Digital Signal Processing and its Applications
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Lecture 26 B
Specifications of Low Pass Filter & Phase Response & Summarization of Discrete Piecewise Constant Filter Design

So, what are the specifications then for the low pass filter?

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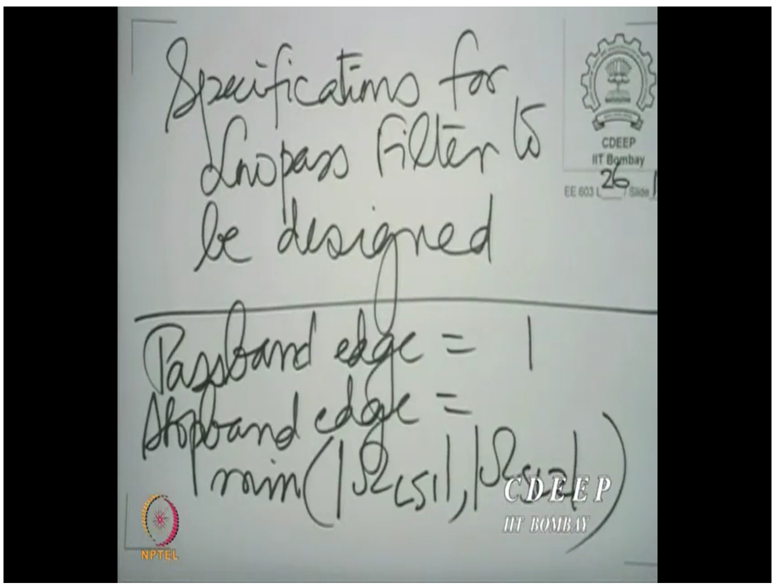

Passband edge 1, stopband edge is the minimum of $\left|\Omega_{Ls1}\right|$ and $\left|\Omega_{sL2}\right|$. The one which is closer is important, that puts a more stringent transition band. Tolerances carried as they are.

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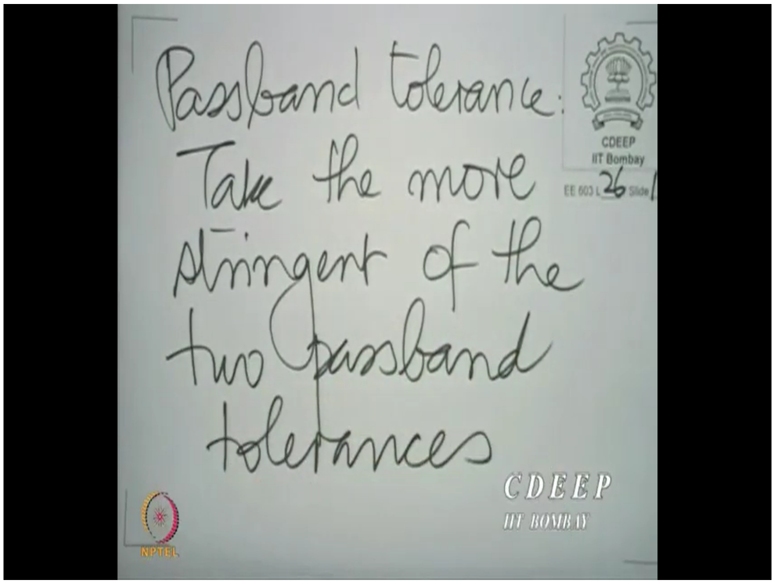

Passband tolerance, take the more stringent of the two passband tolerances if they are different. Of course, if they are the same, there is no problem, just take the tolerance as it is.

