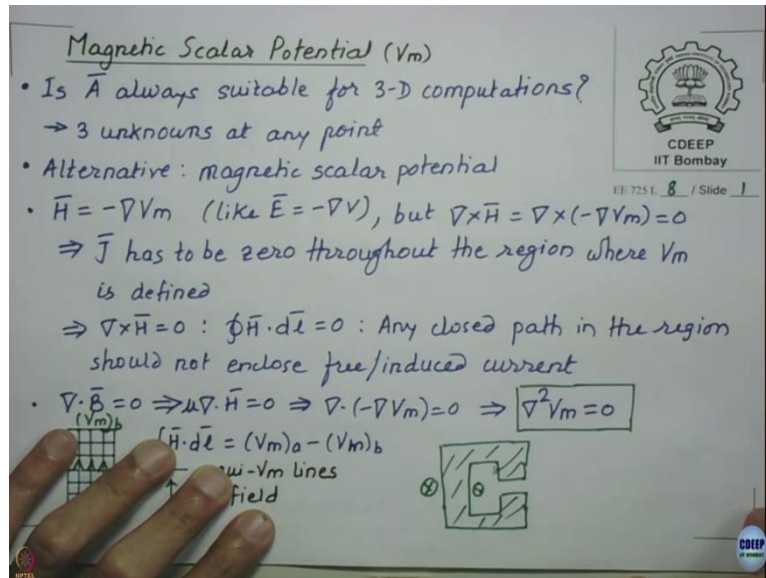


Electrical Equipment and Machines: Finite Element Analysis
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Lecture No. 8
Revisiting EM Concepts: Magnetic Forces and Materials

Welcome to the eighth lecture of this course.

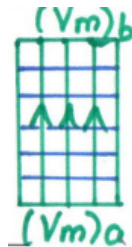
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So, what we saw in the previous lecture was some theory of magnetic vector potential, and its usefulness particularly in two-dimensional magnetic field calculations. Today we will start this lecture with magnetic scalar potential. Although we may not be using this scalar potential in our course, but it is quite useful, many commercial software make use of this scalar potential and that is why we need to understand this concept. The first question that is being asked here is, whether \mathbf{A} is always suitable for 3D computations? No, because when you are doing three-dimensional field computations, you have 3 unknowns at any point in space. For example, if you are using \mathbf{A} formulation then A_x , A_y , and A_z will be 3 unknowns at any point in space. So, then the computations become little bit cumbersome if you are solving a large 3D problem. So, it would be beneficial if we have scalar potential in magnetics. Because for formulation with scalar potential will have only 1 unknown at a point. As it is a scalar quantity, it will have only magnitude, so there is only 1 unknown at a given point in space. So that is why this magnetic scalar potential was coined by earlier researchers and that is an alternative to magnetic vector potential, but there are some limitations, what are those we will see. So now, like $\mathbf{E} = -\nabla V$ in electric fields, can we express $\mathbf{H} = -\nabla V_m$, where, V_m is the magnetic scalar potential.

But we know that if we represent \mathbf{H} as minus gradient of something, then curl of \mathbf{H} will be identically 0. That means wherever we define magnetic scalar potential, then \mathbf{J} has to be 0 throughout the region where it is defined. Second thing is, starting from $\nabla \times \mathbf{H} = 0$ which is the first condition required to use scalar potential, now you apply Stokes theorem and then you will get $\oint_L \mathbf{H} \cdot d\mathbf{l} = 0$.

Normally by Ampere's law you will get $\oint_L \mathbf{H} \cdot d\mathbf{l} = I$. But that is applicable if $\nabla \times \mathbf{H} = \mathbf{J}$. So, here $\nabla \times \mathbf{H} = 0$ because we are defining \mathbf{H} as a gradient of something. So, any closed path in the region should not enclose free or induced current, and this is another condition. So not only \mathbf{J} should be 0, but there should not be any loop which encloses current in that region where the magnetic scalar potential is defined. For example, I will take one simple example as shown in the following figure.



The rectangular piece in the above figure is a magnetic material. And then to establish flux in the vertical direction, as we did in case of magnetic vector potential in which you defined A_1 on the left hand side vertical boundary, and A_2 on the right hand side and then we said, if you make A_2 as 0 then you have to adjust the value of A_1 and the difference of A_1 and A_2 will establish the required flux condition. The same thing we are doing for the above example. Now in this example, to establish the flux in the vertical direction as shown in the figure, on the two horizontal boundaries you have to define magnetic scalar potential (V_m) , for boundary a, $(V_m)_a$ is the boundary condition and on the top boundary b, $(V_m)_b$ is defined and $(V_m)_a > (V_m)_b$. Now the \mathbf{H} field lines go from a to b as shown in the figure so it is same as electric field vectors in a parallel plate capacitor, now you can observe the exact analogy. In the sense, here in electric fields $\mathbf{E} = -\nabla V$ and here $\mathbf{H} = -\nabla V_m$. Also, in case of electric fields, \mathbf{E} goes from higher to lower potential, similarly here \mathbf{H} field goes from higher V_m to lower V_m .

So, $(V_m)_a > (V_m)_b$ the \mathbf{H} field lines are vertically up as shown in the figure, and the horizontal lines will be equi- V_m lines, like equipotential lines in case of electrostatic fields. The other thing is, we just saw that $\oint_L \mathbf{H} \cdot d\mathbf{l} = 0$ but the $\int_L \mathbf{H} \cdot d\mathbf{l} = 0$ will be $(V_m)_a - (V_m)_b$ which

simply defines the potential difference and that is what we have established to generate flux as shown in the figure.

