**Digital Communication Using GNU Radio**

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**Week-01**

**Lecture-06**

Hello, welcome to this lecture on Digital Communication using GNU radio. In this lecture, we are going to take a look at the complex baseband representation of signals. In terms of complex baseband equivalent, we will be defining what are baseband and passband signals. We will then distinguish between real and complex baseband signals. We will then see what the complex baseband equivalent signal is, its significance and how to construct it and finally, up conversion of the complex signal to passband and how you can recover the complex signal from the real passband signal by down conversion. So, what are baseband and passband signals? Roughly speaking, baseband signals are those that occupy frequencies near DC, that is they generally contain lower frequency components.

Passband signals on the other hand occupy a narrow band of frequencies closer to the so called carrier frequency. The carrier frequency is generally much greater than the bandwidth of the actual signal. As an example, 2.4 GHz may be the carrier frequency that may be carrying data signals of 10 MHz bandwidth.

In this situation, the 10 MHz signal is the baseband signal and the carrier frequency is 2.4 GHz. On what basis are these values chosen? So, this is dependent on various parameters and of course, how the standard is designed. So, there are propagation characteristics, licenses available for transmission and several other factors that determine the passband signals characteristics both in terms of the carrier frequency as well as the bandwidth. Visually if we inspect the spectrum, the baseband and passband signals can be very distinctly characterized.

In this example, let us say that

$s_1(t)$

is a baseband signal and its spectrum or Fourier transform is

$S_1(f)$ .

$S_1(f)$

is available over here and as you can see, it occupies a frequency between

$-W$ and $W$

generally closer to the lower frequency range. If you now consider the signal

$s_2(t)$

that is constructed as

$s_1(t)\cos(2\pi f_c t)$

where

$f_c$

is chosen to be larger, generally much larger than

$W$ ,

we obtain the passband signal

$S_2(f)$ .

In this particular situation, if you remember the Fourier transform of cos, it has two impulses, one at

$f_c$

and one at

$-f_c$ .

Therefore, the convolution of the spectrum of

$S_1(f)$

will give two copies, one around

$f_c$ ,

one around

$-f_c$

which is why this passband signal

$S_2(f)$

occupies the frequencies between

$f_c - W$ and $f_c + W$

and the corresponding negative frequencies.

One issue however is in this particular picture is that if

$s_1(t)$

is a real signal, you will recall that its spectrum has the conjugate even property. That is

$S_1(f)$

is the same as

$S_1^*(-f)$ .

This means that the information contained in the positive frequencies of

$S_1(f)$

allows you to directly infer the information contained in the negative frequencies of

$S_1(f)$

which means that keeping only one of them is enough. Of course, you may argue that for a real signal, the conjugate even property has to be satisfied. However at passband, you can see that there are two copies and having only one of these and the corresponding negative is sufficient for you to obtain a real signal that has all the information needed to reconstruct

$S_1(f)$ .

In other words, this particular approach of multiplying the real baseband signal by a cosine is inefficient because the information is duplicated in the passband. So for efficient usage of the spectrum, how do we remove this redundancy and how do we actually use the bandwidth between or the frequency range between

$f_c - W$ and $f_c + W$

to maximize the information transmission. This is what motivates the so-called complex baseband representation of signals. The key idea here is to take two real valued baseband signals for reasons that will be clear shortly. We will call these signals

$s_c(t)$ and $s_s(t)$ .

Remember both are real baseband signals that occupy the same frequency range of

$-W$ to $W$

and we call that a bandwidth of

$W$ .

As these signals are real, both of these signals have conjugate symmetric Fourier transforms that is

$S_c(f) = S_c^*(-f)$ ,

$S_s(f) = S_s^*(-f)$ .

We now construct the complex baseband signal

$s(t) = s_c(t) + j\, s_s(t)$ .

In general, unless there are some specific properties that

$s_c(t)$ and $s_s(t)$

may have, this is a complex signal and it does not have a conjugate symmetric Fourier transform. In other words, we will shortly see that this particular construction eliminates the so-called redundancy in the frequency domain and allows you to transmit two real valued signals within the same bandwidth

$W$

if you permit the use of complex signals.

So, to convince ourselves that

$s(t) = s_c(t) + j\, s_s(t)$

is not generally real is very simple. We know that the real nature of

$S_c(f)$ and $S_s(f)$

means that

$S_c(f) = S_c^*(-f)$ ,

$S_s(f) = S_s^*(-f)$ .

Then by the linearity of the Fourier transform, we can directly write

$S(f) = S_c(f) + j S_s(f)$ .

Then let us see if

$S(f)$

satisfies the so-called conjugate even property. If you take the conjugate of

$S(f)$ ,

we get

$(S_c(f) + j S_s(f))^*$ .

Using this puts the conjugate on the

$j$ ,

so you get a

$-j$

over here. You get

$S_c^*(f) - j S_s^*(f)$ .

Using the properties of our

$S_c(f)$ and $S_s(f)$

we find above, we get

$S_c(-f) - j S_s(-f)$ .