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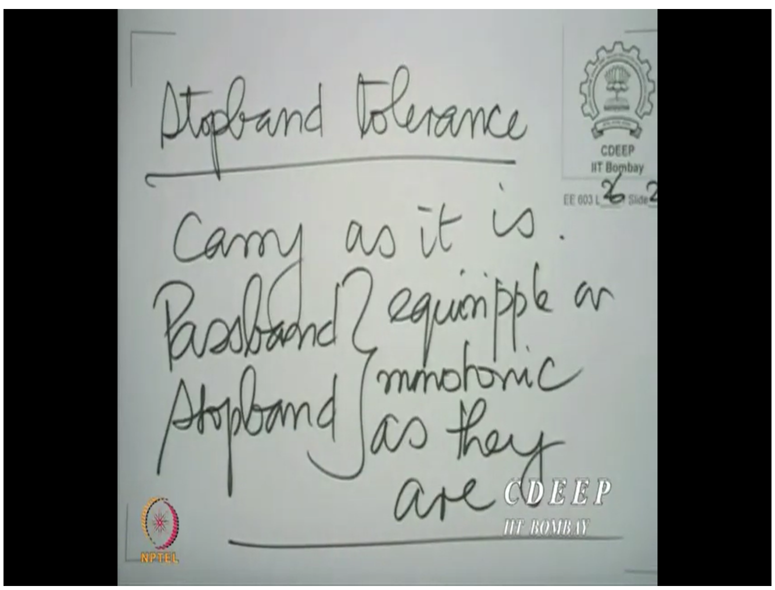

The stopband tolerance carried as it is. Passband and stopband nature carry as they are, equiripple or monotonic, as they are. Naturally, you cannot have different kinds of natures for the two passbands or two stopbands in the bandpass and bandstop filter, they are to be carried as they are. Finally, yes, please there is a question.

So, the question is - when we design, we are ultimately going to design with the same tolerance, yes we are going to, are we going to design with the same tolerance, yes indeed we are going to design with the same tolerance but that tolerance could come from initially different tolerances. That means that, obviously when you have different tolerances, one of the tolerances is more stringent than the other. There is a very ordering principle between tolerances which is not there between natures. One nature is not better than the other nature, or more difficult that the other nature.

And secondly, one nature cannot transform into another nature. You see, unlike tolerance. Tolerances, if you have definitely obeyed a tolerance of *0.1*, you obey the tolerance of *0.12*. But that is not true with natures. So, you can possibly add different tolerances and satisfy both of them by taking the more stringent one. But you cannot do that for natures.

So, the nature must be the same but the tolerances can be different. Yes, there is a question. So, the question is, that is a good question - can Ω_0 fall outside Ω_{s1} to Ω_{s2}, that is very interesting. Now, you see, Ω_0 is the geometric mean of Ω_{p1} and Ω_{p2}, so all that we can say is that it is between Ω_{p1} and Ω_{p2}. But then, of course, you can, yes in principle, choose Ω_{s1}, Ω_{s2} so that it is just on one side of this geometric mean. In principle, yes it is possible but in practice, that will be very rare.

But then, there is trouble. If you do that, then you have the whole, all this transformation goes haywire. That means if we do indeed put Ω_{s1} and Ω_{s2} just on one side of the geometric means, then we have trouble in this transformation. That is a good question.

So, can Ω_{s1} and Ω_{s2}, of course you are free to choose it anywhere between Ω_{p1} and Ω_{p2}. And then Ω_0^2 is Ω_{p1} times Ω_{p2}, so it is the geometric mean. But if you choose Ω_{s1} and Ω_{s2} on one side of the geometric mean between Ω_{p1} and Ω_{p2}, then this design procedure falls flat, you cannot do this. Because both the mapping of $\Omega_{p1}$and Ω_{s1}, Ω_{s2} will come on the same side of the low pass frequency axis.

So, then in fact, I leave this to you as a food for thought. What will you do? Can you change your strategy a little bit if indeed this pathological situation arises? So, I put this, this is a good question that has been asked and we will put that as a challenge.

