Digital Signal Processing and its Applications
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Lecture 29A
Recap of Window Function Inference

A warm welcome to the 29th lecture on the subject of Digital Signal Processing and its Applications. In this lecture, we continue with the subject of FIR filter design by using the window approach and we need to recapitulate a few ideas that we have learned in the couple of lectures that have passed on the subject of FIR filter design. The most important idea that we have seen in the previous lecture is the role of the spectrum of a window function.

You see it was a fundamental question; why you should choose a window function other than the very simple option of a rectangular window? And the rectangular window seems easy to do. Just retain a few samples and reject the rest from the impulse response. Why should we multiply that set of samples that are retained by some other sequence which has some quote unquote desirable properties?

And what are those desirable properties? We have seen a little bit of this in the previous lecture and we need now to substantiate by actually giving examples, but let us recapitulate what we have seen in the previous lecture. We have seen that the windows spectrum has an effect on 3 things.

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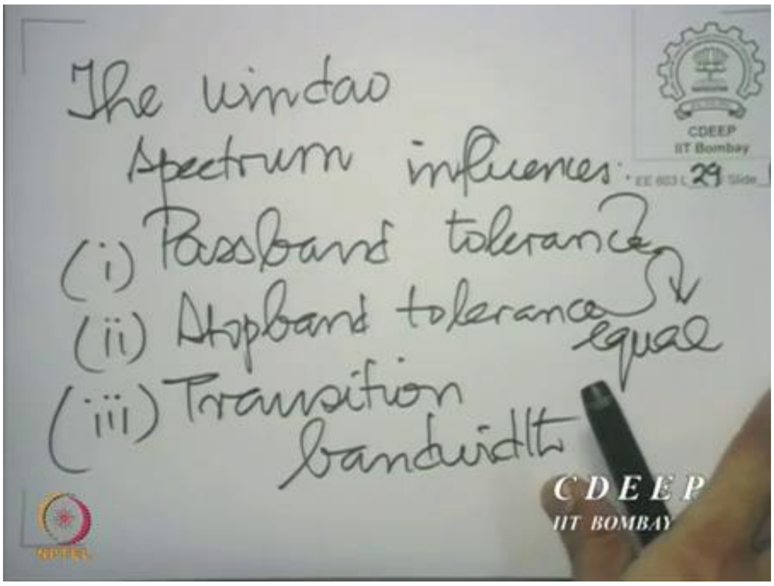

The passband tolerance, the stopband tolerance and the transition band or transition bandwidths from the window approach we do not have separate control on the passband and the stopband tolerance. In fact, the passband and the stopband tolerance are equal in the windowed approach and we have seen that in the previous lecture. The cause of passband and stopband tolerance is the area under the side lobe.

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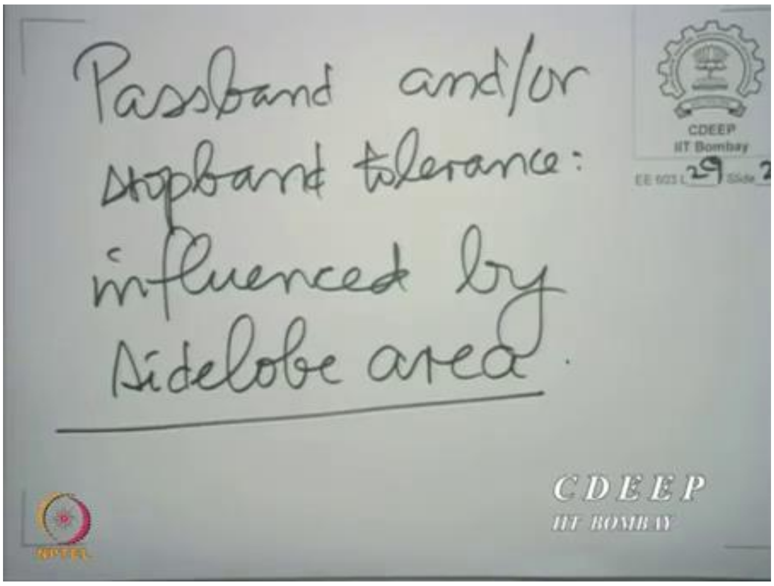

The passband and stopband tolerance influenced by side lobe area. The reason for this is simple. The tolerance is caused by side bands entering and leaving the ideal pass band in the periodic convolution between the ideal DTFT or the ideal frequency response and the windows spectrum for the pass band is the region where the main lobe has completely entered the ideal passband and now the side lobes are entering and leaving.