If we do the following mathematical manipulation for $\nabla \cdot \mathbf{B} = 0$

$$\nabla \cdot \bar{\mathbf{B}} = 0 \Rightarrow \mu \nabla \cdot \bar{\mathbf{H}} = 0 \Rightarrow \nabla \cdot (-\nabla V_m) = 0 \Rightarrow \boxed{\nabla^2 V_m = 0}$$

then you simply get Laplace's equation. Now, if you have a 3D problem, in which if you know you want to analyse some region where current sources are not be there, you can excite that structure by using boundary conditions of this magnetic scalar potential, establish the field condition and then you can probably analyse the flux distribution in that given domain in greater details.

But you have to again remember that, this is applicable only for magnetostatic case because even in this core material you cannot even analyse eddy currents. So it is mainly for 3D magnetostatics analysis. But you know, this has two severe restrictions, there should not be any current or any loop that encloses any current in that domain. For example, for the following magnetic circuit, if you take any closed loop in the core in hashed region, it does not enclose any current, so that second condition also not violated.



Then the questions come into your mind are whether these conditions not too restrictive, or how to exploit the usefulness of magnetic scalar potential. That is why, this magnetic scalar potential is used in conjunction with some other potentials. In the regions of the geometry of a 3D problem domain, where currents are not there, nor any loop in it encloses current, there you define magnetic scalar potential. In the rest of the zone, you use some different potential.

We will not go into details of such formulation but if you actually see the research papers you will find researchers have formulated 3D problems by combining magnetic scalar potential with other potentials. Such methodologies are called as hybrid formulations.

See, when you use a hybrid formulation, that means you are using two potentials; one is say magnetic scalar potential and some other potential and then there will be an interface within

your problem domain wherein on one side of the interface you have one potential, on the other side you have another potential, so at the interface, you can obtain continuity of potential by imposing correct boundary conditions. But this is matter of more details and unless you are interested in using this magnetic scalar potential, we will not get into details of that. In this course, we are not going to use it. But at the same time, I thought it is important for us to know such advanced topics. Now another example wherein this could be useful is like a magnetic circuit with a gap as shown in the above figure. Suppose the current source is represented by cross and dot as shown in the figure.

Now inside this core, if you are not interested to find the eddy currents, then there is neither \mathbf{J} in this hashed portion, nor there is any loop which encloses current. That is why the application of magnetic scalar potential is possible, because there is a cut here, if this was a complete circuit (without air gap) then you could always have one loop like which will enclose the current.

So, here in this core portion you could use magnetic scalar potential, in rest of the problem domain including these current sources, you could use another potential. What is that other potential and the boundary conditions, you will have to see some research papers and get more details, but such possibilities definitely exist.

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Magnetic Forces

Lorentz force Eq: $\vec{F} = Q(\vec{E} + \vec{u} \times \vec{B})$

$\vec{F}_e = Q\vec{E}$ (Q can be stationary or moving)

$\vec{F}_m = Q(\vec{u} \times \vec{B})$ \vec{B} exerts force on a moving charge

* $\vec{F}_m \cdot d\vec{l} \Rightarrow \vec{F}_m \cdot d\vec{l} = 0$

Magnetic forces: do no work \Rightarrow Charges are neither accelerated nor decelerated: speed and KE remain constant \Rightarrow only direction of velocity vector changes

Force on a current element: $I d\vec{l} = dQ \vec{u}$, $d\vec{F}_m = dQ(\vec{u} \times \vec{B})$

If I flows through a closed path:

$$\vec{F}_m = \oint I d\vec{l} \times \vec{B}$$

For volume current density: $F_m = \int \vec{J} d\vec{u} \times \vec{B} = \int [\vec{J} \times \vec{B}] d\vec{u}$

Force Computations: $\vec{J} \times \vec{B}$, Maxwell Stress Tensor, Virtual Work Method

force density

* Ref: D.J. Griffiths, Introduction to Electrodynamics, Pearson Education 4th Edition, 2012.

Then we go to magnetic forces. Now, the Lorentz force equation $\mathbf{F} = Q(\mathbf{E} + \mathbf{u} \times \mathbf{B})$ which we have seen, probably in one of the very first slides of this course when we discussed Maxwell's equations and at that point of time probably I said, that this is an additional equation which is very useful in addition to Maxwell's equations for computing many useful parameters in

electrical machines and equipment. So, this equation has two terms; one is $Q\mathbf{E}$, other is $Q(\mathbf{u} \times \mathbf{B})$. So, the first term $Q\mathbf{E}$ is representing the electric force which acts on charges. Now charge could be either stationary or moving because the equation does not say whether the charge is stationary or is moving. The force on any positive charge at a point in space will be along the corresponding electric field at that point.

The second term, $Q(\mathbf{u} \times \mathbf{B})$, says that \mathbf{B} exerts a force on the moving charge because of $\mathbf{u} \times \mathbf{B}$, where \mathbf{u} is the velocity. And, now this \mathbf{u} being in direction of the path of the charge, and $\mathbf{u} \times \mathbf{B}$ will be perpendicular to \mathbf{u} , and that is why force is perpendicular to the $d\mathbf{l}$ because the cross product will be orthogonal to the plane formed by the involved 2 vectors. That means, the magnetic force will be always orthogonal to the path of the charge and $\mathbf{F}_m \cdot d\mathbf{l}$ will be always 0. So that is why there is an important conclusion that magnetic forces do no work and this sort of puzzles many.