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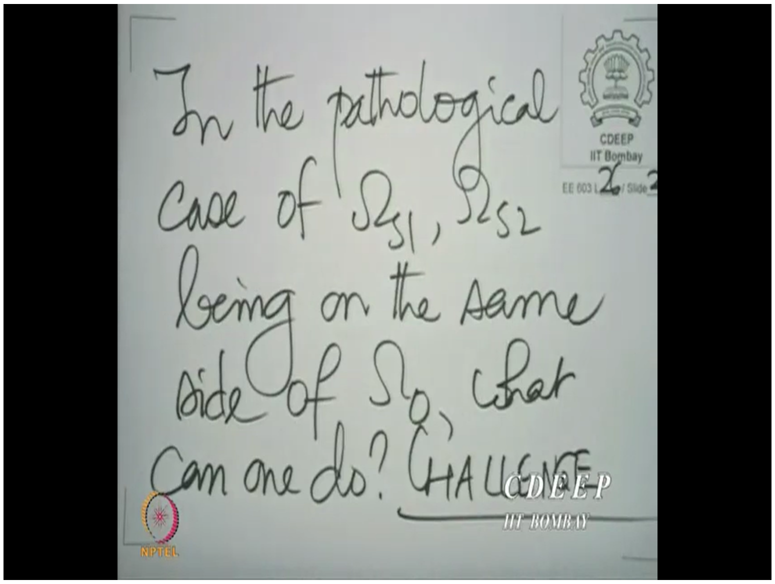

In the pathological case, Ω_{s1} and Ω_{s2} being on the same side of Ω_0, what can one do?

One would need, of course, to modify this procedure. Now that we are anyway discussing challenges, I would like to put one more challenge before the class.

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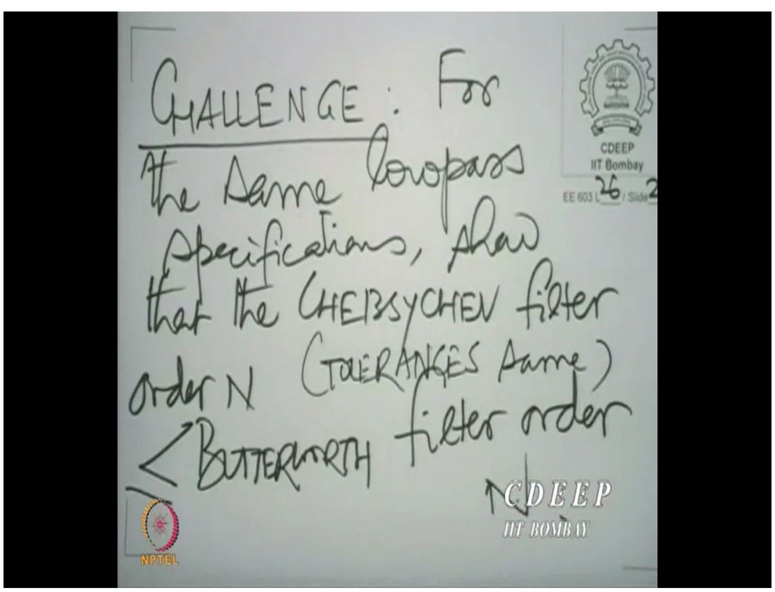

Now the challenge is, for the same lowpass specifications, show that the Chebyshev filter order is always less than equal to the Butterworth filter order. What I mean by the same low pass specifications, is only tolerance. Tolerance is same. On the passband stopband edges is the same. But of course, the edges are different.

That means you are asking for an equiripple passband or monotonic stopband and putting the same passband edge and stopband edge and the same tolerances. Then you ask for a monotonic passband and a monotonic stopband with the same passband edge, same stopband edge and same tolerances. And you find the order in both these cases. And the challenge is to show that the Chebyshev order will always be less than or equal to the Butterworth order.

So, it is very interesting. There is a reason therefore, why we would choose one nature or the other. But I told you that filter design is essentially a game of approximating the ideal. And approximations always involve compromises, nothing comes for free. So, it might seem that when you choose an equiripple passband, you should always be doing better, why at all should you use a monotonic passband if the order is more? The answer lies in the phase response, what you brushed under the carpet.

So, a phase response on the Chebyshev filter happens to be worse in many ways. And worse in what way? Worse in the sense of group delay and phase delay. You would like the phase delay and the group delay to be as constant as possible over the passband, at least over the passband anyway. And that poses a problem.

So, in fact, it would be interesting, it is not easy to prove this, it is a little involved. But one would, one should, the designs that one would do in this course, you would actually go through a design for the Butterworth or the Chebyshev filter, it would be definitely worthwhile to plot the phase response for, in fact, in the class, different people would perhaps look at Butterworth designs and Chebyshev designs. And therefore, it would be worthwhile to plot the phase response in different cases and compare the phase responses.