If

$S(f)$

were corresponding to a real signal, then the conjugate even property would have meant that

$S^*(f) = S_c(-f) + j S_s(-f)$

which is not the case. Therefore, this so-called complex baseband construction or representation allows you to very easily just design two real signals and combine them by just using the complex addition operation that is you take the

(first signal) $+j\times$ (the second signal )

and you end up with two real signals that occupy the same bandwidth

$-W$ to $W$

as a complex signal.

As a visual cue, if we have a

$S_c(f)$ and $S_s(f)$ ,

we are depicting the spectrum being similar as having a hatch over here and a hatch over here. This particular information over here and this particular information over here, they are the same information except that they have the conjugate even property and similar properties hold over here. But in case you construct the so-called complex baseband equivalent signal, you end up with a resulting signal

$S(f) = S_c(f) + j S_s(f)$ ,

the spectrum of that signal that is that need not have the property that the real part and the imaginary part are just mirror images of each other. That is we occupy the same bandwidth

$-W$ to $W$

and now have a complex signal in the form of two real signals that occupy the same bandwidth range. But this is nice, but we need to be able to transmit this signal as a real signal because whenever you have any transmission medium, you have to be able to send a real signal.

The question arises as to how we can make this into a real passband signal that can be transmitted over any media. To do this, we have the baseband to passband transformation which is very simple. We will now start dealing with

$s(t)$

directly where

$s(t)$

is now a complex signal that occupies the frequency range between

$-W$ and $W$ .

In other words, it occupies a bandwidth of

$W$ ,

but it is a complex signal thus it consists of two real signals. We are now going to define

$s_p(t)=\Re\{\sqrt{2}s(t)e^{j2\pi f_c t}\}$ .

For a minute, we can ignore the root 2 which is just a scaling factor, but what we are doing is we are multiplying or modulating

$s(t)$

with the complex exponential

$e^{j2\pi f_c t}$ .

If you remember your Fourier transform properties, multiplication with

$e^{j2\pi f_c t}$

convolves the spectrum of

$s(t)$

and places it around

$f_c$ .

Therefore, this signal

$s(t)e^{j2\pi f_c t}$

is going to occupy the frequency range between

$f_c-W$ to $f_c+W$ .

The real operation just places a copy at

$-f_c$ .

We will soon see that

$s_p(t)$

is a real pass band signal that has information of

$s_c(t)$ and $s_s(t)$

together.

The bandwidth footprint as we mentioned is between

$f_c - W$ and $f_c + W$

of course it is a real signal. So, there is a corresponding conjugate at

$-f_c - W$ to $-f_c + W$ .

Pictorially what happens is that this spectrum base band spectrum which is unsymmetric and corresponds to a complex signal is brought to around

$f_c$

and a copy of the same with the conjugate operation is also brought to

$-f_c$ .

Therefore, the conjugate symmetry property is definitely satisfied, but you can clearly make out that all the information about both the positive frequency part and the negative frequency part of the base band signal are present very much and there should be a way to recover them. Of course, the way we motivated this was by taking a complex signal and then taking real part

$e^{j2\pi f_c t}$

and so on.

One question that arises naturally is that can we get

$s_p(t)$

directly from

$s_c(t)$ and $s_s(t)$ .

$s_c(t)$ and $s_s(t)$

are just a proxy for

$s(t)$

because

$s(t)$

essentially has

$s_c(t)$

as its real part and

$s_s(t)$

as its imaginary part. So, can we construct

$s_p(t)$

directly from

$s_c(t)$ and $s_s(t)$ .

To do this we consider

$s_p(t)$

as

$\Re\{\sqrt{2}s(t)e^{j2\pi f_c t}\}$

and we can expand this by writing

$e^{j2\pi f_c t}$

as

$\cos(2\pi f_c t)+j\sin(2\pi f_c t)$ .

So, let us now perform this operation.

If you now expand

$$e^{j2\pi f_c t}$$

as

$$\cos(2\pi f_c t) + j\sin(2\pi f_c t)$$

you will find that the real part that remains after this expansion is

$$s_c(t)\cos(2\pi f_c t) - s_s(t)\sin(2\pi f_c t) \quad .$$

Therefore, the way to construct your pass band signal

$$s_p(t)$$

is to modulate

$$s_c(t)$$

with a cosine at

$$f_c$$

and to modulate the

$$s_s(t)$$

with a sine at the same frequency and add or subtract them. Therefore, by performing this operation you are able to construct

$$s_p(t)$$

directly from

$$s_c(t) \text{ and } s_s(t) \quad .$$

The c and s subscripts should now become somewhat clear because

$$s_c(t)$$

rides on

$$\cos(2\pi f_c t)$$

in the sense of being modulated by

$$\cos(2\pi f_c t)$$

and

$s_s(t)$

rides on

$\sin(2\pi f_c t)$

which is why the subscripts make sense

$s_c(t)$

is for cos

$s_s(t)$

is for sin. Conventionally because of the fact that

$s_c(t)$

is multiplied by

$\cos(2\pi f_c t)$

we refer to

$s_c(t)$

as the I-component or the in-phase component and

$s_s(t)$

as the Q-component or the quadrature component.

