Digital Signal Processing and its Applications
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Lecture 23
Midsem Discussion

So, warm welcome to the 23rd lecture on the subject of Digital Signal Processing and its Applications. We used this lecture to discuss the solution to the mid semester examination that was conducted in this course. Of course, the detail solution is available on the course webpage; but it is useful to discuss the questions and their answers instructor solution and possible variance, more than from the point of view of evaluation, from the point of view of understanding some nicety, some finer points in the material that was studied. So, you know more than just looking at the evaluation aspect, which of course has been done routinely.

It is important to understand some finer ideas that were been communicated, through the questions that were asked in the mid semester examination. So, what I shall do in this lecture is to present the question first and then discuss the instructor's solution to it and then of course ask for any clarification if there are any questions, or clarifications that have to be made. Now, so you see the we are going to look at; now we look at the first question. I shall project the relevant question or I shall briefly explain the question; and then give its response.

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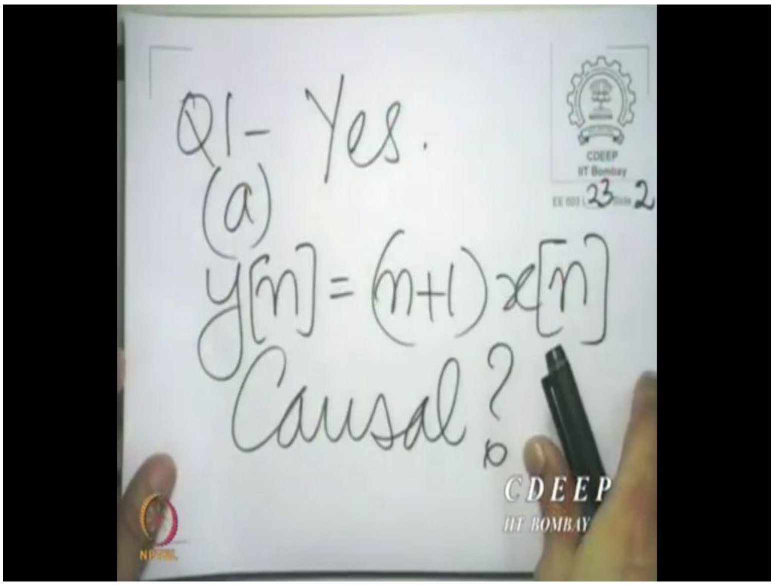

So, the first question I will just mention the main point, of course we note the question paper and instructor solution on the webpage; but will put the main points here. We do not intend to discuss every step of the solution; but we wish to discuss some important or salient points of the solution, which are important from the point of view of your concept. So, here we are go for the first question; here the first question was whether the system response. Now in all the questions in all the parts in the question number 1; we used x[n] to denote the input of the system, and y [n] to denote the output.

And therefore the question was does this describe a causal system? And of course the answer is yes. That is because although this is n+1 and that was the depicted part. This is n+1 that does not affect the causality; the causality only relates the fact that y of n depends on the current sample of x, and in no other. It does not matter what function you have performed on the index before you apply that to the sample; that does not affect the causality. So, the system was indeed causal.

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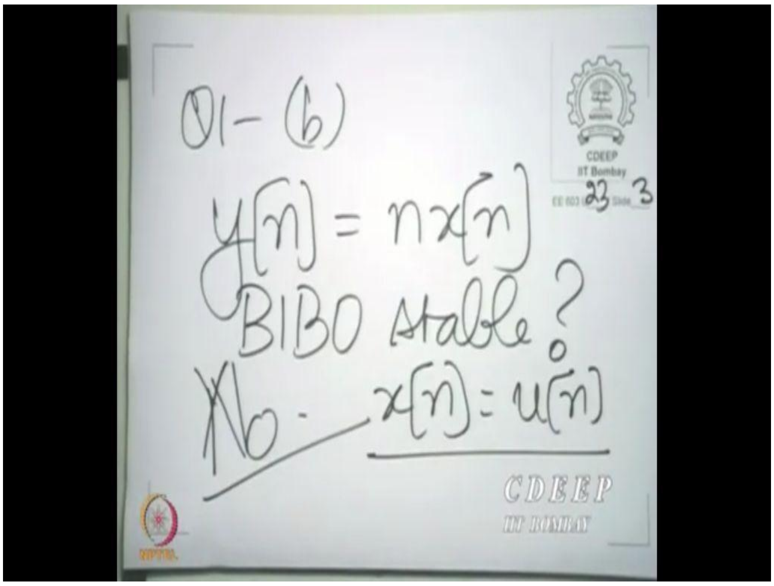

Question 1 part 2, part b asked whether y[n] = n x[n] is a bounded input bounded output stable system? And the answer is no. That's because you could give a counter example by taking y[n] = u[n], this is bounded. But, the output is clearly unbounded, so you have bounded input, you can have unbounded output; and that is how it serves as a counter example. If there are any questions please post them right then and there.

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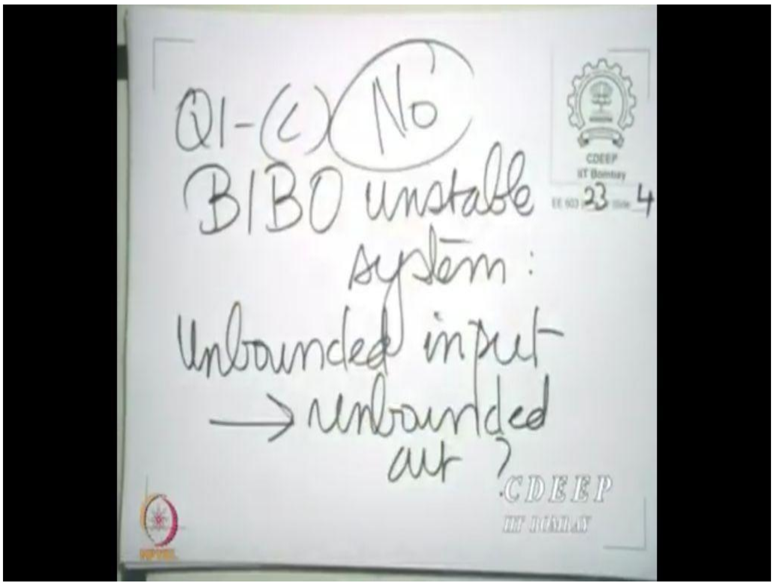

So, question 1 part c asked whether in a BIBO unstable system does a unbounded input necessarily produce a unbounded output; and the answer is no. You see in unstable system, you guarantee that there is at least one bounded input, which produces an unbounded output. But, you are not guaranteed either that every bounded input produces an unbounded output; or that an unbounded input necessarily produces an unbounded output. In fact, we can give a very simple counter example by exploiting the z transform.

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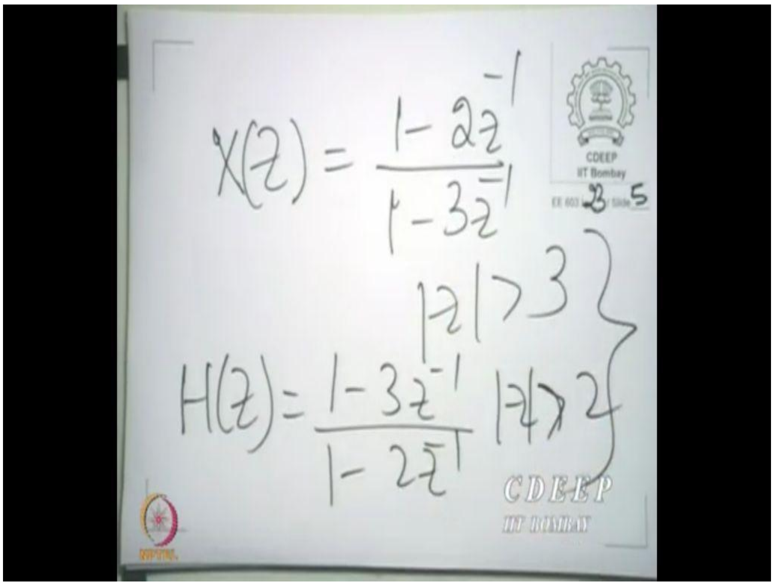

So, you see if you take the system, you know very if you take this system, if you take this input; 1 - 2z. And I am showing the z transform of the input; and if you consider this system.

The system with system function H(z) = $\frac{1-3z^{-1}}{1-2z^{-1}}$|Z| > 2. Clearly, this input is unbounded. the system is unstable.

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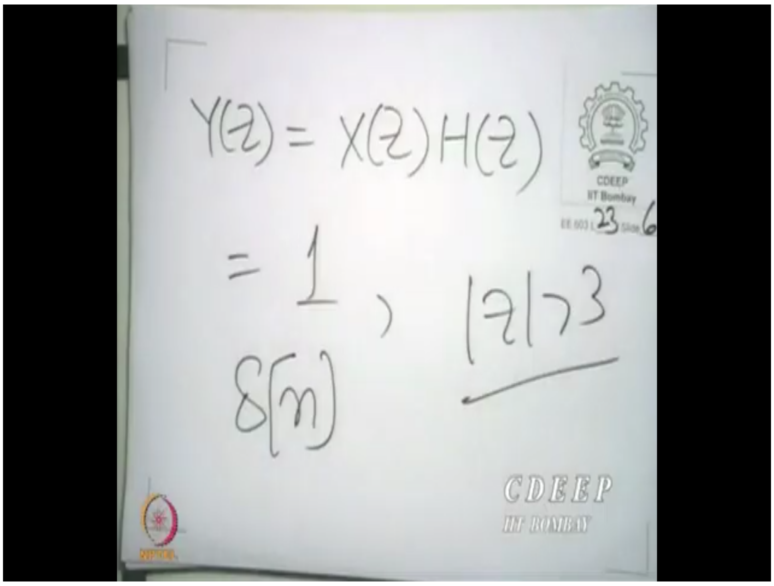

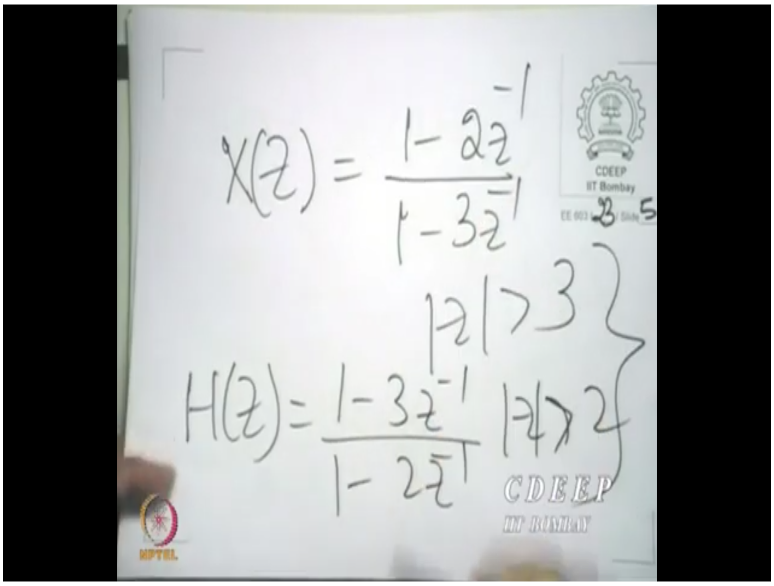

But, when you find the output, it is essentially your impulse; and of course you do have a valid region of convergence. So, you have at least |Z| > 3 as a region of convergence and therefore, if you take the inverse z transform of this X(z) as the input and the inverse; and this as the system, which is clearly unstable. Then this input applied, this system is clearly de-bound stable; this input is unbounded. That is clear because if you look at this is essentially an exponential with a factor of 3. The output is however then to be an impulse.