So, there is a good amount of discussion on this aspect in the book titled *Introduction to electrodynamics* by Griffiths, I encourage you who are interested to know various examples wherein this statement can be proved, you can see them in this book. What I will do is, when we see Faraday's law and the motoring action, there I will sort of intuitively prove that magnetic forces do not do the work, somebody else is doing the work. So, till that time we assume that what is being said is right and of course mathematically it is proven here, that indeed since $\mathbf{F}_m \cdot d\mathbf{l}$ will be always 0, because \mathbf{F}_m will be perpendicular to $d\mathbf{l}$.

As no work is done, the charges in this case are neither accelerated nor decelerated by this magnetic force because the force is in the direction which is not along the path in which the particle or charge is moving. So, the speed and kinetic energy remain constant. But remember, that the direction is changing because velocity is a vector, and it has magnitude and direction. This magnetic force cannot influence the magnitude but it can change the direction. Whereas, the electric force, $F = Q\mathbf{E}$ has the capability to impart kinetic energy to a particle. Because it is simply $\mathbf{F} = Q\mathbf{E}$, so charges will get accelerated or decelerated by the electric field. So, it is always the electric field which is doing the work. Going further, we will derive force on a current element ($I d\mathbf{l}$). We can say, $I d\mathbf{l} = dQ\mathbf{u}$ because, $I = \frac{dQ}{dt} \Rightarrow I d\mathbf{l} = \frac{dQ}{dt} d\mathbf{l} = dQ \frac{d\mathbf{l}}{dt} = dQ\mathbf{u}$, so $I d\mathbf{l}$ can be written as $dQ\mathbf{u}$.

And then $d\mathbf{F}_m$ equation can be written as $d\mathbf{F}_m = dQ\mathbf{u} \times \mathbf{B}$. And then using the relation $I d\mathbf{l} = dQ\mathbf{u}$, you get a famous equation $d\mathbf{F}_m = I d\mathbf{l} \times \mathbf{B}$ which is very useful in rotating machines. For

example, suppose I flows through a closed path (a coil), then of course the total force is calculated by evaluating the following equation.

$$F_m = \oint_L I d\vec{l} \times \vec{B}$$

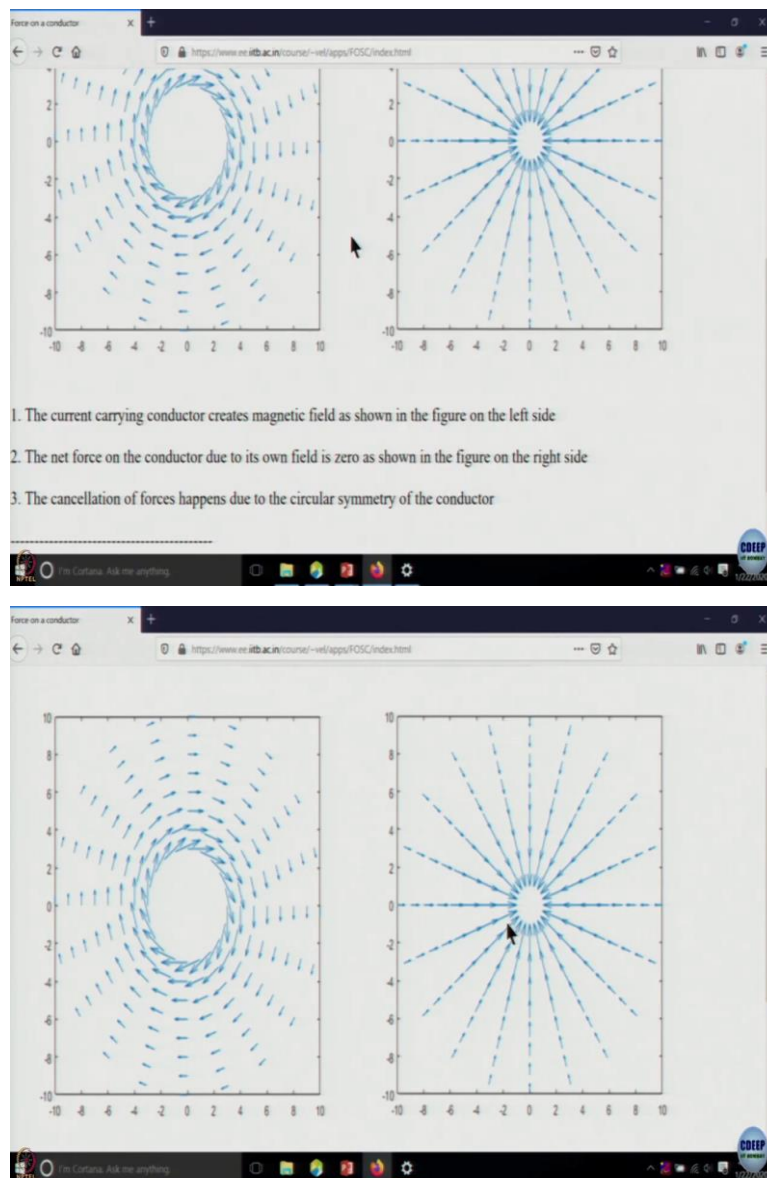
For volume current density, then the above equation is modified by replacing $I d\vec{l}$ with $\mathbf{J} dv$.