It would also be worthwhile to take the same specifications and compare the orders that one is getting for the Butterworth filter and the Chebyshev filter to verify the truth of what one has been saying here in this challenge. So, that would be an interesting thing to do.

Anyway, so much so then for. Yes, please, there is a question.

Student: When we consider the bandpass filter, we have defined Ω_0 and the bandwidth with respect to the bandpass filter, Ω_{p1} and Ω_{p2}. Why do we not do the same when ...

Professor: So, that is an interesting question.

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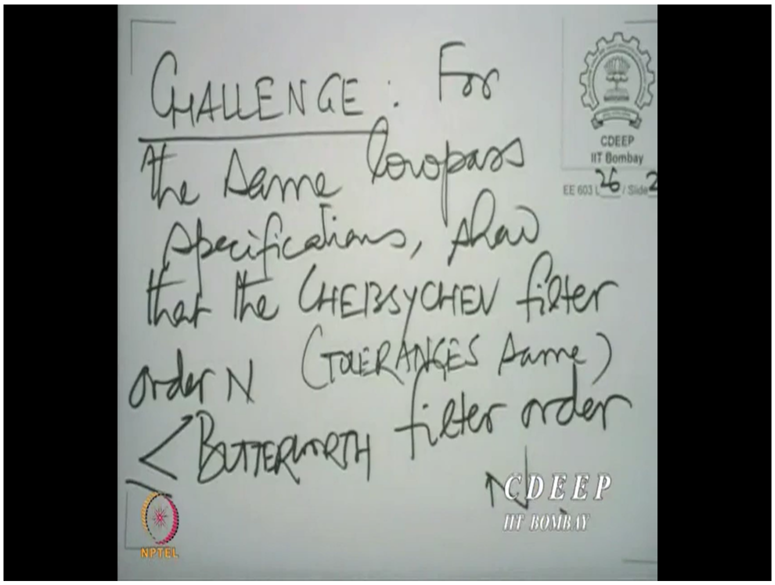

In fact, maybe there, the question is - when you took the band pass filter, you had used Ω, the passband edges to define the Ω_0, the center frequency, Ω_0^2. So, why can you not do the same thing for the bandstop filter. That is a very good question, in fact, maybe that is just an answer to a challenge that I posed.

Anyway, I would certainly urge you to reflect on what has been suggested. The suggestion is that, instead of using Ω_{p1}, Ω_{p2} geometric mean as Ω_0, could you use the geometric mean of Ω_{s1} and Ω_{s2} as Ω_0? How would the whole design change? And that, perhaps, is a very interesting alternative to consider.

Anyway, now we are well equipped then, to design bandstop filters as well. And in fact, now let us summarize the process of any kind of discrete time filter design.

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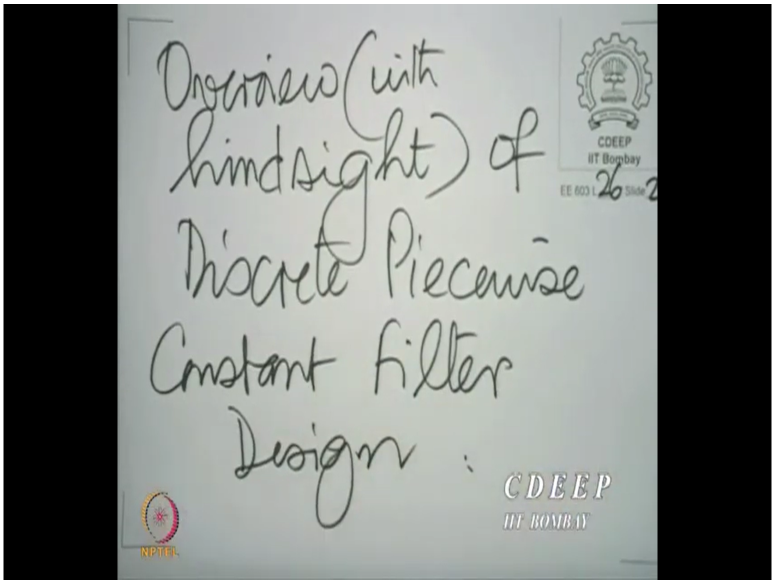

The overview with hindsight, as they say, hindsight means having now traversed the territory of discrete filter design, discrete piecewise constant filter design.