Now, this is of course just 1 counter example; you are welcome to give any other counter example. But, in general we are not guaranteed, it shows it cannot guarantee tricky thing; that is very tricky. You are not guaranteed the BIBO unstable system gives an unbounded output when the input is unbounded, yes.

Student: (()) (06:39)

Professor: So, what is the region of convergence? At least |Z| > 3. So, question is why why we are taking |Z| > 3? So, the answer is that you see the region of convergence is at least the intersection, if not more. So, here we have shown that there is some part in the region of convergence; |Z| > 3 is the minimum part. Of course The region of convergence may expand, in this case it does.

In this case in fact the region of convergence goes all over the z plane. But, we are sure that there is a region of convergence to begin with; so maybe what we should say is |Z| > 3 at least is the region of convergence. So, we have something to begin with. Alright.

Now, question 1 part 4, so the question is in the previous system where you have y[n] is n times x[n]. could you have that given a bounded input which produced the bounded output? Yes indeed. So, for example if you give x [n] = δ[n] the unit impulse; it produces a bounded output ; in fact this is all 0. So, in unstable system it is not necessary that every bounded input produces an unbounded output; but, there is at least one bounded input which produces an unbounded output, this illustrates the idea.

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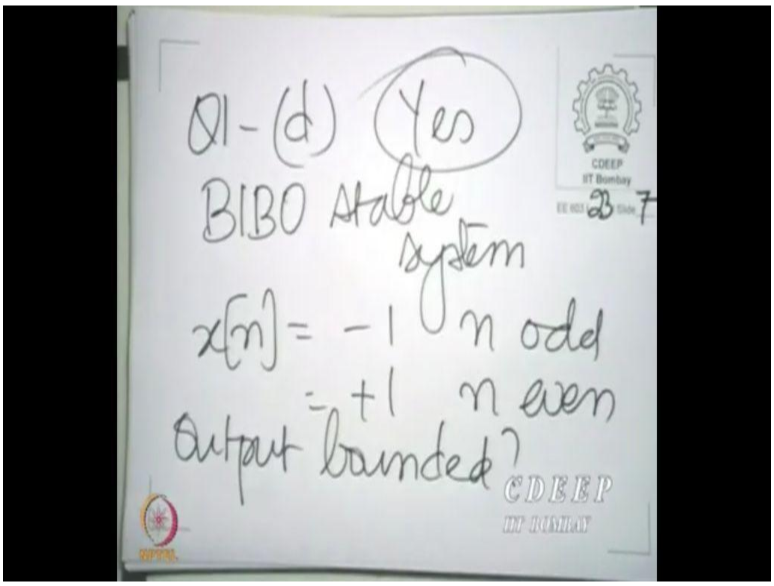

So, question 1d was if you give the BIBO stable system, the input x[n] = -1, for n odd; and 1 for even n is the output bounded and of course the answer is yes; that's because the BIBO stable system, the input is bounded by 1 and therefore the output must be bounded. So That is simple.

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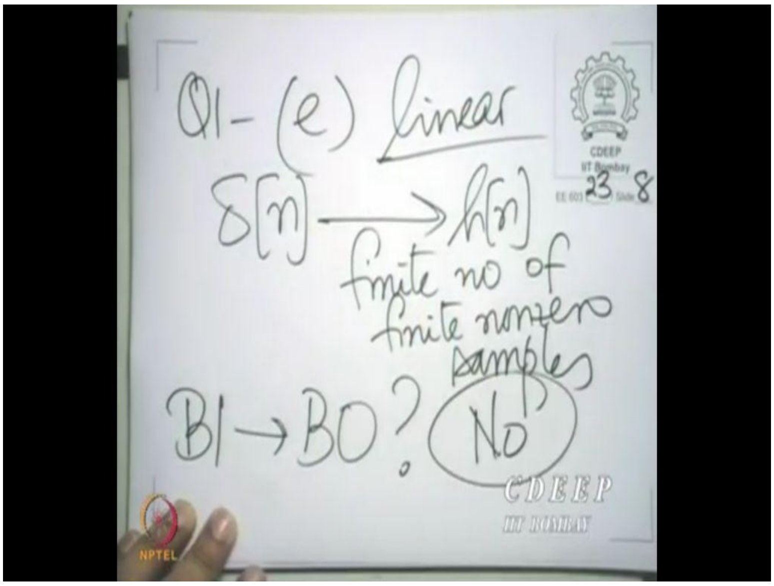

On the other hand, in question 1 part e, you are asked whether a linear system response to the standard. You have told that δ[n] produces some output h[n]; which has a finite number of finite samples, system is linear. Now, the statement was that if you give a bounded input where it produce a bounded output in general and the answer is not necessarily, that's because you are not told the system is shifting invariant. So, although the impulse located 0 produces a finite number of finite samples; the impulse located somewhere else might not and in fact, not just the impulse, some other combination might not.

And in particular for a linear system, you can use impulses located at various places to see what the system does. So, it's quite possible that the input located somewhere else, may not and then you have trouble; so it is not necessarily. So, the answer is no, it's not correct; So, much let us so for question 1. Now, are there any doubts or any difficulties about question 1 beyond this? Yes please.

Student: The only point is that the, this some other which shifted, shifted the impulse to be corresponds to this (()) (10:08).

Professor: Yes, so the question is question asked is in the part e of question 1 was the point that if the impulses were located elsewhere, they could be an unbounded output. The answer is yes; the system being linear; but not shifting invariant, could have a different response to an impulse located somewhere else; and that could be unbounded. Now, we come to question 2.

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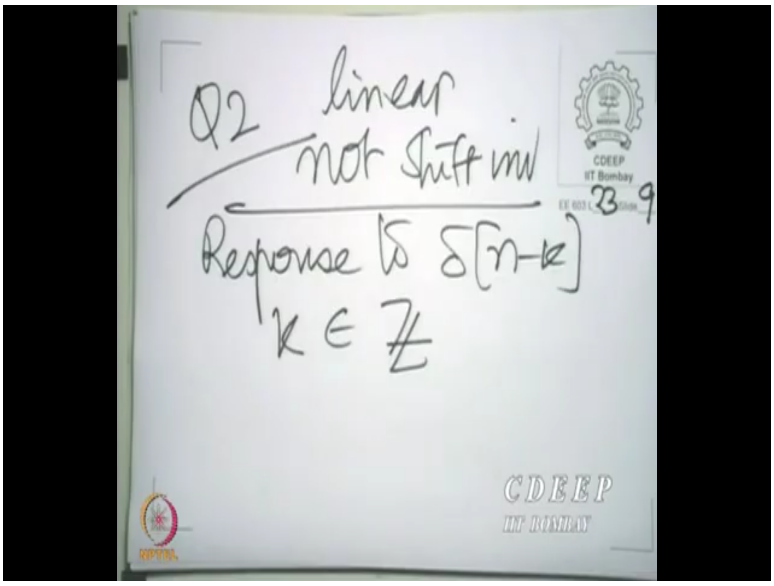

Here,You are given the response to δ[n-k] ; for k over the set of integers. You are told that for k < 0, the output is all 0. For k = 0 and 1, the output is given. So, now we will quickly put them down.

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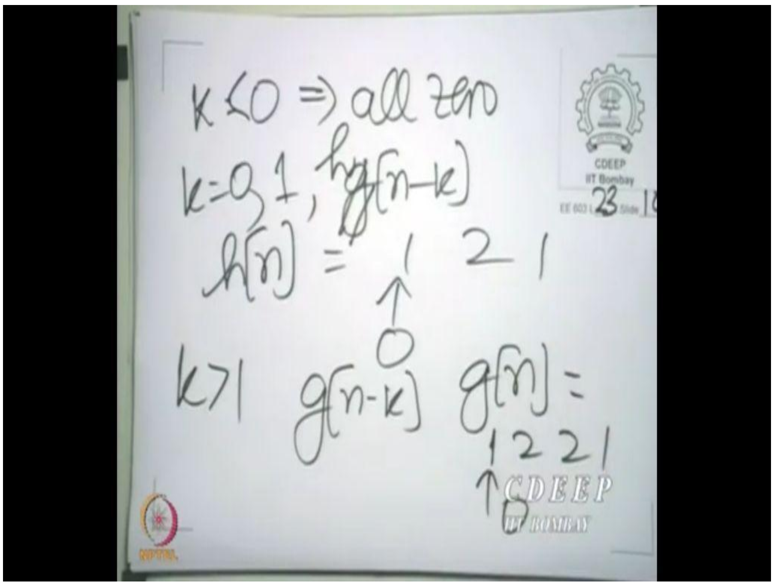

K < 0 all 0, k = 0,1 its g [n-k]; and g[n], I am sorry h [n-k] ; and h[n] is specified to be 1 2 1 with this 1 placed at 0. and for k > 1 its g[n-k] and g[n] is specified to be 1 2 2 1 with this 1 located at 0. So, the first question is to write down the output y[n] in terms of the input x[n] in general.

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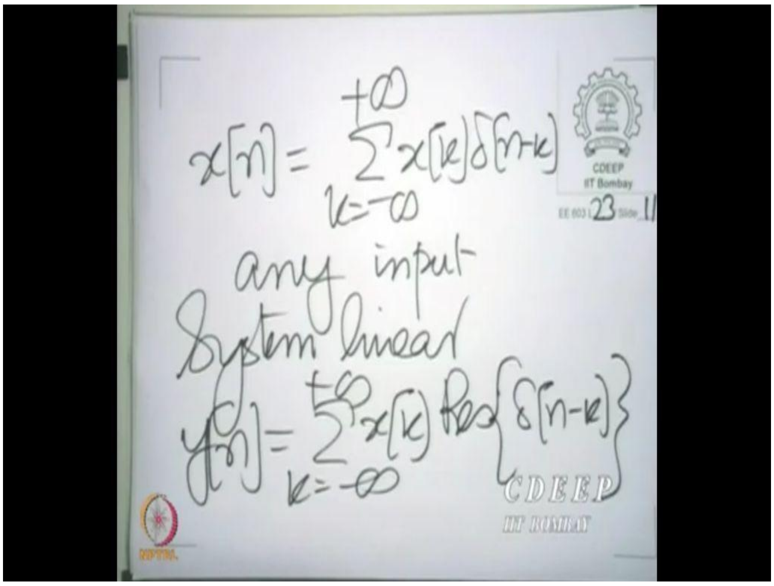

Now, you see when a system linear of course any input can be written in terms of the impulses. This is true for any input and this system is still linear. Then one thing is clear

$y[n] = \sum_{k=-\infty}^{+\infty}$ x[k] So, x[k] response to δ[n-k]; this is true because the system is linear. So, on account of additivity, you can take the response over each of these; and on account of homogeneity, you can take the response in into the x[k] and therefore we can now write down y [n] in terms of all the responses to different impulses; that is true for any linear system.

