close here and if there are any questions, you can ask me now.
Thank you.