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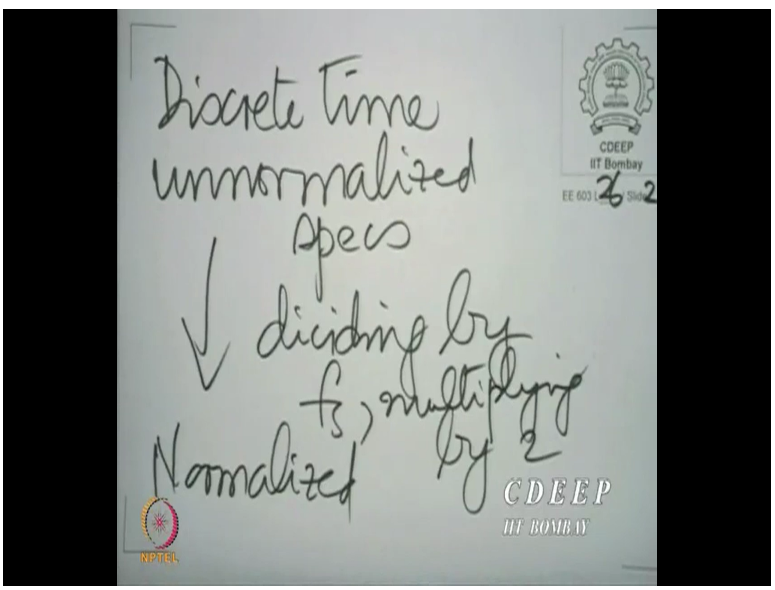

Of course, the first step is discrete time unnormalized specifications. This is what is given to you. This must first be transformed into normalized specifications by dividing by the sampling frequency, dividing by f_s and multiplying by 2π, you get the normalized specs.

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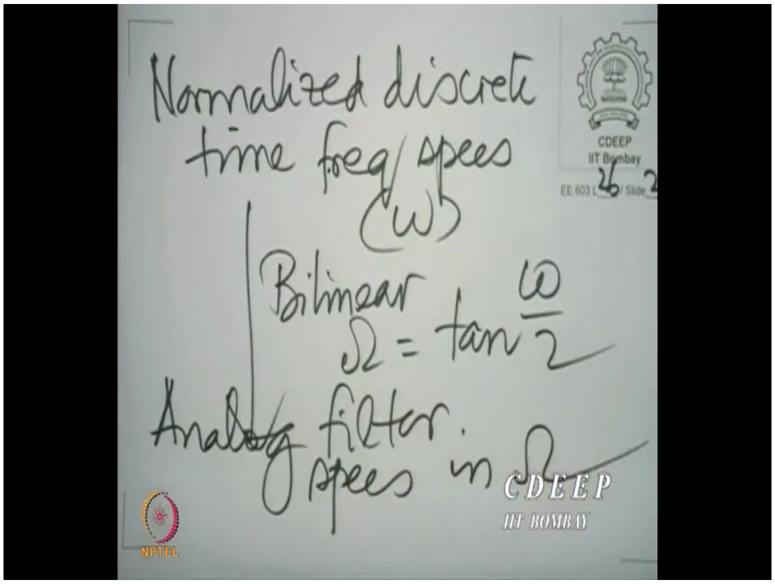

The next step is to go from the normalized specifications to the corresponding analog filter specifications. So, normalized, discrete time specs, ω that is.

We used the bilinear transform, bilinear frequency transform, Ω is $tan\frac{\omega}{2}$ and that gives you the analog filter specifications of appropriate nature in Ω. Of course, here the analog filter has the same nature. So, high pass goes to high pass, bandpass to bandpass and bandstop to bandstop and of course, lowpass to lowpass.

The next step is to use the frequency transformation.