The only catch is that for a linear system which is non shift variant; this cannot be written in terms of a single h [n].

Now, this question was meant to test one's understanding of the proof that LSI systems have an impulse response that characterizes them completely. So, the system is linear but not shift invariant what is it that is required to characterize. So, one dilutes the proof so to speak to come to weaker requirement. So, it is meant, i used to stress in the class that its not enough simply to understand the proof; but, also appreciate all its variance and that means you one must learn to read between the lines in what it is, in one what is what one is doing in the classroom.

So, you see that was what was been tested in this question; and that is of course is going to be true in the future as well. Unless you learn to look beyond simply what is said obviously in the class; it will not enable you to understand the subject in depth. Now, one can work this out and one can show but in general.

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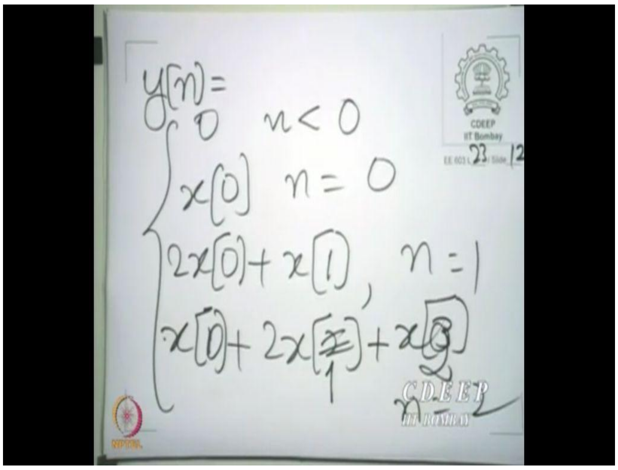

This is the response that you get $y[n] = 0$ for $n < 0$. $x[0]$ for $n = 0$, $2x[0] + x[1]$, for $n = 1$. So, you can expand this, you can expand this output and get; so I am just putting down the response. $X[1] + 2x[2] + x[3]$, for I am sorry; $X[1] + 2x[1] + x[2]$, for $n = 2$ and we can continue this.

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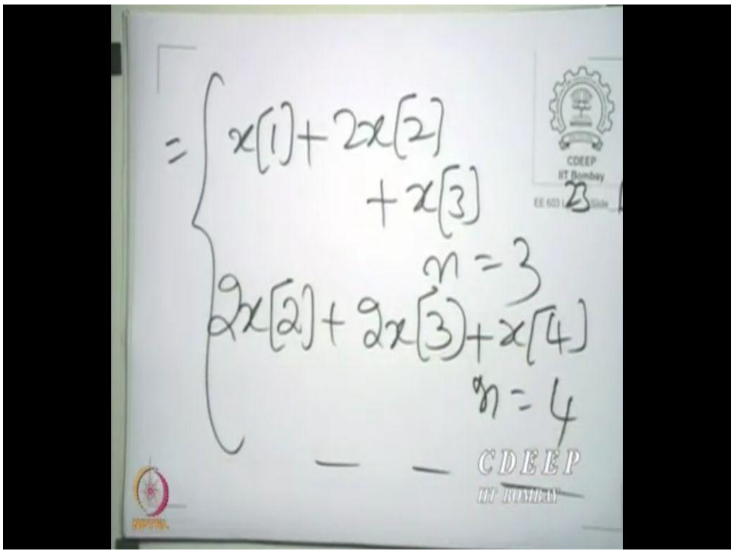

It is equal to x[1] + 2x[2] + x[3] , for n = 3; 2x[2] + 2x[3] + x[4]. So, you see it is clearly shift variant, the behavior changes for n = 4 and again for n = 5, we need to write a separate expression; and beyond that you can write a common expression.

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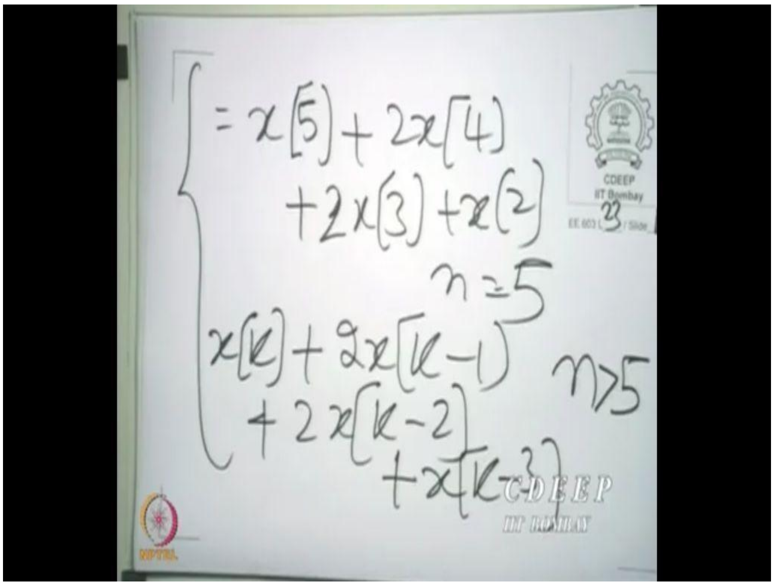

X[5] + 2x[4] + 2x[3] + x[2], for n=5 ; up to 5 we have to write separate expressions and beyond that one can write x[k] + 2x[k-1] + 2x[k-2] + x[k-3], for n > 5. Now, of course this expression also holds for 5 and you may therefore take it from 5 onwards.

Now, this follows by adding the individual responses to different impulses; so I will leave it to you to workout the details. But, the idea is that in such a linear shift variance system, you need to write down the output at every point in terms of the input samples and from here you can also see that for some samples, there could be a combination of a fewer number of input samples and for some more and this is what was also discussed in question 1; in one of the questions, where is the system was not shifting invariant.

So, it could be possible that for 1 impulse, impulse located at one place; you may have a finite length response and for another place, the response may be of infinite length; that can happen, if the system is not shifting invariant there. So, this is in general the expression. Now, of course one can: the next part of the question was to write down the output for a specific input.

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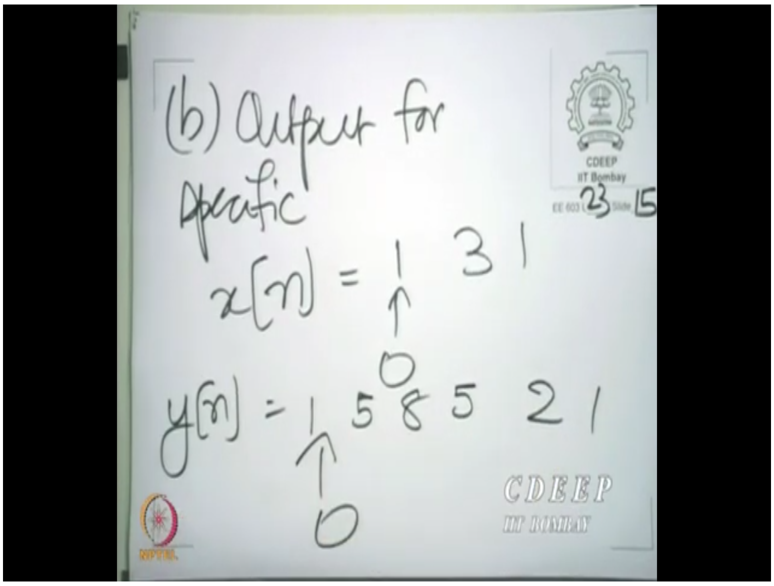

And the specific input was this and of course one can simply substitute and one can get the following expression. So, this is for specific input, if one just substitutes the values of x[0], x[1], x[2]; and puts them in the respective samples to get the output.

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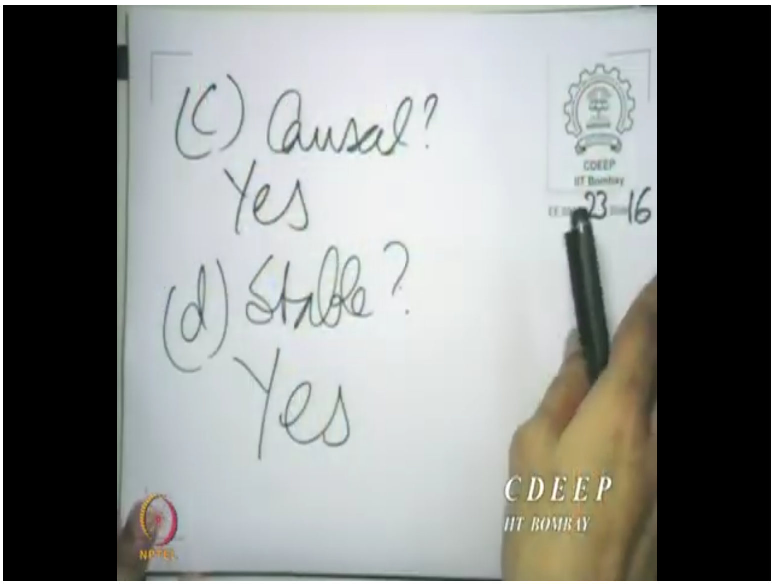

Now, question 2 part c was to answer whether the system was causal? And of course the answer is yes indeed. That is because if you look at the expression for y[n] in terms of x[n]; every sample involved either the current sample or past. Future samples were never involved and therefore one could see the system was causal.

The next part was to answer whether the system is stable? and the answer is again yes. Luckily in this particular system every output sample was a combination only of a finite number of input samples, or different at different places and if the input was bounded, then you would have a finite combination of bounded samples and therefore the output needed to be bounded everywhere, so the system was stable. Now, I is there any questions on this, then we can answer them before we proceed to question 3. Alright, Since there are no questions, we will proceed to question 3.