$$F_m = \int_V \vec{J} dv \times \vec{B} = \int_V [\underbrace{\vec{J} \times \vec{B}}_{\text{force density}}] dv$$

And just see the difference between the two expressions of force, in the former case it was a closed loop integral, but in the later case, there is nothing like closed volume or open volume, so we will always simply say, over volume v . So over volume v , $\iiint_v (\mathbf{J} \times \mathbf{B}) dv$ defines magnetic force, because I have already explained you earlier, $I d\vec{l} \equiv \mathbf{K} dS \equiv \mathbf{J} dv$. So here that is why we are replacing $I d\vec{l}$ by $\mathbf{J} dv$ and $\mathbf{J} \times \mathbf{B}$ is the force density and this is popularly used in calculations of forces.

Now, later on, in one of the last modules of this course, we will also be doing force computations using finite element method. So there actually we will see 3 methods; one is force calculation by $\mathbf{J} \times \mathbf{B}$, the second one uses Maxwell stress tensor, and the third one is by virtual work method. $\mathbf{J} \times \mathbf{B}$ we have understood now, but we will differ the discussion of Maxwell stress tensor and virtual work method. So along with the FEM computation, we will understand Maxwell stress tensor and virtual work method.

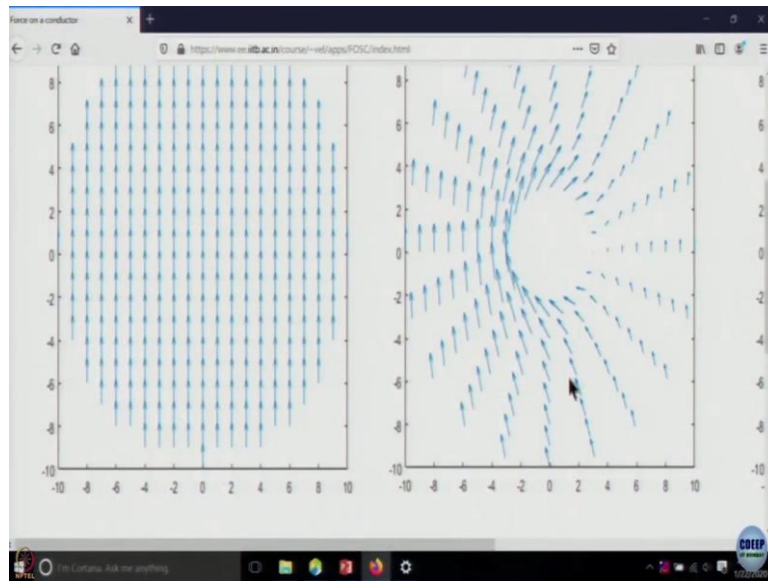
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Now let us see, 1 or 2 experiments in the virtual electromagnetic lab. In this virtual lab there is one case study on force on a conductor. So, consider an isolated conductor shown in the above two slides. That means, it is not in the vicinity of any other field and it interacts with its own field, so the field lines generated by the conductor is shown in the figure on the left hand side of the above slide. The force acts on that conductor because there is current and \mathbf{B} in the conductor region.

So, if you calculate $\mathbf{J} \times \mathbf{B}$, you will find that the force direction is sort of into that conductor as shown in the right hand side figure of the above slide. But here the point to note is the net force on this conductor is 0 because all these forces will be cancelling out. But there is a force at each point on the conductor. So, this the first case.

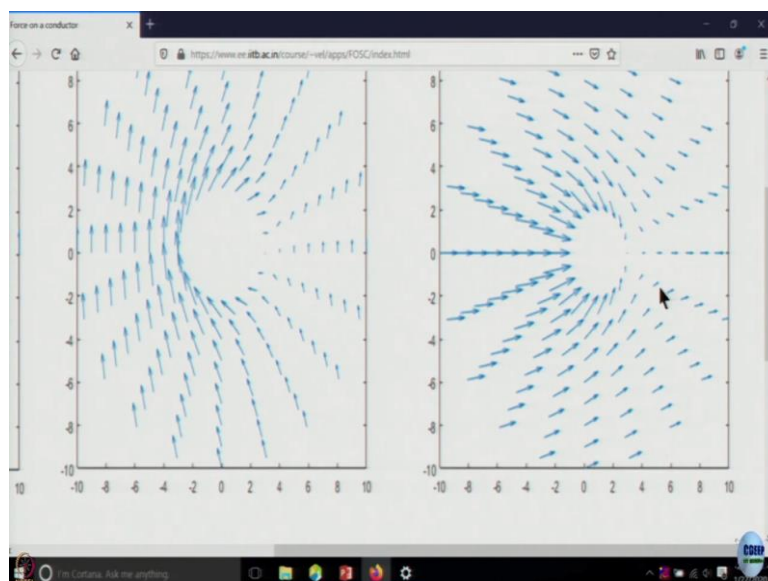
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Now let us understand the second case. Now there is some uniform field, as shown in the figure on the left hand side of the above slide. And now the same current-carrying conductor is placed in that field. Now you can see the field of this conductor and this uniform field will get superimposed and the net field will be something like as shown in the figure on the right hand side of the above slide. So always remember when you see this kind of resultant field, the field you can consider like a stretched object, because on one side you have more field, and on the other side field is less.