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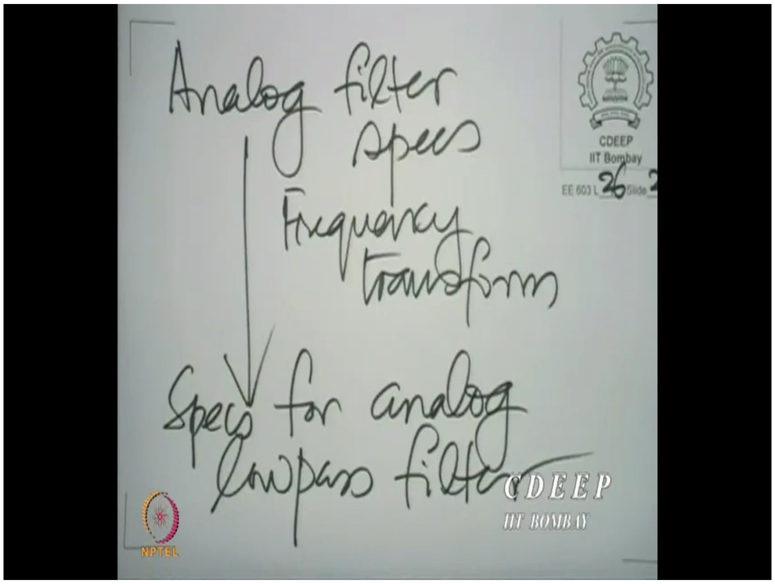

The analog filter specs use the frequency transform to design to get the specs from the corresponding analog lowpass filter. So, you know how to do that for the bandpass filter and the bandstop filter. In our method of design, we have put the passband edge at *1* and the stopband edge is chosen to be the most stringent that emerges from the frequency transformation.

We have of course put the more stringent of the tolerances on the passband and the stopband and the nature is carried as it is. We can design that analog lowpass filter.

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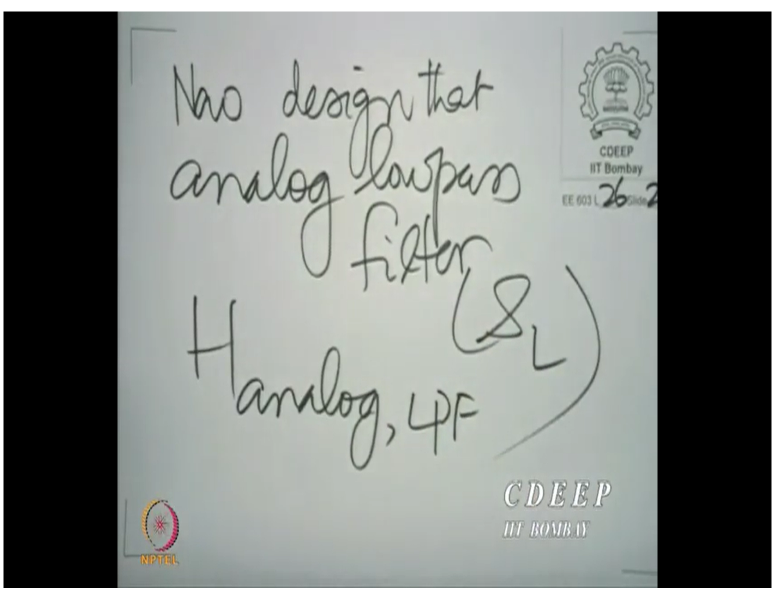

Let us call it $H_{analog,LPF}(s_L)$. s_L is the complex frequency variable in the analog lowpass. And of course, now we know what to do. In fact, we have two routes that we can follow.