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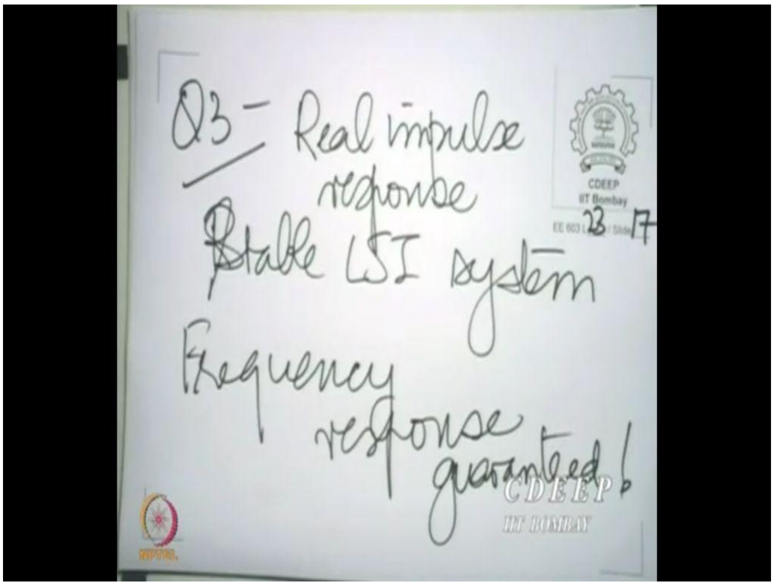

Now, in question 3 in question 3 the object there were 2 objectives and the question was as follows. Given a real impulse response rational LSI system, not necessarily rational actually I am sorry, stable LSI system. So, you have a real impulse response and you have a stable LSI system; so you are bound to have a frequency response. So, the frequency response is guaranteed to exist. Not only that, the frequency response is going to be magnitude symmetric and phase anti-symmetric. That's because the frequency response is conjugate symmetric; so the magnitude is symmetric and the phase is anti-symmetric.

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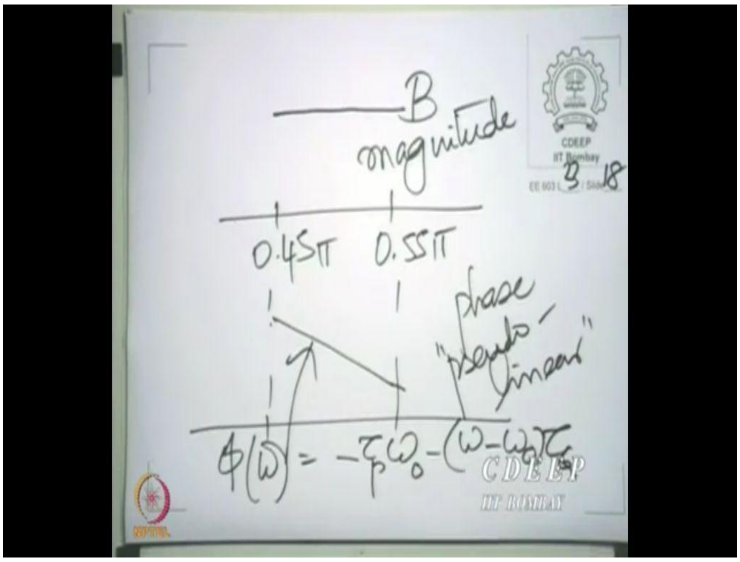

And you are given the magnitude in the range 0.45π, you see the magnitude between point. You are only given the magnitude between 0.45 and 0.55π and you are told the magnitude is constant at B, and now told that the phase is essentially a linear phase. If not one one call would not call it linear, actually, one should call it pseudo linear. Pseudo linear because it does have straight line expression, but it does not pass through the origin.

So, it is a phase like this, it is a straight line kind of function and the phase response was specified to be $-\tau_p\omega_0 - (\omega-\omega_0)\,\tau_G$; where, τ_P and τ_G are positive constants.

And ω_0 is called the omega frequency, ω_0 is also positive constant called the center frequency. Now, you were told to consider a combination of two sinusoids; in fact the input given was of this nature.

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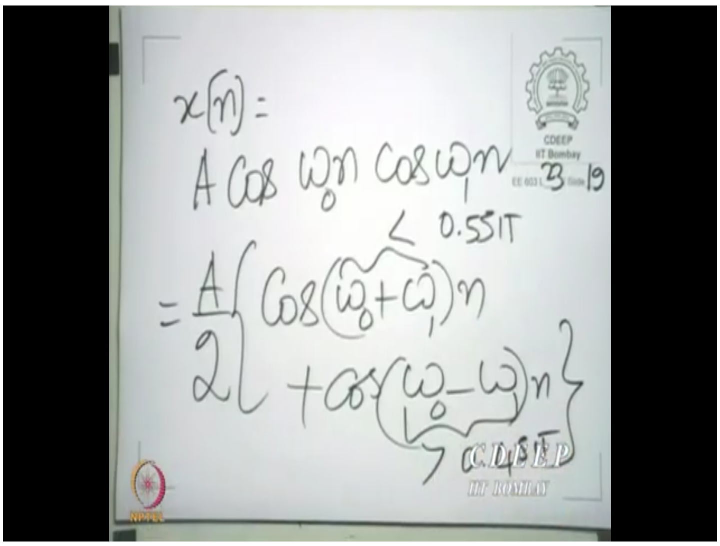

Now, product of sinusoids is always expressible as a sum of two sinusoids. So, its cos $(\omega_0 + \omega_1)n$ + cos $(\omega_0 - \omega_1)n$. So, product of two sinusoids which was given to be the input is actually the sum of two sinusoids as well; with frequencies $(\omega_0 + \omega_1)$ and $(\omega_0 - \omega_1)$.

Now, you are also told that $(\omega_0 + \omega_1)$ is strictly > 0.45π and you are told it is < 0.55π. What it really says is that these two sinusoids are well within the band of interest. So, you are in a position to use the frequency response to find out the output; and of course the system is linear shifting invariant.

The input is sum of the two sine waves, so you can find the output by using the frequency response; that is that is the idea here. Now, one can easily calculate the output in this case by using the frequency response.

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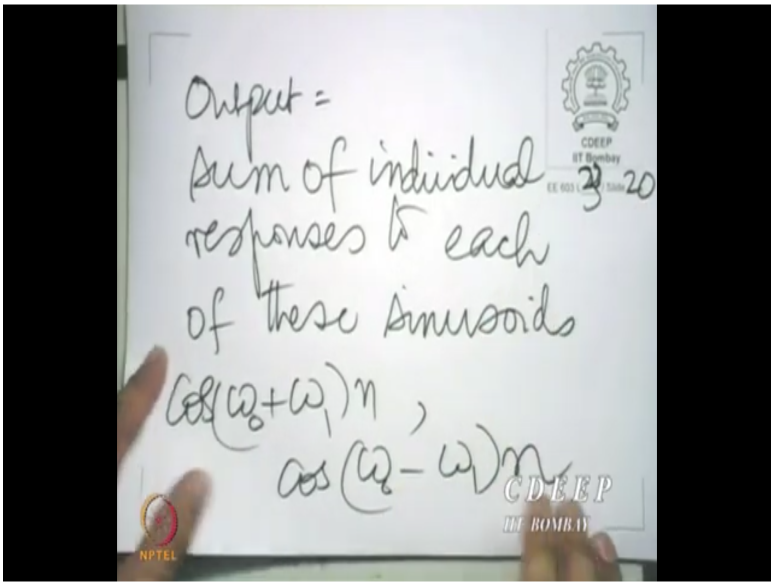

And we are we have here, the output is going to you see so essentially the input is sum of two sine waves. The output is the sum of individual responses to each of these sine waves. So, cos $(\omega_0 + \omega_1)n$ and cos $(\omega_0 - \omega_1)n$ you are guaranteed that they lie within the band, you reach the frequency response is specified. Now, I must again stress that here, you do not need to worry about the frequency response on the negative side of ω. Because, it is going to be conjugate symmetry; that is given the impulse response is real.

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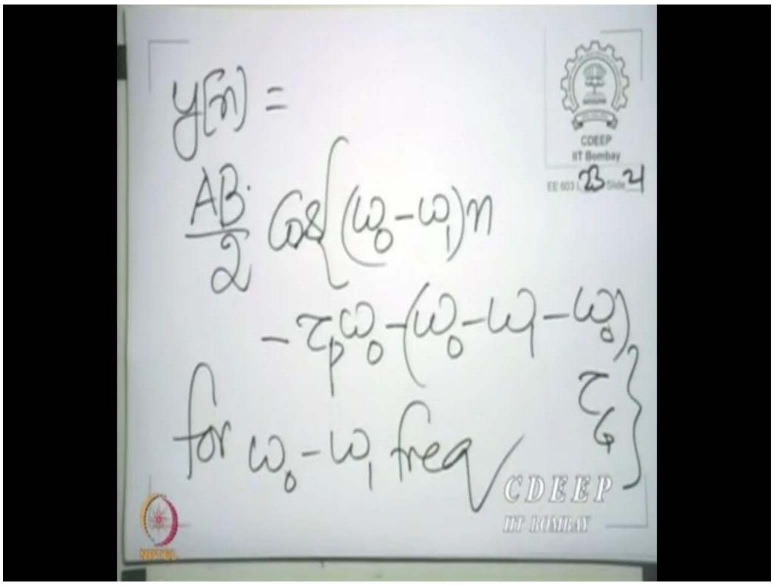

So, therefore the output turns out to be $\frac{AB}{2}$; So, you see $\cos(\omega_0 - \omega_1)n - \tau_p\,\omega_0$ you see. this is the change of magnitude and this is the change of phase. For the $(\omega_0 - \omega_1)$ frequency essentially a multiplying the magnitude by B; and we are adding the phase of as given $- \tau_p\,\omega_0 - (\omega_0 - \omega_1 - \omega_0)\tau_G$; so there we are.

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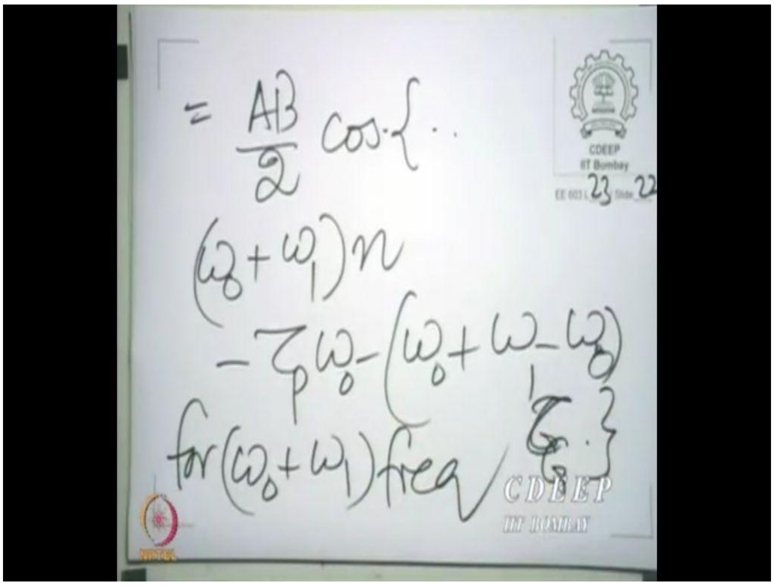

And for the, I mean it is equal to $\frac{AB}{2}\cos\{((\omega_0 + \omega_1)n - \tau_p\,\omega_0 - (\omega_0 + \omega_1 - \omega_0)\tau_{G\}}$. So, here we have again multiplied the magnitude by 2, and you have added a phase of- $\tau_p\,\omega_0$ -. Now, in place of ω you have put $(\omega_0 + \omega_1)$, so $(\omega_0 + \omega_1 - \omega_0)\tau_G$. So, you see when you add these two terms what you get is the following.