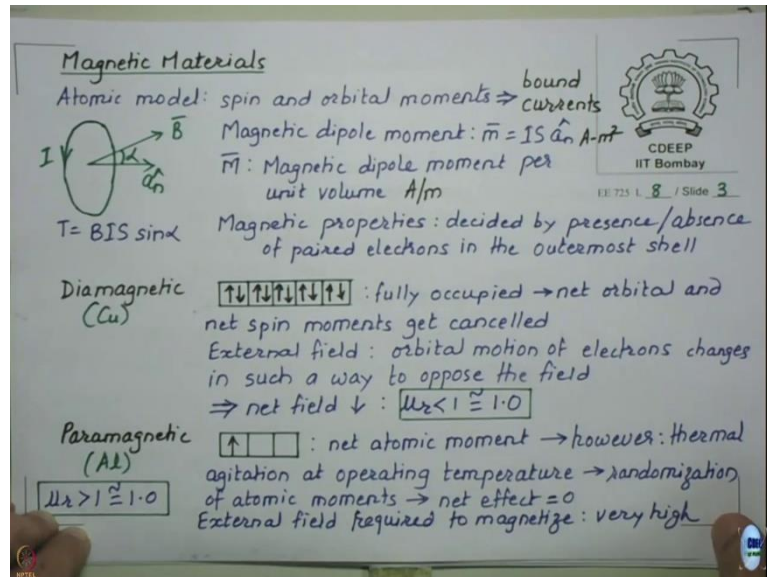
So this is like a stretched rubber string which will be pulled, similarly the conductor is going to get pulled on this side, which is actually will be clear from the following slide.

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So the figure on the right hand side of the above slide shows the force direction. So now you can see the force on the left side of the conductor is more, and force on the right side is less, and the vertical components of these forces are getting cancelled, and there is only a net horizontal force from left to right direction.

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So now we go to the next topic in magnetic fields, which is magnetic materials. Now we will see the classification of magnetic materials, for that we have to get into the atomic model. In an atom, you have electrons spinning about their own axis as well as they are orbiting around the nucleus, so these two rotations give, what are known as spin and orbital moments because they are representing the corresponding bound currents.

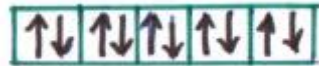
We have already seen the concept of magnetic dipole moment which is given by $\vec{m} = IS \hat{a}_n$, where, \hat{a}_n is the unit normal to a current loop area as shown in the following figure, and this I is representing the bound current which in turn represents the spin or orbital moments.



\vec{M} is the magnetic dipole moment per unit volume because there will be millions of such electron dipole moments in any given magnetic material. Also, magnetic dipole moment is per unit volume and it is a vector quantity. We have already defined torque on a current loop as

$T = BIS \sin \alpha$ in one of the earlier lectures; where S is the area of the loop, α is the angle between the applied field B , and the unit normal ($\hat{\mathbf{a}}_n$) to the current loop.

The magnetic properties of a material are decided by either presence or absence of paired electrons in the outermost shell of their atomic structure. For example, diamagnetic material, most common example is copper, wherein in the outermost subshell you have 5 orbitals as shown in the following figure.



Now you can see in the above figure there are 5 orbitals and each orbital has got 2 electrons in anti-parallel fashion. And they occupy in this anti-parallel way because in this condition the energy gets minimized.

So, in case of diamagnetic materials and copper for example, you have fully occupied orbitals in the outermost subshell. And hence, the net orbital and net spin moments get cancelled because of the antiparallel orientation of the electrons. But this is in the absence of, an external or applied field.

Now when you apply an external field to diamagnetic materials, the orbital motion of electrons changes in such a way that the corresponding effect opposes the applied field, by Lenz's law. And that is why the net field is lower than the applied field and effectively we get the value of relative permeability as marginally less than 1, but for practical purposes, for engineering calculation purposes we consider it as very close to 1 and that is why we take $\mu_r = 1$ for copper.

So, copper, as well as aluminium, which is a paramagnetic material (we are going to discuss), both are considered as non-magnetic materials because their μ_r is very close to 1 and they are as bad as air, because air is also a non-magnetic medium.

Now coming to paramagnetic materials, aluminium being one of the classic examples. The outermost subshell of aluminium has only 3 orbitals and only one of them has an electron and there is no corresponding anti-parallel electron as shown in the following figure

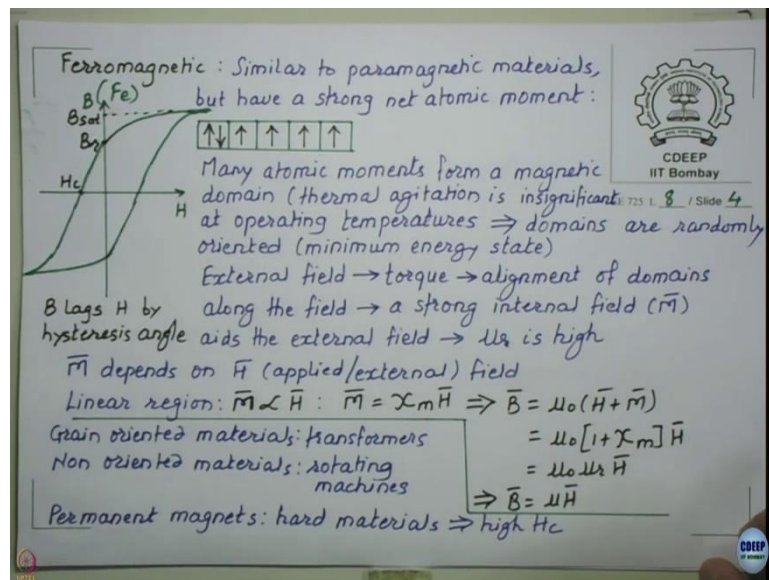


So, there is definitely a net atomic moment due to this single electron. But due to the thermal agitation at operating temperatures where we generally use these materials, there is

randomization of these atomic moments and net effect, therefore, reduces to 0 in absence of applied field.