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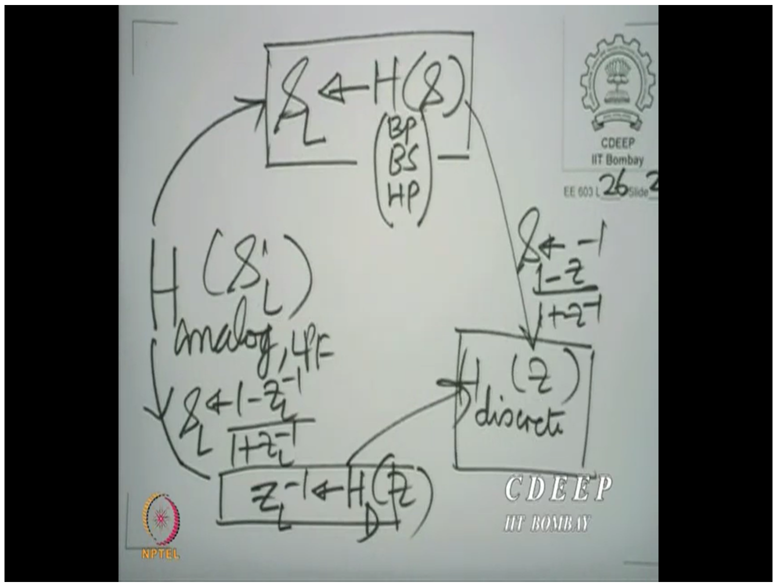

So, we have $H_{analog,LPF}(s_L)$. We can either make $s_L \leftarrow H(s)$, those could be either bandpass, bandstop or highpass, whichever one we desire. So, we make a frequency transformation in the analog domain.

And then we replace s by $\frac{1-z^{-1}}{1+z^{-1}}$, the bilinear transform and we get the discrete edge as a function of z. This is one route that we can follow. And that is the route that we have discussed so far. But you see, there is another route. You could, in principle, replace s_L by $\frac{1-z_L^{-1}}{1+z_L^{-1}}$. And get something in between.

So, make a bilinear transformation first. And then, replace z_L^{-1} by some $H_{discrete}(z)$ and come there. You could also do that in principle. That means for each of these three kinds of filters, the high pass, the bandpass and the bandstop, you could conceive of a discrete transformation, taking you from z_L^{-1} to z or z^{-1}, as you like, z^{-1} really.

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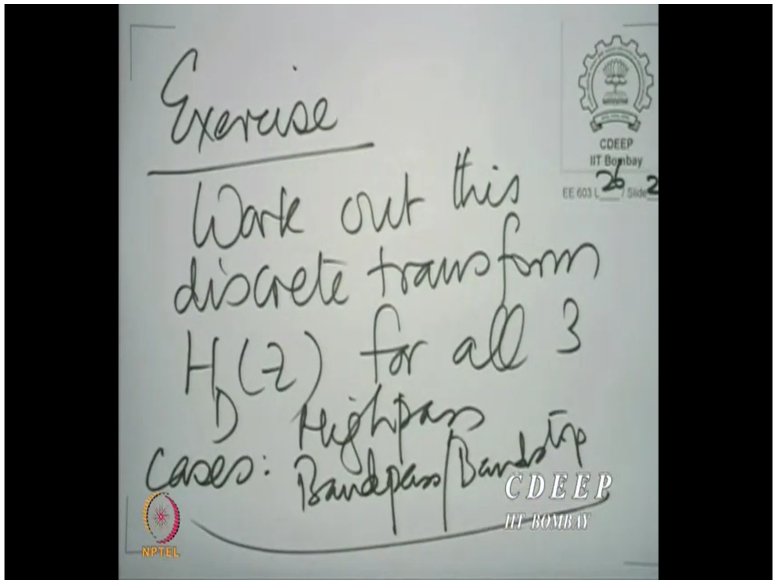

Now, the exercise for you is, work out this discrete transformation $H_D(z)$for all three cases, high pass, band pass and bandstop. And a corollary to that question is, observe carefully the nature of this transformation. Think of this transformation, of course, we expect that transformation to be rational, could not be irrational.

Now, if it is a rational transformation, you could think of it as a rational system in its own right. What property does that system have is a question I leave it to you as an exercise. With that then, we are now all set to carry out filter designs. We are in a position to design the filters that we have been assigned where the bandpass, bandstop or highpass, right from the discrete filter specifications, right up to the discrete system function.

Of course, the infinite impulse response filter can now be designed. We shall now therefore, actually we should now, individually proceed to design the filters that we have been assigned and from the next lecture we shall commence our discussion on the design of finite impulse response filters. So, with that now, we are well equipped and we all urge one another to design the infinite impulse response filters that we are required to. Thank you.