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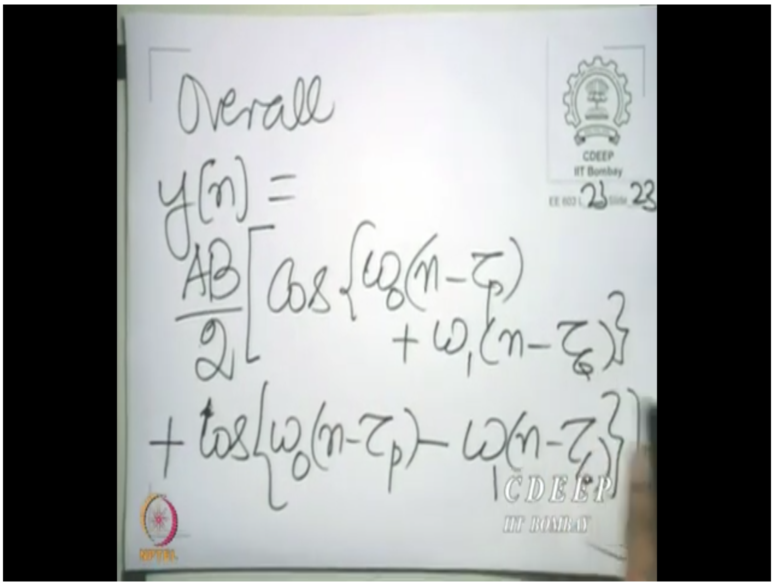

The output is $\frac{AB}{2}$ cos $\omega_0(n - \tau_p) + \omega_1(n - \tau_G)$. In fact, you can take all this in the bracket,cos $\omega_0(n - \tau_p) - \omega_1(n - \tau_G)$. So, again it's a sum of two sinusoids as expected; the catch is that you have a very interesting change of the time. There is a different change of time on ω_1 and on ω_0; that is very interesting. So, you can combine; now, you can put this back.

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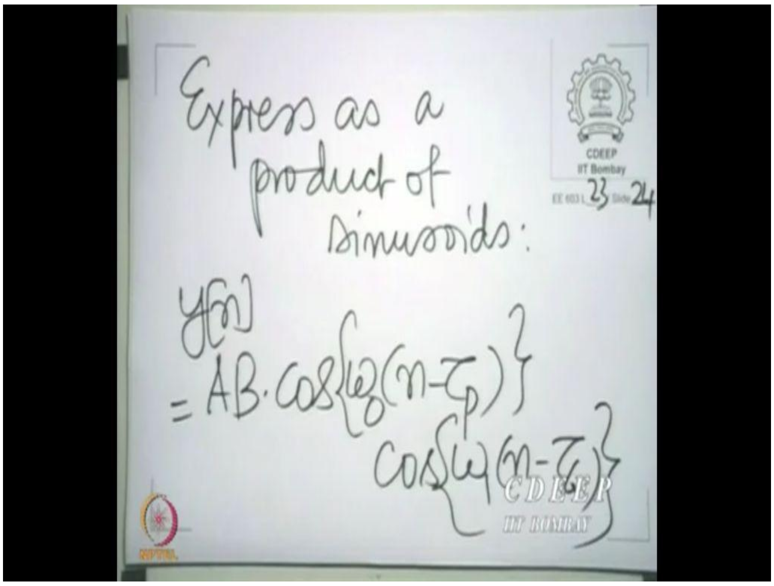

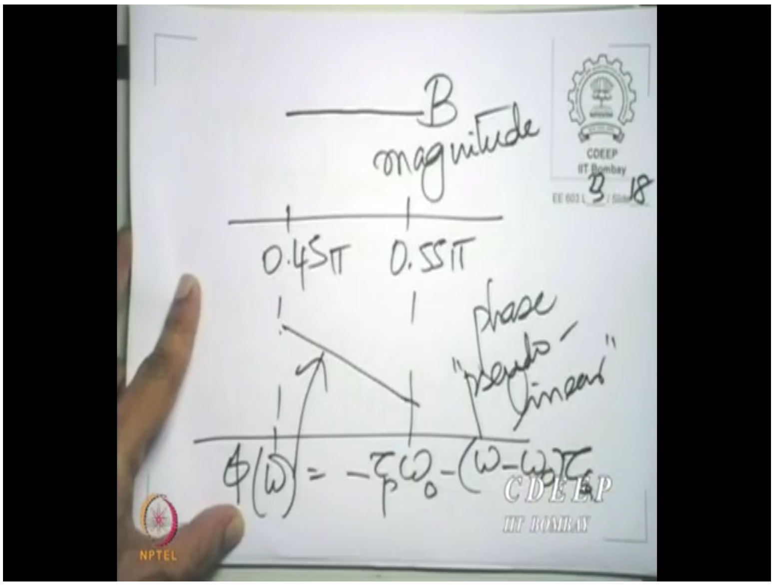

And actually it was asked to express as a product of sinusoids; and that would have of course given you AB $\cos\{\omega_0(n - \tau_p)\}$ $\cos\{\omega_1(n - \tau_G)\}$ Now, you need to interpret this; it is very interesting. We have product of two sinusoids initially of frequencies ω_0 and ω_1; and you know one is assuming here or even though it is not explicitly said in the question.

Let us take the situation where ω_0 is considerably larger than ω_1; so you have to take an example of ω_0 could be 0.5π. Here, this must be in the range, it has, because $\omega_0 - \omega_1$ and $\omega_0 + \omega_1$are both in the range 0.45 to 0.55π.

So, the ω_0 has no choice but to be inside that range, let us; it has taken to be 0.55π and let us take ω_1 to be let say 0.01π; so it is much smaller than ω_0. Now, this is like a situation where you have what is called amplitude modulation for communication.; For those who are familiar with it. In other, in any case you have a very low frequency sine wave multiplying a high frequency sine wave in that case.

In communication this is called amplitude modulation; So, you change the amplitude very slowly in accordance with the message and here the message can be taken to be of frequency ω_1 , and the carrier which carries the message is of frequency ω_0 and what we have done is to take a pseudo linear phase; if what I mean by a pseudo linear phase, you have taken the phase to be approximated by a straight line around the carrier.

Now, what you mean by approximating a phase by a straight line? You see if you have a very small band signal, a very narrow band signal around the carrier and if the phase is continuous, then the magnitude non-zero. If you are taking the signal to be very narrow band compared to the center of frequency of the carrier. So, in that narrow band if the frequency response is analytic at that point, then you make a Taylor series approximation.

And if the magnitude if, if you are not operating on what is called the transition band region; that's important. You should not be operating on a transition band. If you are operating somewhere in the passband, you can assume that the magnitude response does not change too much around the carrier, if its narrow band and the phase response even though it may change; if it is analytic in that region, you may approximate it by a Taylor series with only the first term.

That is what you are essentially saying. So, take a Taylor series approximation of the phase with only the first term; that's what we have done here. Now, what it shows is that in that circumstance, they there are 2 two delays. You see

The first term of a Taylor series, how do you how do you construct the first term of the Taylor series?

The value of the function. So, you go back to the expression that we wrote for the phase right in the beginning. See, in the expression of the phase has a constant term here, and a term which varies ω. Your your all took your a Taylor series expansion around $\omega = \omega_0$; so, this is the first term of the Taylor series. So, this is essentially the derivative of the phase; and this is the 0th term or the constant term in the Taylor series; this is essentially the value of the function at that point.

So, when you take you can always divide that value by ω_0, and called that τ_p. So, the phase $\tau_p\omega_0$ divided by ω_0 is called the phase delay and the derivative of the phase at $\omega = \omega_0$ is called the group delay and the significance of the group delay is that it acts on the message, and the significance of the phase delay is that it acts on the carrier. That's for what we need to demonstrate by this question here.

Now, here it demonstrated exactly when you took a message which was exactly a sinusoid.; A very narrow band message with a sinusoid. But, you could have of course extend this idea is the message had combination of sinusoids but narrow bands. So, the again is the question was to illustrate, so through a very closed process, to a through a very direct process; the meaning of the group delay and phase delay, when you have a phase response.

We had been saying at some stage the phase response is necessary way; and what this illustrates is the precise effect the phase response has, when you have a narrow band message operating on a high frequency carrier. Are there any questions on this? So, there are none we can then take the next question, question 3 b.

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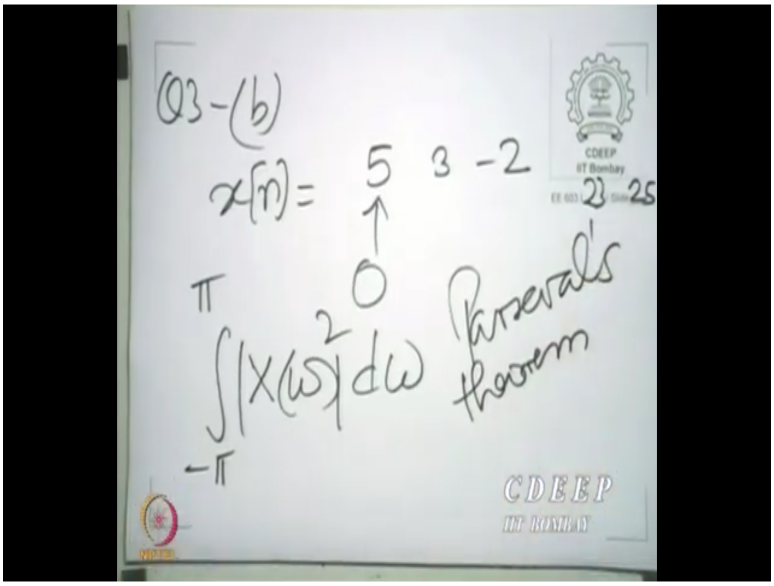

Now, question 3 b give you the sequence x[n]; this is the sequence was given and you are required to evaluate $\int_{-\pi}^{\pi} |X(\omega)|^2 \, d\omega$ of course This is easy to do; one can use the Parseval's theorem for doing it.

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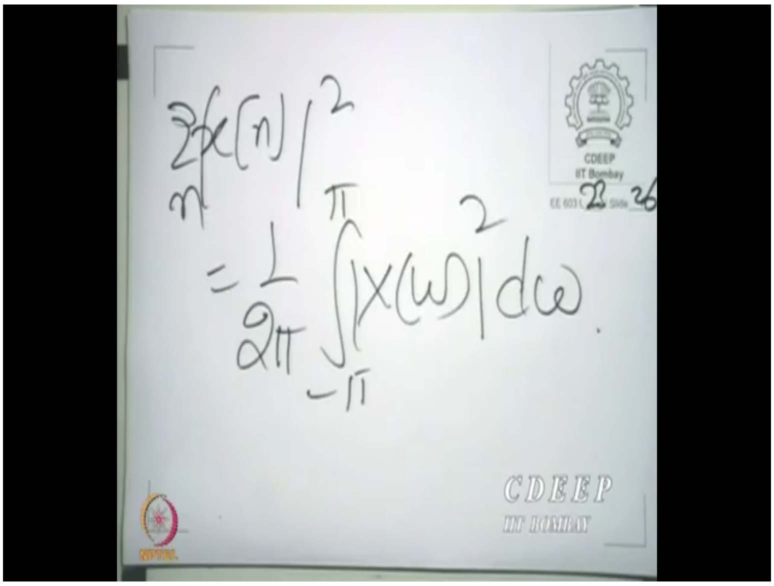

Because, you know that $|x[n]|^2$ some there were all n is $\frac{1}{2\pi}\int_{-\pi}^{\pi}|X(\omega)|^2\,d\omega$. So, the idea was that without without evaluating the discrete time Fourier transform; you are required to find this quantity. Which is easy to do because you can take πto the other side, we are done.

