Digital Signal Processing and its Applications
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Lecture 08 D
Causality and Memory of an LSI System

Having discussed the properties of systems, we are now going to focus our attention largely on linear shift-invariant systems and we know why. We have also convinced ourselves why now. We know that if a system is linear and shift-invariant, on giving it a complex exponential rotating phasor, it produces a rotating phasor of the same angular velocity. The catch is that we need $H(\omega)$ to converge. And what we have seen is that for stable systems it definitely converges.

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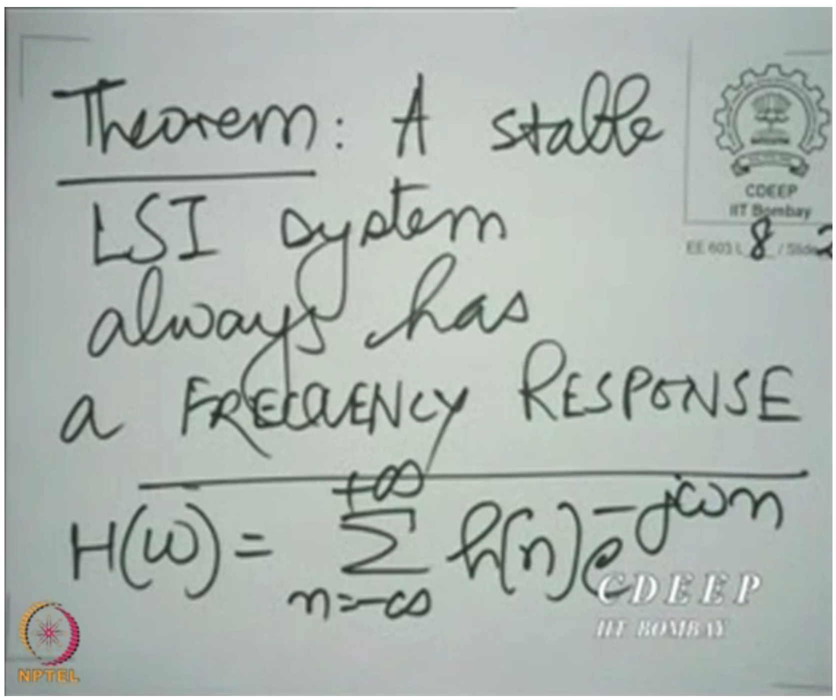

Formally, we can say that a stable LSI system always has a frequency response, given by:

$$H(\omega) = \sum_{n=-\infty}^{+\infty} h[n]\, e^{-j\omega n}$$

Here we have introduced this new term, frequency response. Frequency refers to the frequency of the rotating complex number. So we call this the frequency response of that LSI system.

Please note that we have to be very careful in our wording. We have said that a stable LSI system always has a frequency response, we are not saying that only stable systems have a frequency response and that is a very subtle point which we do not want to get into at this point in time but we shall dwell on it a little later as we proceed in the course.

For the time being let us look at systems which have a frequency response, and definitely stable LSI systems are such systems. The term frequency response is meaningful only if you have a complex exponential giving you a complex exponential of the same frequency, otherwise that term has no meaning. It is very clear that we have a very interesting situation here and we will now give it an entirely different interpretation.