The stopband is a region where the main lobe is completely outside and the transition band is caused by the main lobe actually entering and the whole process of the main lobe entering the ideal pass band and ultimately going entirely inside is the cause of the transition band and therefore, the main lobe width influences the transition band. We have seen that in the previous lecture.

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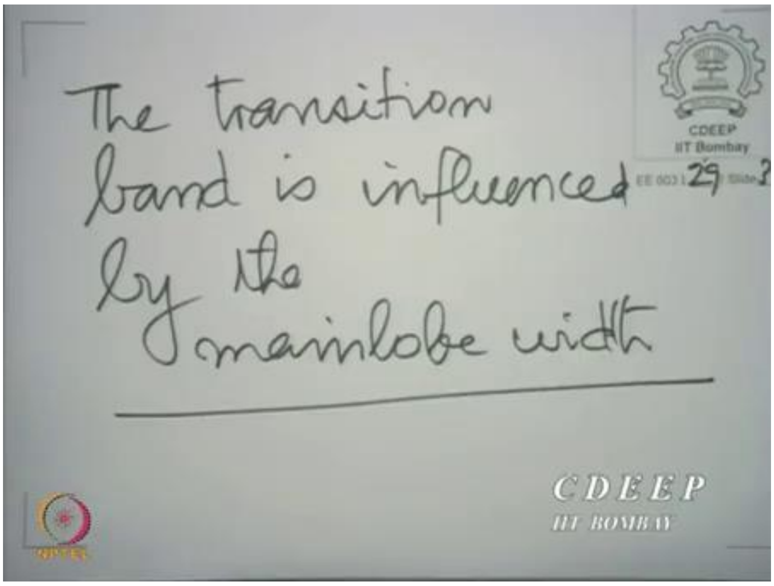

Transition band is influenced or is caused by actually influenced by the main lobe widths. It is the main lobe width and the side lobe areas that play a role. The main lobe area has only a peripheral influence and the side lobe width again have only a peripheral influence. The influence of the main lobby area is only to determine the average height of the resultant pass band.

Now that is not a terribly important issue because the height can be changed by simply multiplying all the coefficients by a constant. Now, even when we say the passband and our stop band tolerance is influenced by side lobe area, we should qualify this by writing relative side lobe area.

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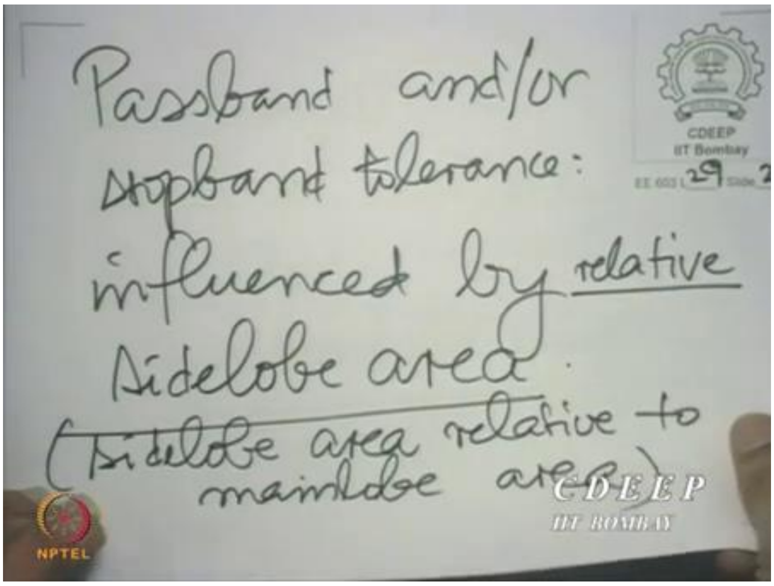

Copy side lobe area relative to main lobe area; it is the relative side lobe area which really plays a role not the absolute side lobe area, by relative site love area, we mean the ratio of the side lobe area to the main lobe area, because the main loop area is what determines the average height of the passband and superposed on this height are the oscillations caused by the side lobe area.