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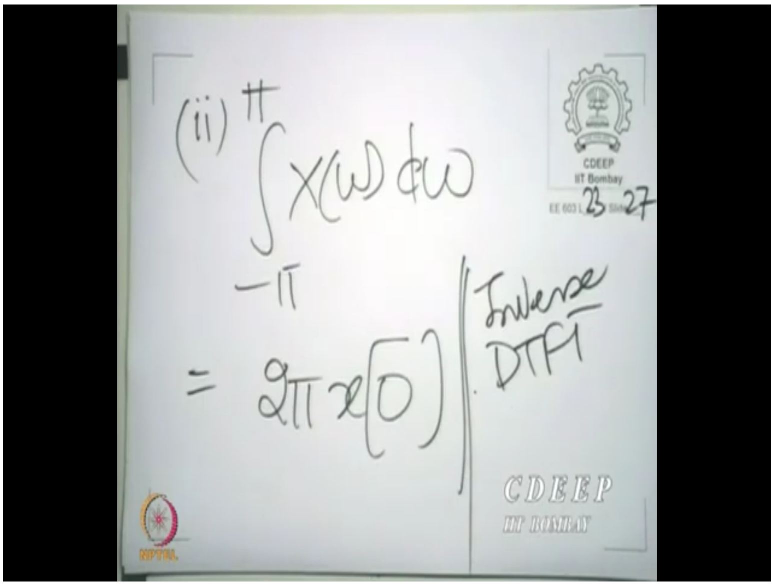

Similarly, you are asked to evaluate the quantity $\int_{-\pi}^{\pi} X(\omega)\,d\omega$. and that is easily seen to be essentially $2\pi x[0]$; that's easy to evaluate, by using the inverse discrete time Fourier transform. So, essentially this is inverse discrete time Fourier transform evaluated at n = 0, and multiplied by 2 π. So, that is easy to do; once you have the samples of the sequence, you have nothing more to do; it is in fact given to you. So, any doubts from this question, question 3 part b? So, I leave off course the calculations to you. Now, the last question 4 was a very interesting question.

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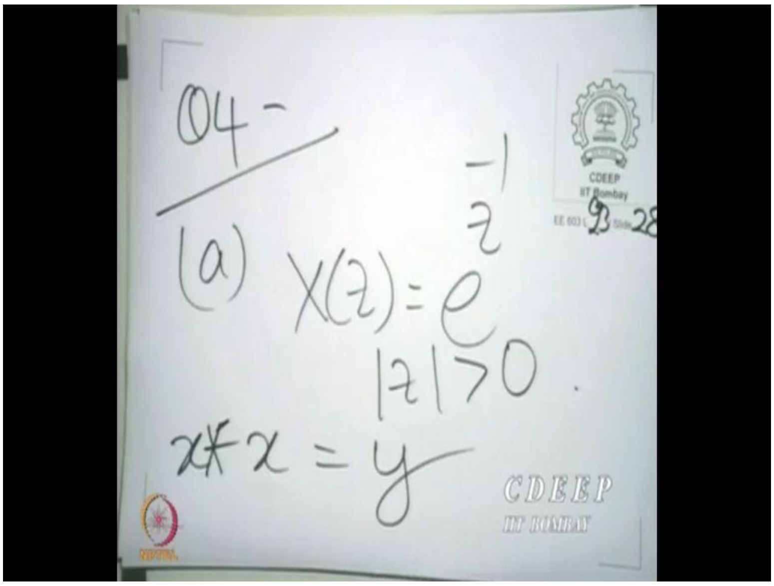

In part a, you are given X(z) = $e^{z^{-1}}$ |z| > 0 and wheater is that x(n) was convolved with itself to give y. So, of course you are asked to find out y as a product of x and another sequence j. So, you were told that y which is the convolution of x with itself should be written as the product of x also with another sequence G.

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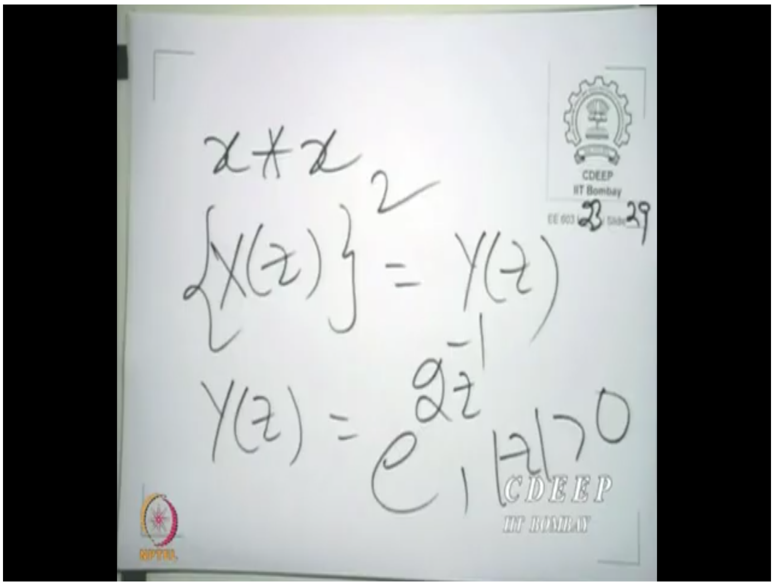

And that is easy to do because when you convolve, when you convolve the sequence x with itself with z transform is multiplied. So, you get Y(z) ; and therefore Y(z) is simply $e^{2z^{-1}}$, with the same region of convergence;|z| > 0 and just as you can expande$^{z^{-1}}$, you can also expande$^{2z^{-1}}$; that is very easy to do.

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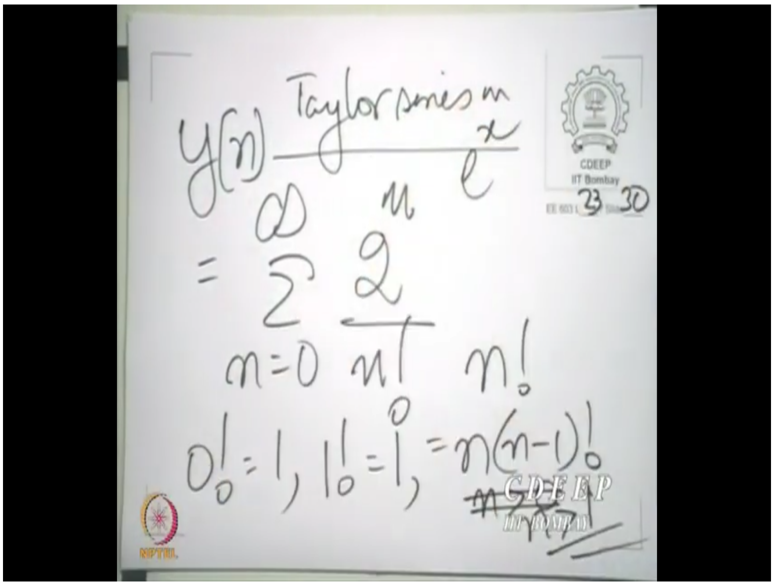

And that is gives you y[n] is essentially $\sum_{n=0}^{\infty} \frac{2^n}{n!}$ where 0! = 1, 1! = 1 and n! = n(n-1)! for n >1

Now, this is from the Taylor series expansion, the Taylor series expansion on e^x. We discussed this in the class, we discussed this as an example of irrational z transform; and we essentially need to use the Taylor series expansion. And interestingly this expression here.

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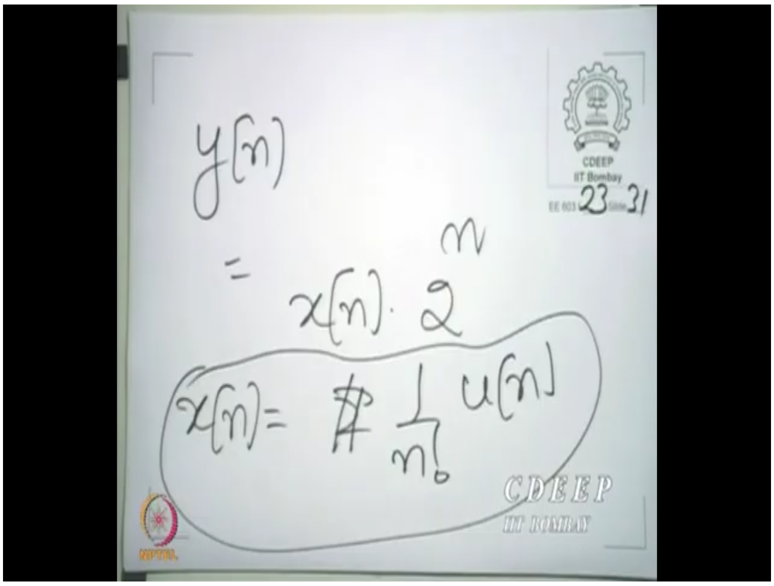

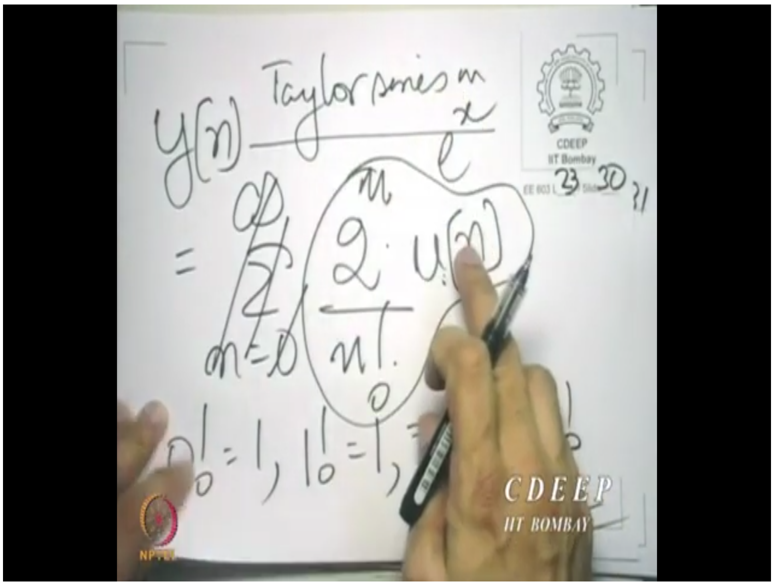

is clearly x[n] 2^n. Now, you know x [n] itself is of course I mean I must make correction here is the not summation n going. You see, this is $\frac{2^n}{n!}$u[n]; that's how you must write, the correction here. Because you are not writing the z transform, you are writing the expression sample by sample. So, y[n] = $\frac{2^n}{n!}$u[n]; where the factorial is defined in this way and what I wrote initially was the z transform y z; and there I showed it is 2^{-n}.

