Digital Signal Processing & Its Applications
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Lecture No. 04 c
Introduction to Phasors

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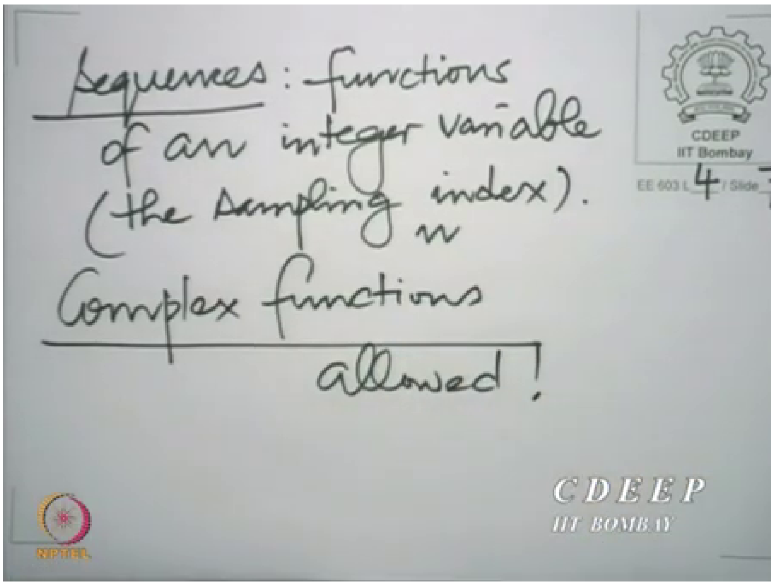

Sequences are essentially functioning of an integer variable. And the physical interpretation of the integer variable is the sampling index. I said functions maybe a little carefully, but we need to qualify whether I am talking about real functions or complex functions. Well, I am going to allow complex functions.

And again, the natural question is what is the physical meaning of a complex functions? If I sample a real signal, I am certainly not going to have complex values. So, where would complex signals come from? Like complex signals for the moment should be thought of as hypothetical extensions of real things. So, you could think of them as two real signals, one for the real part, and one for the imaginary part.

And we will use complex signals as a tool to deal with real things. For the moment, let us think of it with this reassuring note. So, we do not really have complex signals in nature. But we bring in, we allow the sequence to be complex because sometimes it helps us to deal with real signals by bringing in complex signals for a while, doing some work with the complex signals, and going back to the real. Now, in a minute, we are going to use a complex signal. So, you see, the complex signal comes in as follows.

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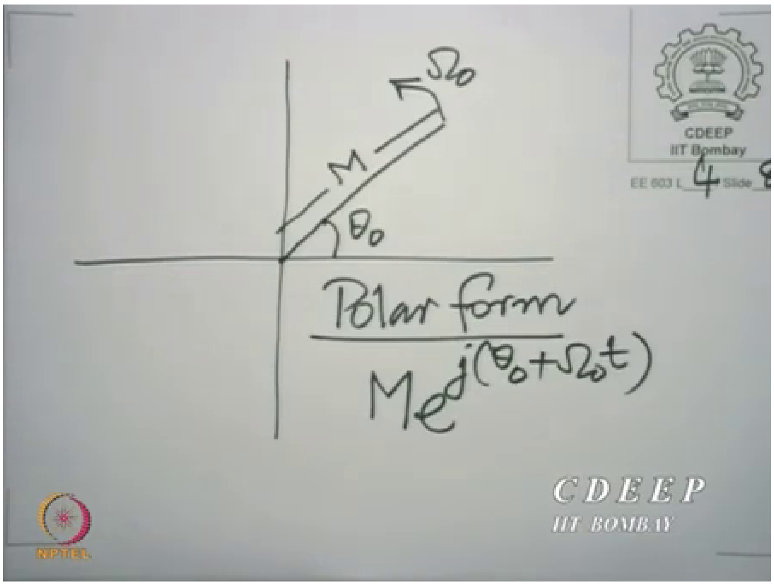