Also, the external field required to get a sufficient magnetization will be very high. And in other words, μ_r is just marginally greater than 1. And this happens because of the thermal energy which is predominantly active in the operating temperatures. And hence, the external field required is very high to set up a given flux density.

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Now we will come to ferromagnetic materials, which are quite important for electrical machines and equipment. So, in some textbooks they are called as a class of paramagnetic materials, so they are similar to paramagnetic materials. But the main difference between the two is, ferromagnetic materials have a strong net atomic moment.

For example, if we take iron (Fe), which is a ferromagnetic material, Fe has again 5 orbitals in the outermost subshell like in case of copper, as shown in the following figure.



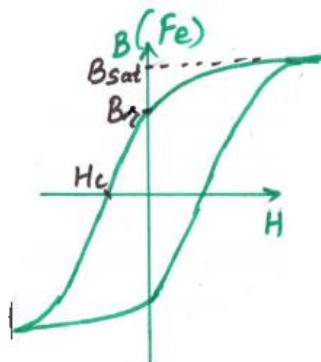
But in the above figure, there are 4 orbitals with single electron and that is why you have a net atomic moment which is quite strong. And many such atomic moments form what are known as magnetic domains and these domains get formed because thermal agitation energy in ferromagnetic materials is insignificant at operating temperatures. In fact, it becomes very high and significant only at temperatures like 800°C. Only at those temperatures like 800°C randomization would happen. So, that is why these magnetic domains get formed but these

magnetic domains are randomly oriented in the absence of applied field. Now, this random orientation is to attain a minimum energy state in the absence of applied field.

So now when the external field is applied, atomic moments experience corresponding torques, as we have seen in the last slide, and now that torque acts on the current loop, now that current loop is representing one whole domain which has many magnetic moments so the response to the applied field is significant, in case of ferromagnetic materials. And you have got a torque and then you have the corresponding alignment of domains along the field, and that sets up a strong internal field, which is \mathbf{M} , magnetisation vector. And this strong internal field aids the external field, that is why we say μ_r is high.

What does it mean, when you apply an external field to set up a given flux density, less current is drawn from the source, because the internal field is aiding the external field. And that is why the requirements from the source is reduced and this is beneficial in many applications.

Now if you consider a typical ferromagnetic material like iron, that material exhibits what is known as hysteresis characteristics as shown in the following figure. So the material exhibits not only non linear characteristics but it also exhibits, what is known as hysteresis phenomenon which is represented in terms of B versus H or it can be also be represented as V versus I , in terms of circuit parameters. So, B lags H by, what is known as hysteresis angle.



Now for example in the above figure, H goes to 0 first, when you traverse this hysteresis loop in the first and second quadrants. When you are reversing the field, H is already 0 at B_r but B is not yet 0. To make B value as 0, you have to apply H in the negative direction and coercive field H_c is required to make $B = 0$. So, the two important parameters are remanent flux density B_r , at which H is equal to 0 and coercive field H_c , at which B is equal to 0.

So, these are the two important parameters for any hysteresis loop. Another thing is the saturation B which is also important. So, all the electrical machines including transformers are

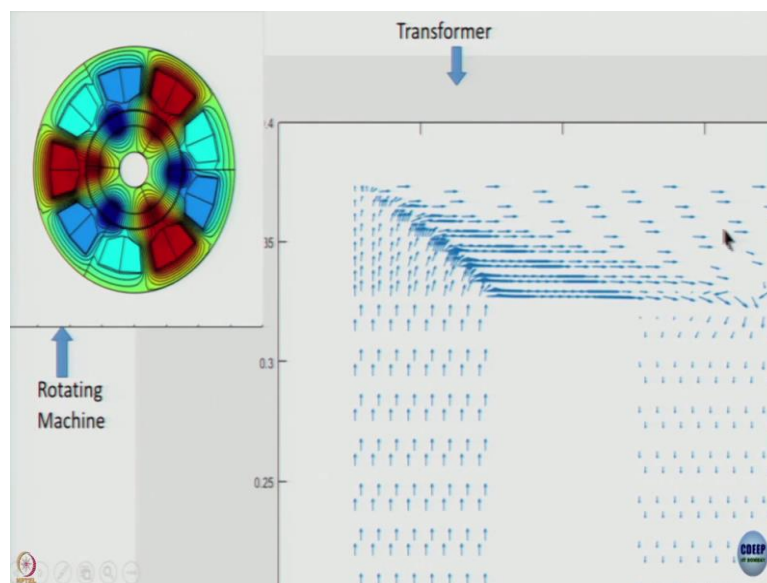
operated below saturation level. For example in case of transformers, saturation flux density is 2 T and its typical operating flux density is around 1.7 T.