But, nevertheless if I remove the 2^n here, what I have left? Namely, $\frac{1}{n!}$u[n]. So, $\frac{1}{n!}$u[n] was x [n]; the original x[n] and so you can see that y[n] = x [n] 2^n; you can equally able to write as y[n] is 2^n u[n] x[n]. Now, therefore the sequence g[n] can be specified for $n \geq 0$; for $n < 0$, the sequence g[n] can be arbitrary.

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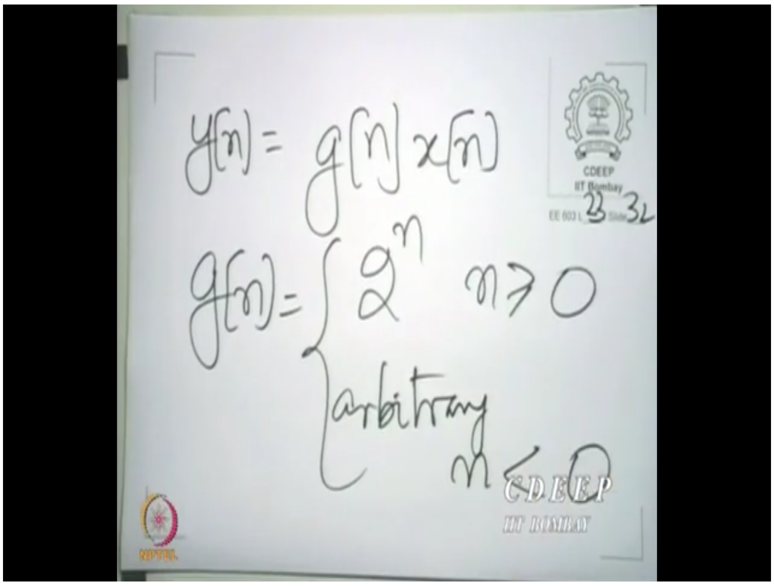

So, indeed y[n] = g[n] x[n]; where g[n] is clearly specified to be 2^n, for $n \geq 0$; an arbitrary for $n < 0$. You can of course extend it to 2^n and also for $n < 0$; it doesn't matter.

But, of course one would give full credit if one has written 2^n ; any acceptable sequence is fine and the simplest thing that comes to mind is either 2^n itself, or 2^n u[n]. either of them is what comes to mind immediately and both of them are acceptable.

Now, any any doubts about part a of question 4? If not then we go to the very last of the questions of mid semester exams and that was question 4 part b.

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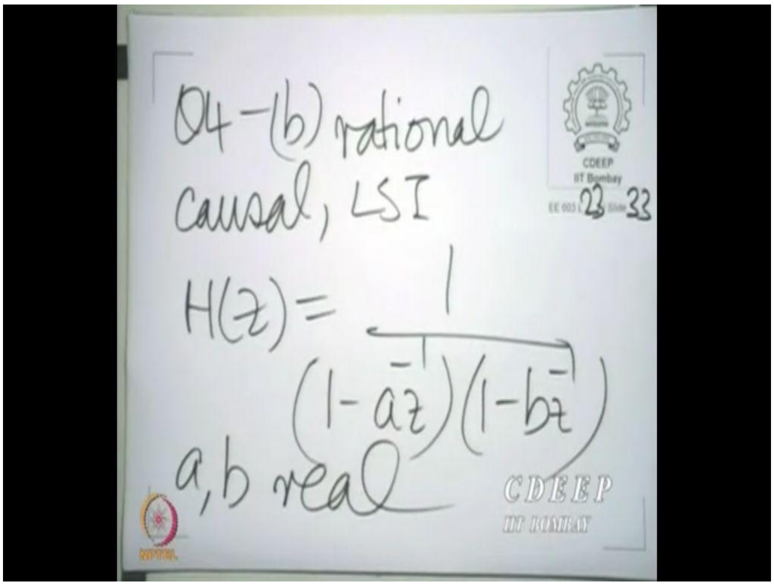

And there you are told that there is a system given to you; it is a rational causal LSI system and the system function was given to you H(z) = 1$\frac{1}{(1-az^{-1})(1-bz^{-1})}$; and a and b are real constants. You do not know whether a and b are equal or not equal, but you know that they are real.

You do not know whether less than 1 or greater than 1 nothing else. But Of course, you know the system is rational, of course that is obvious; but it is also causal and LSI and LSI and rational have to be adjusted; otherwise there is no system function. But, you know the system is causal. So, you know that the region of convergence is very clear; mod z greater than the greater of mod a and mod b.

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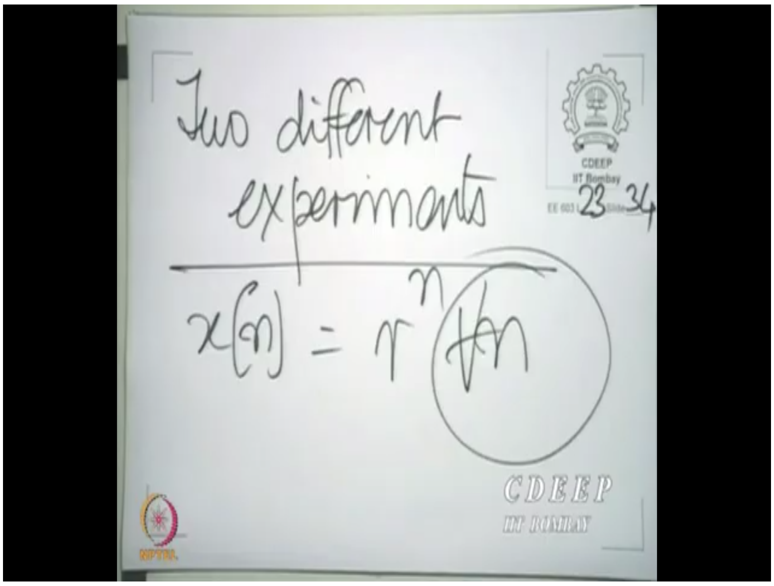

Now, you have given a response in two different experiments, so, two different experiments were conducted. Each of the experiments the input was of the form r^n, for all n; now please note, this r^n for all n. Not r^n u [n], there is a very big difference. r^n u [n] gives the difference response and r^n, for all n gives in fact r^n and of course you know it is obvious that if you got a finite output here will see in a minute; if you got a finite output; r must have been in the region of convergence, otherwise it could not have let see.

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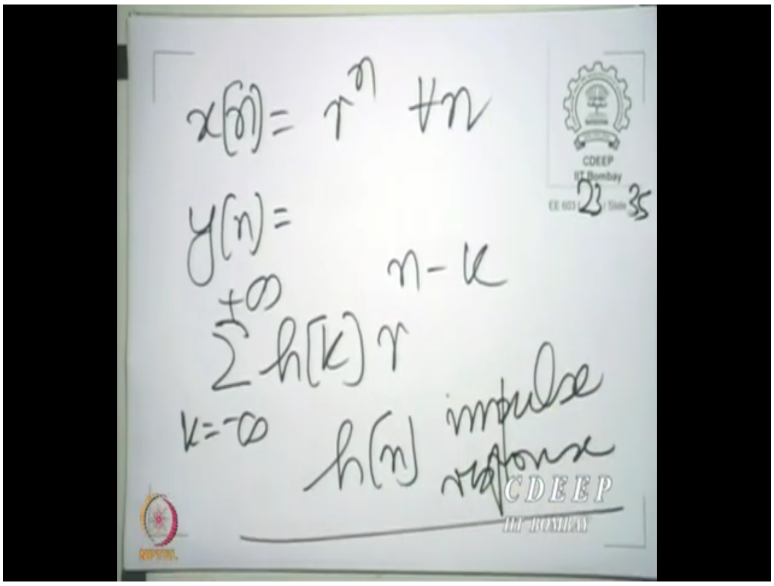

So, what is the output when the input is r^n for all n? In fact, we saw this. The very moment we began our discussion on the z transform. The output would be $\sum_{k=-\infty}^{+\infty} h[k]\, r^{n-k}$ where h[n] is the impulse response and we can split this r^{n-k} is r^n to r^{n-k}; that can be written as.

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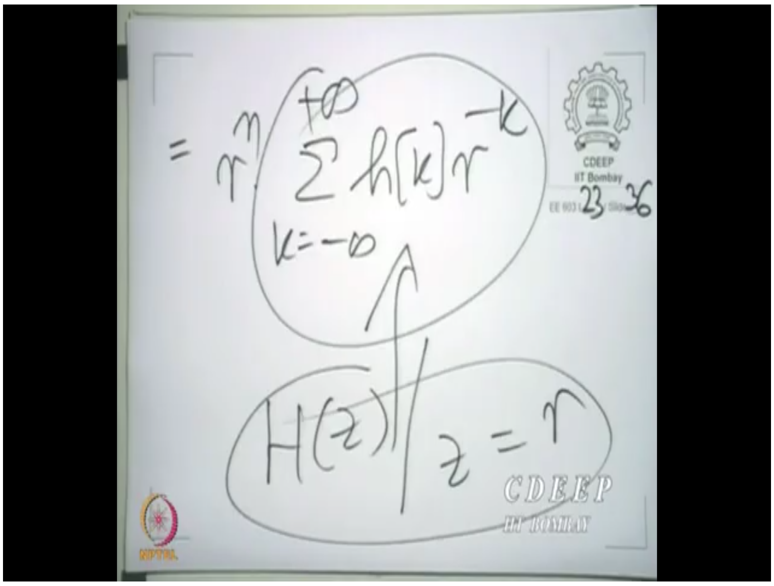

$r^n \sum_{k=-\infty}^{\infty} h[k]r^{-k}$ you go and this is nothing but H [z] evaluated at z = r. So, essentially this illustrates the idea of what are called Eigen sequences; r^n is an Eigen sequence of the LSI system.

It goes into the system and is multiplied by a constant; no other change takes place. So, you see moreover the constant by which it is multiplied is the value of the z over the system function evaluated at z = r. Needless to say, r must be in the region of convergence; otherwise this is not acceptable. This would not converge otherwise, so r must be in the region of convergence.