And if you multiply the whole set of impulse, the resultant impulse response coefficients by a constant both of these are going to get multiplied by the same constant. So the relative side lobe area is not affected by multiplication by a constant. So, you see, these 2 things play a very crucial role and we have seen 2 examples in the previous lecture, the rectangular window and the triangular window.

We saw that in the triangular window, we have less relative. Now, we did not show that but we gave an intuitive argument for it, we said that you know in the triangle of window, we definitely have a reduction of side lobe height that we showed the relative side lobe height, but what you now put the challenge before you.

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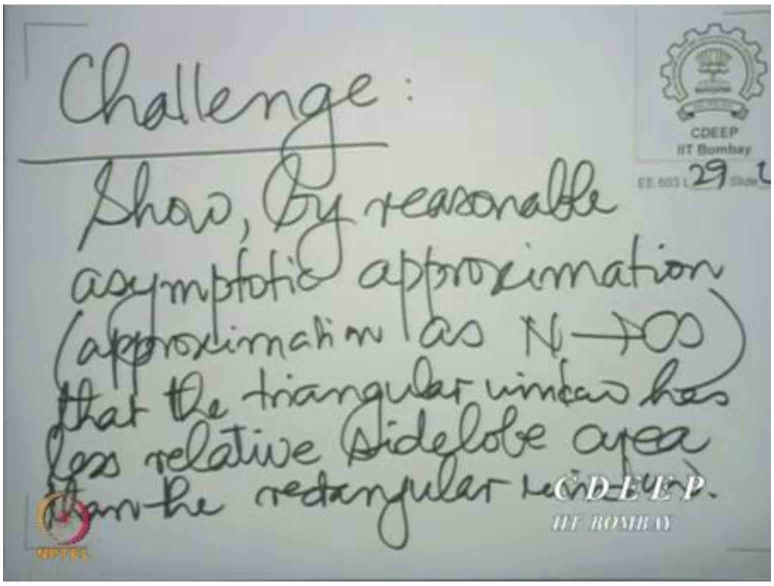

Show by a reasonable approximation, by reasonable asymptotic approximation, by asymptotic approximation, I mean approximation as n tends to infinity, that the triangular window has less relative side lobe area than the rectangular window. Now, the hint to solve this challenge is that you can treat the side lobe asymptotically like (rect) triangles, where the base is equal to the width of the side lobe and the height can be determined by an asymptotic approximation.

You may treat them very close to triangles and find out the area under the triangle and then see how the area asymptotically changes as the length of the window goes to ∞. Now, of course, I told you this comes with a cost. The cost is that the main lobe width expands. So it is not for free. Although the oscillations in the past panic stop and decrease they do so at the cost of an increased main lobe of width, which means a larger transition band.

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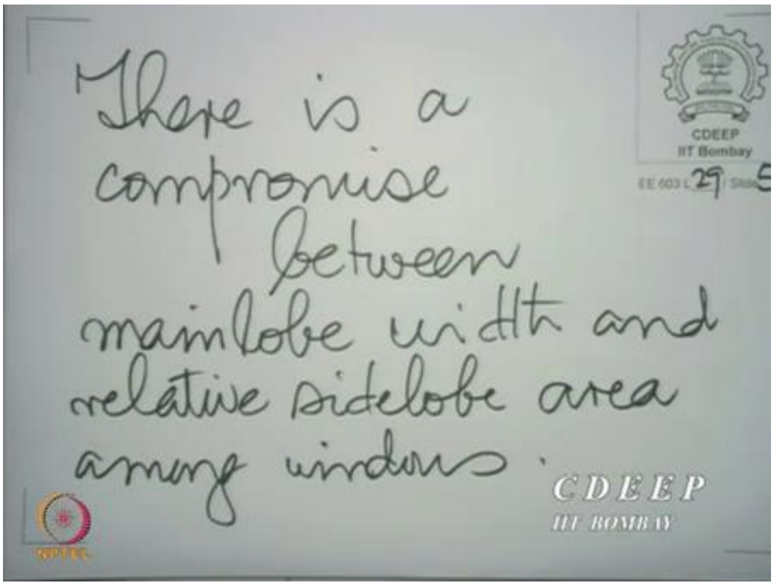