This should be consistent with the nomenclature that is used in the context of circuits and power systems phases and so on. To understand this pass band transition better we will take a small detour that considers combining these signals from the base band to obtain pass band signals. We will now take a detour to make a very simple experiment in GNU radio wherein we will construct a complex base band signal using very simple cosine and sine. We will take the base band signals

$\cos(2\pi f_0 t)$ and $\sin(2\pi f_0 t)$

implicitly assuming that

$f_0$

is small and therefore close to dc thus these can be treated as base band signals. Since both of these are real signals they have conjugate symmetric Fourier transforms and we are going to use the conventional approach of taking the first signal plus j times the second signal and observing the spectral characteristic and showing that the symmetry is no longer present thereby indicating that you have a baseband signal that has two distinct real signals.

Let us begin by first adding a signal source. We will use the conventional approach control f or command f type signal grab a signal source place it in our flow graph. We want a real signal therefore we will double click this and change the type to float. Since we also want a sign we can pull in another signal source or we can just select the signal source by clicking on it and hitting control c or command c and control v or command v to produce a copy. We then double click the signal source and change this to sign and say ok.

Our next course of action is to construct the first signal plus j times the second signal. GNU radio offers a convenient approach to do this. We will pull in the float to complex block control f or command f type float and we have the float to complex. Next we connect the signal source one to the real part the second one to the imaginary part and we are ready with our complex baseband signal. But we wish to view the spectra of this signal along with the original signals.

So we would need a complex QT GUI frequency sink. We will press control f or command f, f, r, e, q get the QT GUI frequency sink. Let us not forget the throttle control f or command f throttle. We can connect our signal to the throttle and we want to visualize three signals in the frequency sink.

So we will double click it. We will say grid yes, auto scale yes and we will say three inputs and say ok. We will connect the throttle to the third input. Now to view the signal source in the complex GUI frequency sink, we need to convert the signal source again to complex. For that we will again use float to complex but we will keep only the real part. An easy way is to just select this float to complex, hit control c or command c to copy and control v or command v to paste, control v to paste again so that we get two of them.

Connect the output to the real part, connect this output to the real part and then we will play a trick to make sure that the signal which comes out has only a real part. We will create a constant source that emits zero. So, control f or command f, c, o, n, s, d, const. We will get a constant source. The constant source that always outputs the real number zero.

So, double click this, change it to float, say ok and we connect the output of this constant source to the complex imaginary part over here and the imaginary part over here. We can then connect the first source to the input zero, second source to the input 1 and our flow graph is ready. Let us execute this flow graph. Now, what is the interpretation? Let us inspect only the first signal by disabling the second and third ones.

This is our cosine. As expected, you see two peaks, one at 1 kHz and the other at minus 1 kHz. If we then see the sine, it is essentially overlapping with the cosine because both the sine and cosine have similar magnitude spectra but they differ only in phase. In fact, to get a finer line, we can middle click, hit the control panel and change this to a rectangular window in which case you will see the lines distinctly. But when you now bring the third signal, the third signal only has a peak at exactly 1 kHz and no peaks elsewhere. Why is this the case? This is because

$$\cos(2\pi f_0 t) + j\sin(2\pi f_0 t)$$

is actually

$$e^{j2\pi f_0 t}$$

which has a Fourier transform of

$$\delta(f - f_0) \quad .$$

Therefore, as expected, it has a single distinct peak at 1 kHz. But from the perspective of baseband signals, here is a baseband signal

$$e^{j2\pi f_0 t}$$

that embeds

$$\cos(2\pi f_0 t) \quad \text{and} \quad \sin(2\pi f_0 t)$$

which are two real signals and does not have a complex conjugate which is equal, thereby having a complex baseband signal. So, this is an example of a very simple complex baseband signal. As a final step, let us also convert this to a passband signal by performing the operation

$$\Re\{s(t) e^{j2\pi f_c t}\} \quad .$$

To do this, let us take a complex signal source say control f or command f, signal source.

This signal source is going to be

$$e^{j2\pi f_c t} \quad .$$

So, let us choose the carrier frequency as say 6000 Hz. We will then multiply this

$s(t)$

that we obtain by combining the cos and sine with this. So, control f or command f, we get the multiply block, we multiply the signal, multiply the other signal and all we need to do is to take the real part of this and look at the spectrum.

So, let us now take the real part. So, we say control f or command f and type real, we get the complex to real block, connect the output to here and we get a real QT-GUI frequency sink. Control f or command f, grab the frequency sink over here, we double click it and change it to float, connect it over here, hit run. Now, as you can see the complex baseband signal was

$e^{j2\pi f_0 t}$ .

When you now modulate it with a carrier at 6 kHz, you will get a copy at 7 kHz and minus 7 kHz. The reason is because 6 kHz is the center frequency and your signal appears 1 kHz to the right and over here 1 kHz to the left of minus 6 kHz.

This is a very simple example of obtaining a passband signal from a complex baseband signal. We will soon expand this to more sophisticated signals in the next detour. Thank you.