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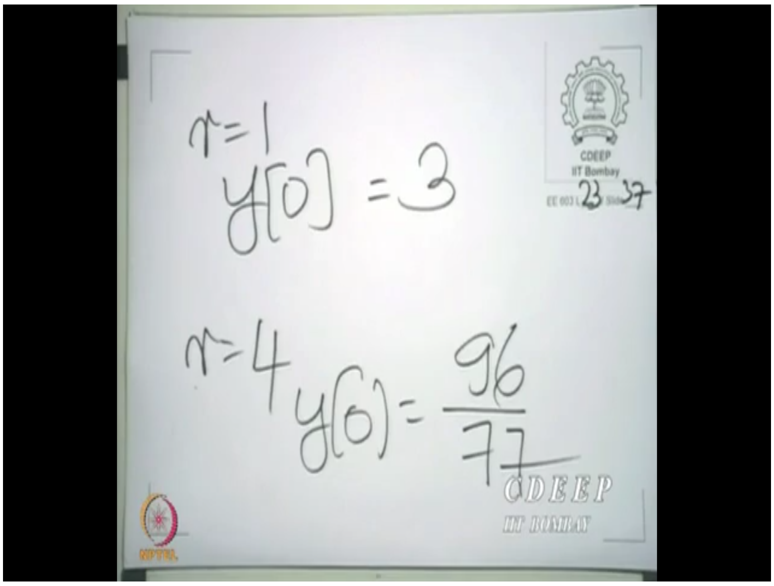

And here you are not given two different cases here, and given the case r = 1 and r = 4 and you are told that the sample y[0] = 3, for r =1; and for r = 4, the sample y[0] = $\frac{96}{77}$. So, obviously we have two equations on the two unknowns a and b; because you can substitute r = 1 and r = 4 in the.

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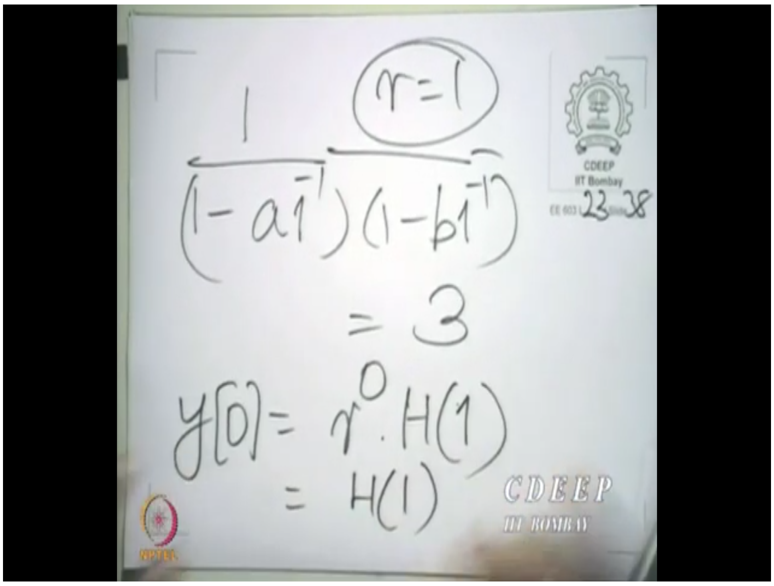

So, you have $\frac{1}{(1-a1^{-1})(1-b1^{-1})} = 3$ You remember y[0] = r^0 .H(1) this is for r = 1. So, y[0] is simply H(1) here for r = 1; this is the first equation that you get.

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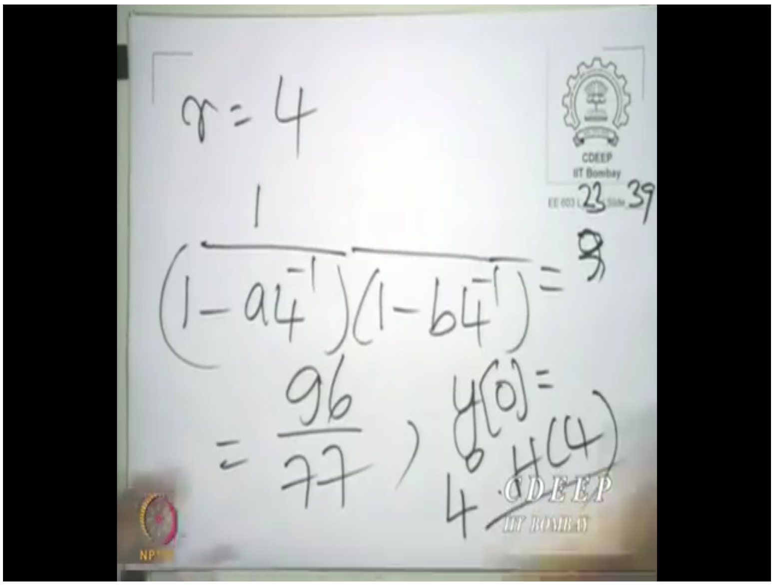

And the second equation that you get, when r = 4 is that $\frac{1}{(1-a4^{-1})(1-b4^{-1})} = \frac{96}{77}$. Because, y [0] here will come out to be 4^0 H(4) So, you have 2 equations and 2 unknowns; and you can now solve them. Let me write down the 2 equations that we have once again.

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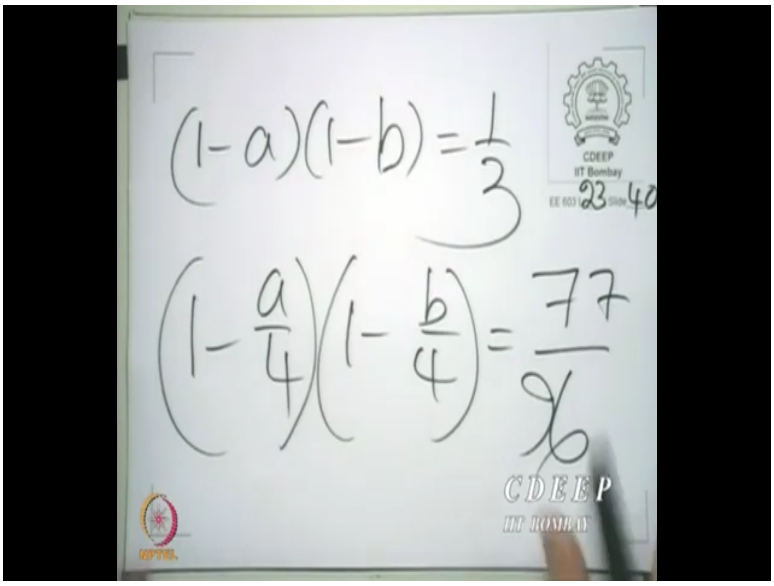

The first equation is (1-a) (1-b) = $\frac{1}{3}$; and the second equation is $(1 - \frac{a}{4})(1-\frac{b}{4}) = \frac{77}{96}$. Now, these are essentially you could treat them as quadratic in either a or in b. So, one can solve this by a quadratic.

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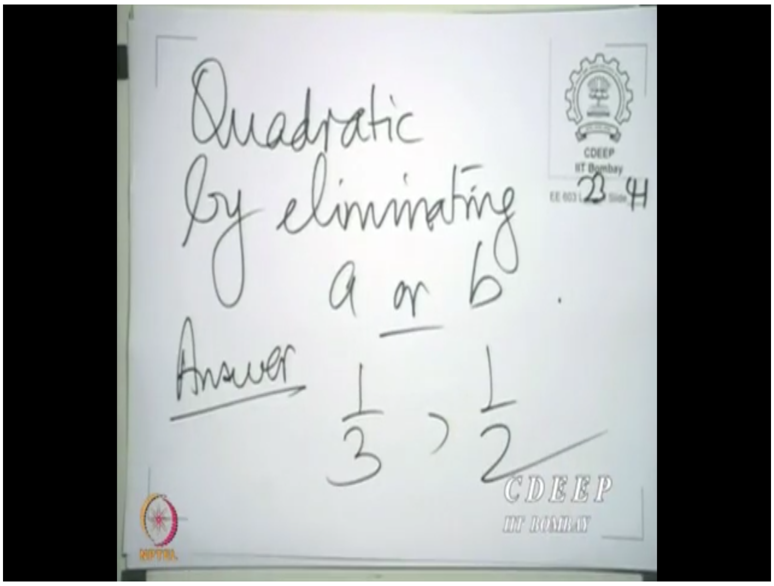

So, you need to eliminate one of two, and you get 2 solutions; you might wonder why you get a quadratic. Here you get 2 two solutions because a and b are symmetric; there they can be the roles of a and b can be reversed. So, the 2 solutions of a are in fact the solutions of a and b; or the 2 solutions of b, when you obtain them are in fact the solutions of b and a; that is how you get a quadratic. Needless to say here it would not have mattered whether a = b or

a ≠ b; that can be obtained once you solve the quadratic. So, you may require to find the constants a and b which you could and it tends at; here the constants evaluate to $\frac{1}{3}$and $\frac{1}{2}$.

So, you can take either of them to be $\frac{1}{3}$and other to be $\frac{1}{2}$; doesn't matter; they are symmetric. Now, of course I do understand a lot of people took the input to be $r^{u[n]}$; and of course that was incorrect that gives a totally different response. Now, this question was meant to illustrate or to test or to strengthen the idea of the z transform. What exactly is the z transform? What significance does it have? That has been examined in this question.

Any questions on this? For the question is what you what exactly did we mean when we talk about symmetry.

Even the role of a and b is indistinguishable here. So, it could be what what you are calling b could be a; and in that case what you are calling a would become b; so a and b are indistinguishable in this. In that sense there is a symmetry with a and b. Yes, any other question?

Student: (()) (47:35).

Professor: So, The question is could you write r^n as $r^{n\ u[n]} + r^{n\ u[-n\ -\ 1]}$ .Yes, you can but it would not offer a significant advantage. You could do that, but it only makes matters more difficult; it doesn't simplify matters in any way. Yes please.

Student: (()) (48:06)

Professor: Yes, So, the very good. So, the question is if you did indeed write r^n as

$r^{n\ u[n]} + r^{n\ u[-n\ -\ 1]}$. How would you deal with that, is exactly the problem? r^n has no z transform. Or, has no z transform in the sense of conventional function; you need to introduce impulses in the z domain to take the z transform r^n. So, even when you take r^n and u[n]; and

$r^{n\ u[-n\ -\ 1]}$ separately. Substitutive the only possibility is that you solve them separately and add; but that is not convenient at all to do.

So, you could find the responses individually in the time domain and add them; but that is not there is no way that is convenient to do, so it is interesting. In this case you use the z transform, but you cannot input and the output do not have z transform; that is the great secret in this question and it was meant to really test the fundamentals of z transform to the core. To conclude, any other questions? There is no if you take them together, there is no region of convergence. So, you cannot really either you deal with them separately and add them in the time domain, not in the z domain in the time domain.

Or you work it out straightaway in time, it was meant.

Now, the aim of many questions in examinations was to test understanding at the core. It is often the case one that where one comes to a graduate program in electric engineering in particular or in any other discipline. One comes with the full assumption that one has dealt with many concepts earlier, and not too much required to review them before an examination of basic nature.

Basics in a subject are often, so vast that you need review more than once. One often proceeds with weak foundations to construct the strong second story.

But, it is also useful once into life to strengthen one's foundations; so that the story is above becomes more stable and I do hope that some of the questions in examinations illustrated that one did need to ask for fundamental questions about fundamental concepts in DSP. Thank you.