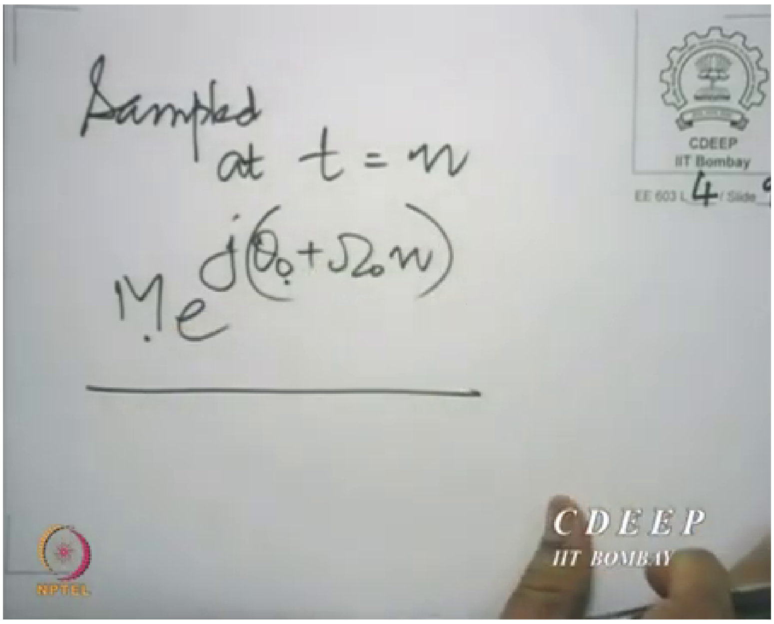

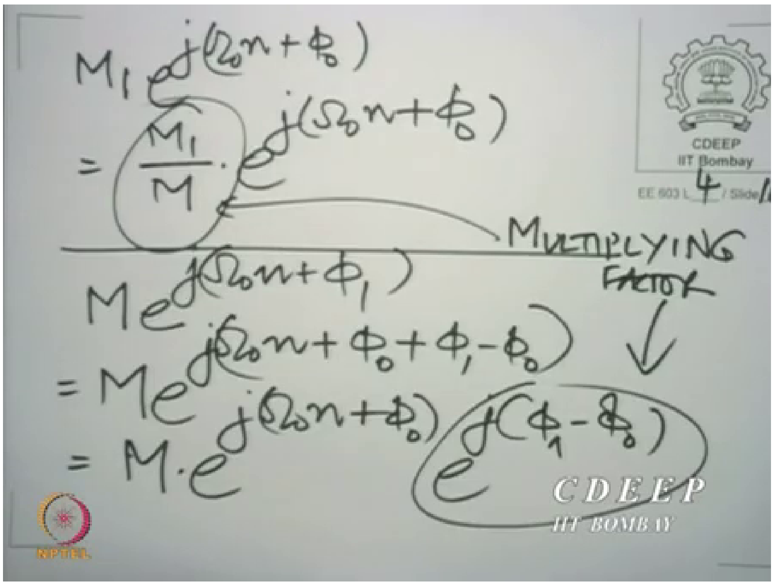

Suppose I were to take a complex number, rotating a complex number, which maintains the magnitude of M, starts with an angle of θ_0, and rotates angular velocity of Ω_0. We know how to write this complex number in polar form. The polar form representation of the complex numbers is very simple, it is $Me^{j(\theta_0+\Omega_0 t)}$. Of course, this is the continuous complex number.

Now, let us assume that I were to sample this complex number with a sampling rate of *1*. I would get $Me^{j(\theta_0+\Omega_0 n)}$. You see all the time. I am making the convenient assumption that my sampling interval is unity. Let us get used to this assumption.

I mean, I am normalizing my time measurement with respect to sampling. Now, you see the beauty of this representation is that if I were to change its amplitude or its phase, so, if I were to change the amplitude or the phase, initial phaseθ_0. If I were to change the amplitude, $Me^{j(\theta_0+\Omega_0 n)}$, is of course, expressible as $(M_1/M)e^{j(\theta_0+\Omega_0 n)}$.

On the other hand, this is, of course, obvious. So, when I change an amplitude to M_1 is equivalent to multiplying that number by M_1/M. Simple enough, but the more interesting thing is the phase. So, if I were to change the phase to θ_1, this could be rewritten as $Me^{j(\Omega_0 n+\Phi_0+\Phi_1-\Phi_0)}$.

And of course, that can be rewritten as $Me^{j(\Omega_0 n+\Phi_0)}e^{j(\Phi_1-\Phi_0)}$. So, in either case, you have a multiplying factor here. Whether you take a change of amplitude or a change of phase, it amounts to multiplying the original rotating complex number by a constant all or be it complex.