So, the next point that we have seen in the previous lecture is that there is a compromise between main lobe width and relative side lobe area among windows. We also remark that this compromise, though generally present can also be to some extent optimized in the sense that one can do well on both fronts to an extent. Now, to what extent is a very very basic and very fundamental question.

How we determine, how best we can do on both fronts together? There are 2 conflicting things happening here, the main lobe width and the side lobe area, relative side lobe area. But you see there is a fundamental limit on how well you can do on both fronts together and although I shall not go into the genesis of this fundamental limit, or even try to explain it mathematically, I shall state the general idea behind this fundamental limit.

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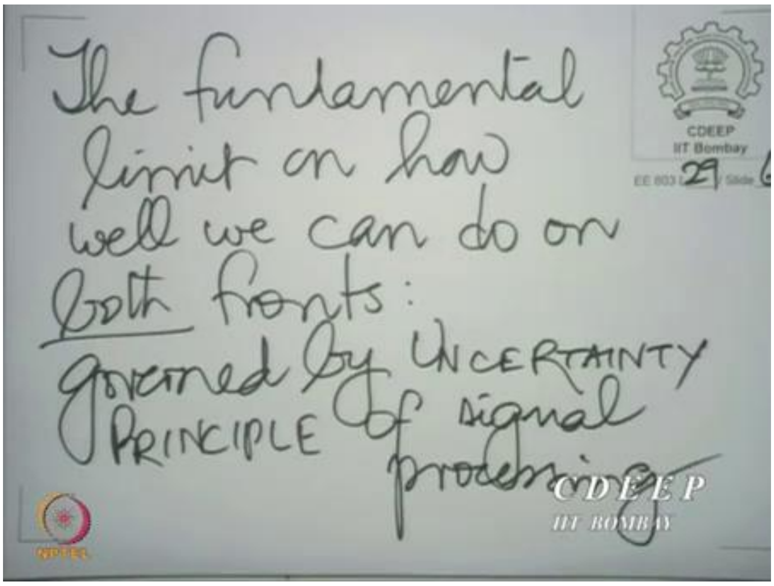

This fundamental limit stemmed from something called the uncertainty principle. The fundamental limit on how well we can do on both fronts is essentially governed by what is called the uncertainty principle, governed by the uncertainty principle of signal processing. I should not just say a couple of sentences to put this in perspective. But, to understand the relationship better or even to understand the principle better, one would need to take a higher level course, on signal processing.

The principle says that there is a basic conflict between the time and the frequency domain representation of a signal. So, if you try and narrow a signal down in time, the tendency for the signal is to expand in frequency which of course, we do not need a principle or uncertainty principle to understand because, we know that, you know, if you take the Fourier transform for signal and then if you contract or expand the signal, there is a respective expansion or contraction in the Fourier transform that is a basic property of the Fourier transform, so that does not require too much of exposure.

What requires a much deeper understanding is that there is a fundamental limit on how narrow you can be in both domains. You can be arbitrary narrow or broad in one domain. But to what extent can you narrow down in two domains together is what the uncertainty principle addresses and in fact, it is very closely related to this whole question of how much you can narrow down the window in terms of main lobe width and side lobe area.

Now, I shall not go entirely into the details here safe to say that there is a, there is a relationship and for those of you who are interested, you will want to look up more about the uncertainty principle or take a higher level course. Or also try and reason out how this in other words expresses a basic conflict between expansivity in time and frequency.