So now suppose if we want to have an expression of relative permeability for these materials, then what we have to do is we take linear region because only in the linear region you have \mathbf{M} proportional to \mathbf{H} . So in some cases, if you can approximate the material characteristics by linear segments then we can write \mathbf{M} as $\chi_m \mathbf{H}$, where χ_m is the magnetic susceptibility.

In the linear region you can express $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$, because, as I already mentioned since \mathbf{M} is aiding the external field that is why total \mathbf{B} will be $\mu_0(\mathbf{H} + \mathbf{M})$. In the absence of magnetic material, this \mathbf{M} will be 0, and that is why in free space you just get $\mathbf{B} = \mu_0 \mathbf{H}$.

So now we substitute $\mathbf{M} = \chi_m \mathbf{H}$ in the expression of \mathbf{B} , and then we represent $1 + \chi_m = \mu_r$ and then finally you get $\mathbf{B} = \mu_0 \mu_r \mathbf{H}$, and finally $\mu_0 \mu_r$ is the magnetic permeability, so $\mathbf{B} = \mu \mathbf{H}$. In case of transformers and rotating machines, different grades of ferromagnetic materials are used.

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So, for example, in the above slide we see a transformer on the right hand side, and on the left side you have a typical rotating machine. The horizontal part of the transformer core is called as yoke and the vertical part of the core which is called as leg.

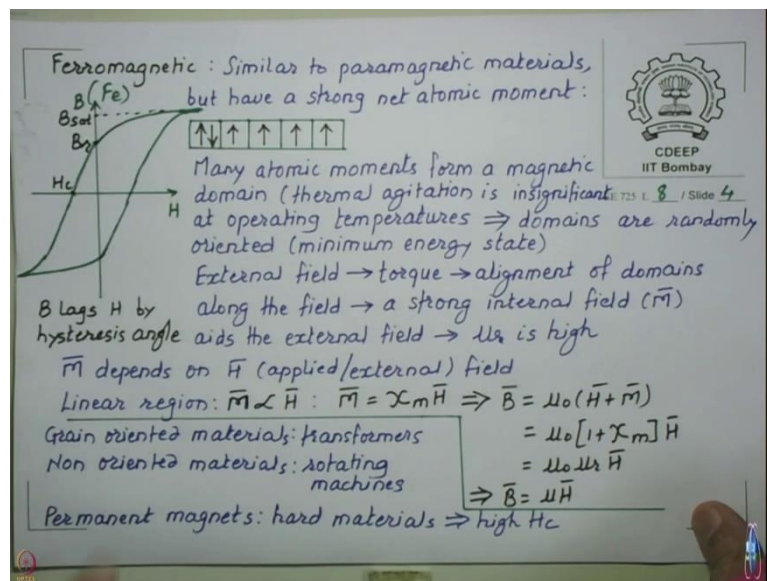
For the leg portion, the flux is in the vertical direction, either vertically up or vertically down. And in yoke portion, flux is either going towards right side and after a half cycle, it just reverses

and goes to the left side. So, in most of the part of the core you have flux direction either parallel or anti parallel to a particular direction. That is why in case of transformers, the grade that is used is grain oriented; that means the material is processed such that the permeability is maximum along the direction of flux.

Whereas in case of rotating machines, you can see in the flux plot on left hand side of the above slide, the direction of flux is not fixed along any part of the core and since the 3 phases are 120° displaced in time, this flux direction is continuously changing with time and there is no fixed direction of the flux. For example, you can observe the change in the flux pattern in the above slide.

So, there is no point here in using grain oriented material in such cases because most of the time the flux will not be in the direction of grain orientation and then we will not be able to exploit better properties of material along the grain orientation. So that is why in case of rotating machines you use non oriented material.

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And finally, permanent magnets which are called as hard magnetic materials. What we saw in the previous slide are grain oriented and non oriented materials, and a hysteresis loop for a typical soft magnetic material is shown in the above slide. The soft magnetic materials are characterized by a low value of H_c , that means the value of H required to demagnetize core is small. Whereas in case of permanent magnets which are called as hard magnetic materials, H_c is very high and the corresponding hysteresis loop is very broad.

And you are aware that permanent magnets have got a lot of applications these days in rotating machines. And we will see one such example later when we get into finite element analysis.

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Boundary conditions: Magnetostatic fields
 $B_{1n} = B_{2n}$: normal component is continuous
 $\mu_1 H_{1n} = \mu_2 H_{2n}$: normal component of \vec{H} is discontinuous
 $H_{1t} = H_{2t}$ (absence of surface currents)
 $\frac{B_{1t}}{B_{2t}} = \frac{\mu_1}{\mu_2}$: B_t is discontinuous across the interface

Inductance
 Flux linkage $\lambda = N\psi$, $L = \frac{N\psi}{i} = \frac{NBS}{Hl/N} = \frac{\mu N^2 S}{l}$
 $= \frac{N^2}{R}$ R : reluctance $= \frac{l}{\mu S}$
 if reluctance of magnetic circuit increases, the corresponding reactance in its equivalent electric circuit decreases

Air gap: almost entire magnetic energy stored in it

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Next topic is boundary conditions. In this lecture we will see boundary conditions for magnetostatics fields and probably in the next lecture we will see time-varying fields also. So without going into the derivation, I am directly stating the following condition which states that the normal component of magnetic flux density is continuous.