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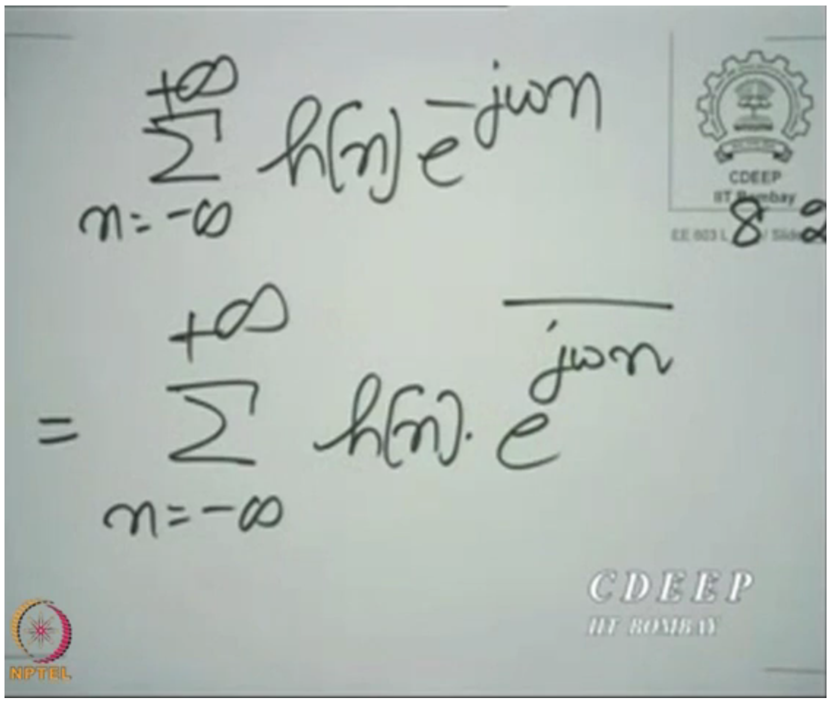

We have a situation where this quantity $\sum_{n=-\infty}^{+\infty} h[n]\, e^{-j\omega n}$ converges, and we can reinterpret this as $\sum_{n=-\infty}^{+\infty} h[n]\, \overline{e^{j\omega n}}$. We are reinterpreting it in this manner because we want to draw a parallel, and we want to give an entirely different interpretation. This number $H(\omega)$ is the number by which that phasor is multiplied when it goes into that stable LSI system.

Now, you see, when a force acts on a system the system responds by the component of the response in the direction of the force. For example, suppose you have an elastic body which may behave differently to deformations in different directions. So, when you have a force which tries to deform an elastic body and if you know how the elastic body responds to deformations in different perpendicular directions, how would you deal with the situation? You would resolve the force into the different directions along which you know the body's response and then calculate the response individually for each of these directions.

Then you multiply the component of that force in a particular direction by the component of the response in that direction. So, the force component in a particular direction multiplied by the response component in that direction gives you the net in that direction. You consider the net in all such directions and you add them to get the overall net.

This is an informal way of explaining how you deal with resolution of forces or agents. Now you must think in terms of agents here. Here you may think of the rotating phasor as an agent. And how do you find the component of a response in a particular direction?

You take a unit vector in that direction and you take a dot product of the response with a unit vector in that direction. How do you take a dot product? You take a dot product by writing each of those vectors in a standard orthogonal axis set and multiply component by component. So for example, let us take 2 vectors in 2 dimensional space.

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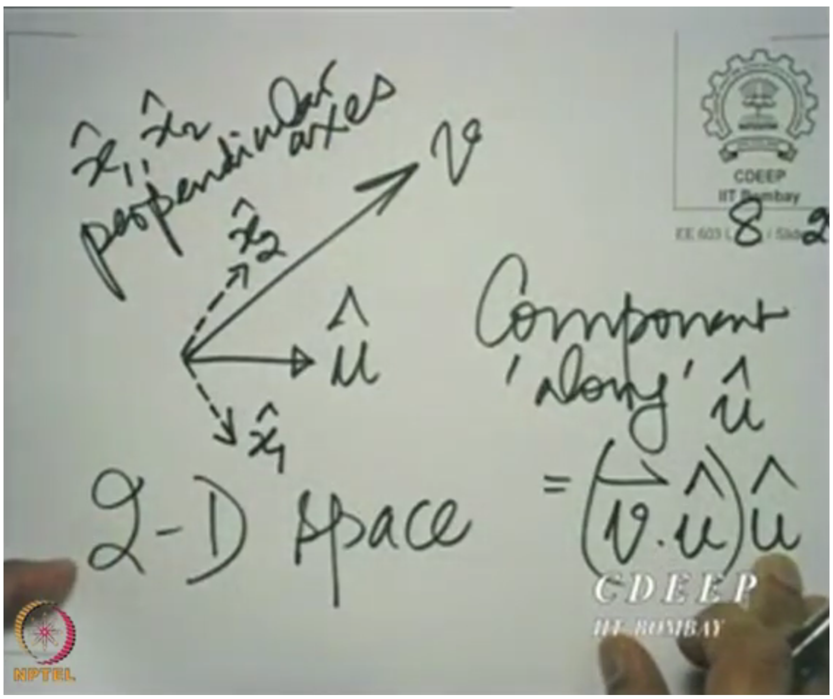