This is a significant observation, unlike a sinusoid. Where, when I change the phase, it is not tantamount, it is not the same thing as multiplying the sine wave by a constant. But in a sine wave, of course, when I multiply the amplitude by when I change the amplitude is equivalent to multiplication by a constant. So, therefore, the beauty of dealing with this rotating complex number is that multiplication by a constant factor or be it complex, is a correct description of change of amplitude and change of phase.

And of course, change of amplitude and phase together too. If I change the under independent, if I change the amplitude to M_1 and the phase to $\theta_1$or Φ_1, it is equivalent to multiplying by $M_1/Me^{j(\Phi_1-\Phi_0)}$, so you can take them together. That means when I now put it is very clear if I have a system instead of dealing with sinusoids, now, if I were to deal with these, which we call phasors.

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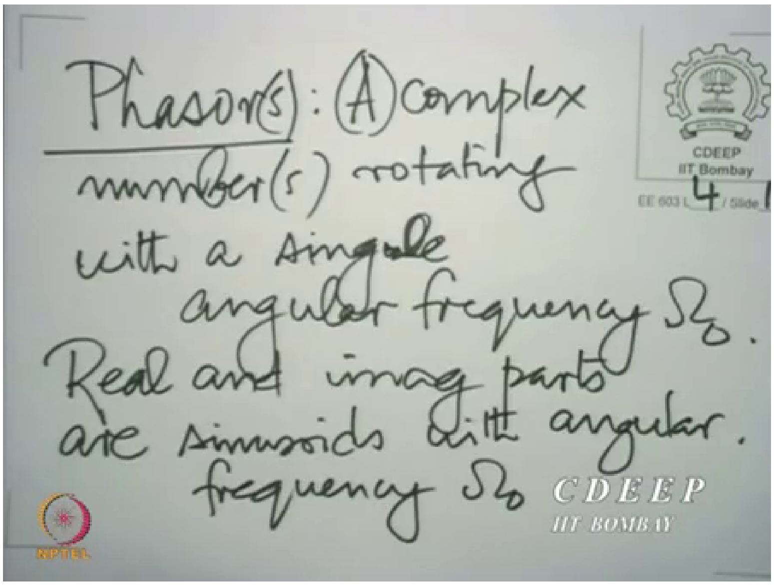

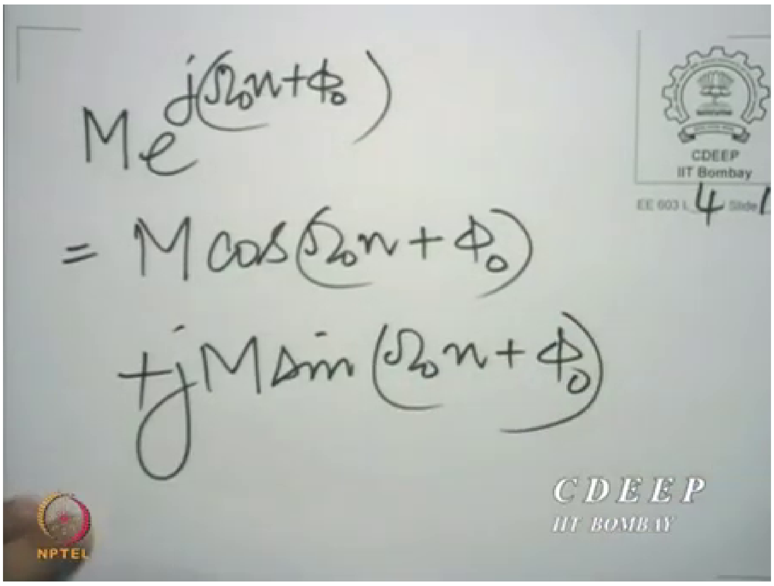

So we call such a rotating complex number a phasor with a single angular frequency it is denoted by Ω_0. Now, why is phasor is important to us, the real and imaginary parts of phasors are sinusoids with frequency Ω , angular frequency Ω_0, is that clear that is straightforward because you can even write down the real imaginary parts.

Indeed, if you have a phasor $Me^{j(\Omega_0 n+\Phi_0)}$, this can be decomposed as$Mcos(\Omega_0 n + \Phi_0)$. This is the real part, $Msin(\Omega_0 n + \Phi_0)$ this is the imaginary part. Each of them is the sinusoid of the same frequency Ω_0. So, now I see the connection.