$$B_{1n} = B_{2n}$$

Similarly, we had $D_{1n} = D_{2n}$ in case of electric fields. And if you substitute $\mathbf{B} = \mu\mathbf{H}$, then the normal component of H is discontinuous as given below.

$$\mu_1 H_{1n} = \mu_2 H_{2n}$$

And then by using the Ampere's law in Maxwell's equation ($\oint_L \mathbf{H} \cdot d\mathbf{l} = I$) and then we assume the absence of surface currents which is valid for quasi-static low frequency field. In the next lecture, when we talk of time varying fields we will bring in this surface current and see how the boundary conditions get affected. In absence of surface currents, the tangential component of H is continuous and you can express this in mathematic form as given below:

$$H_{1t} = H_{2t}$$

If we substitute $\mathbf{B} = \mu\mathbf{H}$ in the above equation, we get B_t as discontinuous across the interface. This condition is mathematically represented as

$$\frac{B_{1t}}{B_{2t}} = \frac{\mu_1}{\mu_2}$$

Now for example, if you see the case of a transformer as shown in the following figure



In the above figure, there are two windings (LV and HV) and core. The leakage field is shown with a contour, and I have shown only one flux contour in the above figure. So the flux is impinging the core and it is coming back by forming a closed path as shown in the figure. Now here, the permeability of the core is in thousands as we saw in the previous slide, it is in the range of 1000 to 5000. Consider the core as the second material, so, μ_2 is very high, so from the above relation of tangential components of flux density, the ratio of $\frac{B_{1t}}{B_{2t}}$ will be very small, so B_{2t} will be also very high because μ_2 is high, so tangential component of B will be high.

Effectively, once the flux enters the core normally, the tangential component will be immediately dominant. That is why in the leakage field plot in case of a transformer, you will find that the flux is seen to be almost entering normally to the core and then it becomes tangential. So, the normal component of the incident field is high and then it turns inside the core, and the tangential component becomes predominant as governed by the magnetic boundary conditions.

So more about these boundary conditions for time-varying fields will be discussed later.

Next topic is the inductance. So, flux linkage is represented as $N\psi$, ψ being the flux. So we can express inductance as

$$L = \frac{N\psi}{I}$$

which defines inductance as flux linkage upon current and flux can be represented as $\psi = BS$ and current $I = \frac{Hl}{N}$, we have not yet seen this relation of current. In magnetic circuits, MMF is

equal to Hl which is equal to NI , so $MMF = Hl = NI$ and the expression of L can be represented as

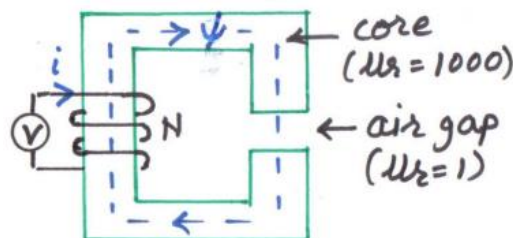
$$L = \frac{N\psi}{i} = \frac{NBS}{Hl/N} = \frac{\mu N^2 S}{l}$$

So, the above expression is another formula for computing inductance. If you can replace $\frac{l}{\mu S}$ where S is area of cross section through which flux is flowing and l is the mean length of magnetic circuit, so then the expression of inductance becomes

$$L = \frac{N^2}{\mathcal{R}}$$

where \mathcal{R} is the reluctance of the magnetic circuit. So, the above-mentioned 3 formulae are commonly used for calculating inductance.

Now you should remember that reluctance in a magnetic circuit and reactance in its corresponding representation of electric circuit do not go hand in hand. Although they both look similar, they are not proportional. For example, consider the following figure which is a magnetic circuit with a gap.



From the previous discussion, the relative permeability of the air gap is 1 and that of the core is 1000.

Effectively what we are saying is, 1 mm of air gap is equivalent to 1000 mm of iron core. So almost entire energy of this magnetic circuit is stored in this air gap only.

Suppose consider that you have the same gap in an electric circuit, representing a capacitor. If we compare the stored energy by this capacitor and the stored energy by the gap in the magnetic circuit for a flux density of 1.5 T because the core will not saturate for this value of flux density, at 1.5 T, you calculate the energy for some number of turns and source.

Now this energy stored in the magnetic gap is something like 1000 times more than the corresponding energy stored in the capacitor and in case of electrostatics there will be the limit of breakdown of the air gap. So, in case of electric field and a capacitor, how much energy you can store is the function of the breakdown voltage.

So if we consider 3 kV/mm which is the breakdown voltage for air and then calculate the corresponding stored energy you will find that, that energy is much less as compared to the magnetic energy stored in this gap. And that is the reason you find, whenever you require a larger amount of forces in practical applications, like plungers, you always use the magnetic energy.


You use electrostatic forces in applications like MEMS (Micro Electro Mechanical devices) etc., where forces required are very small. But this is a thing that I thought I should just mention to you. So I think, what we will do is we will stop at this and continue in the next lecture. I will discuss more about magnetics and then we will go on to time-varying fields.

[Refer Slide Time: 42:06]

L8: Review Questions

Q1: If a test positive charge is moving in a region with electromagnetic fields, which term of the Lorentz force equation will accelerate the charge?

Q2: What is the difference between $\nabla \times \mathbf{H} = \mathbf{J}$ and $\nabla \times \mathbf{B} / \mu = \mathbf{J}$?



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(NPTEL - MOOC course)
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