Suppose you had this vector $\vec{v}$ in the 2 dimensional space spanned by this paper, and you wish to find the component of this vector $\vec{v}$ in the direction of the unit vector $\hat{u}$. You know what it is. The component along $\hat{u}$ is simply the dot product of $\vec{v}$ and $\hat{u}$ multiplied by $\hat{u}$, that is:

$$\text{The component along } \hat{u} = (\vec{v} \cdot \hat{u})\, \hat{u}$$

And how would you find the dot product? Let us assume that your perpendicular axes are as shown in the figure. The axes $\hat{x}_1$ and $\hat{x}_2$ are perpendicular axes. The unit vectors along the perpendicular axes are shown in the figure.

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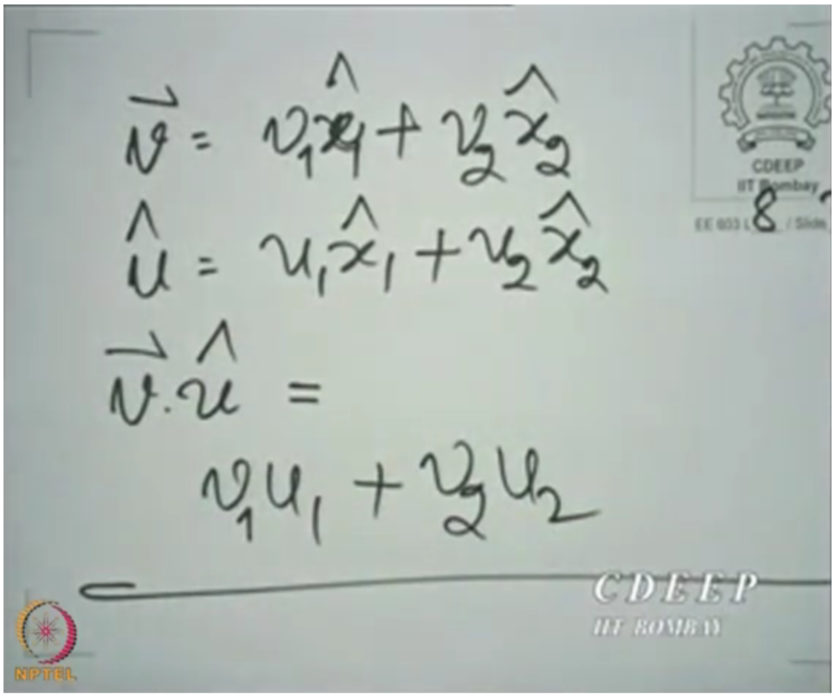

Then you could express $\vec{v}$ and $\hat{u}$ as:

$$\vec{v} = v_1 \hat{x}_1 + v_2 \hat{x}_2$$

$$\vec{u} = u_1 \hat{x}_1 + u_2 \hat{x}_2$$

Then the dot product of $\vec{v}$ and $\hat{u}$ is:

$$\vec{v} \cdot \hat{u} = v_1 u_1 + v_2 u_2$$

So how do you calculate a dot product? You multiply the corresponding components of the vectors in each perpendicular direction and then add up all such products. In the next lecture we wish to do exactly the same, but we shall generalize this idea to the context of systems.

So, we wish to interpret the quantity $H(\omega)$ that we have just arrived at in terms of a dot product, and we wish to then carry on that interpretation further to lead us to an interpretation of how LSI systems behave and how one can decompose the behavior of LSI systems into responses along different angular frequencies ω and then come to a conclusion about the very operation of convolution itself, in a different domain.