Electrical Equipment and Machines: Finite Element Analysis
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Revisiting EM Concepts: Magnetostatics
Lecture 07

So welcome to 7th lecture. Till now we have covered vector calculus and electrostatic fields. For the next two and half lectures, we will see magnetic fields and time-varying fields which are very important for this course because most of the time we are interested in calculation of magnetic fields in electromagnetic devices. So when we talk of magnetostatics, we start with Biot-Savart law.

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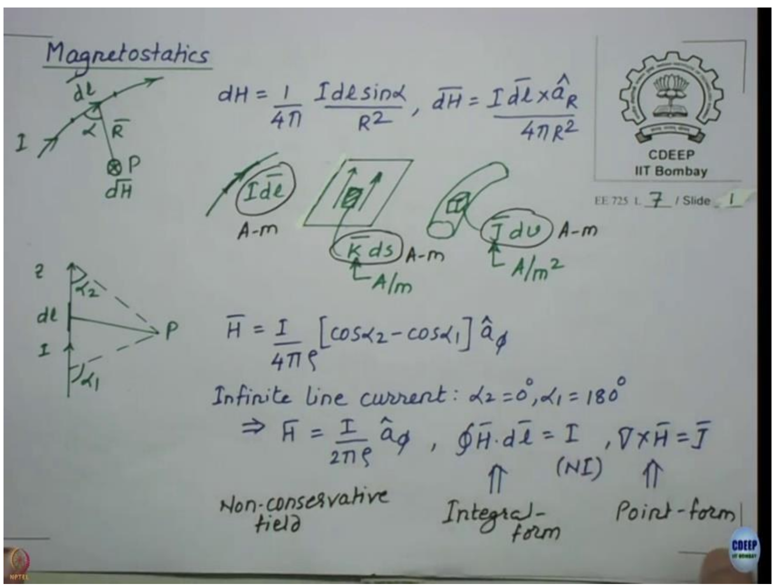
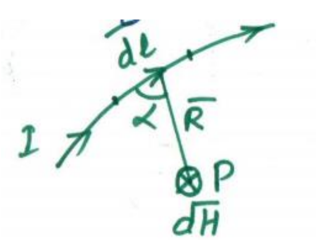


For a line current I shown in the following figure, the value of magnetic field intensity $d\mathbf{H}$ at a point P which is at a distance R is given by the expression:

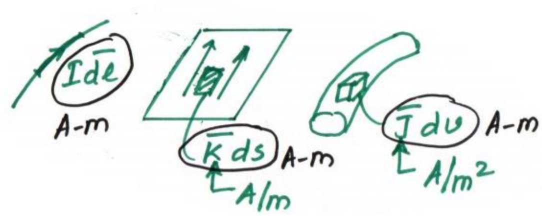
$$dH = \frac{1}{4\pi}\frac{I\,dl\sin\alpha}{R^2},$$



The angle α is denoted in the above figure. In the vector notation, the expression of $d\mathbf{H}$ is modified as:

$$\overline{dH} = \frac{I\, d\bar{l} \times \hat{a}_R}{4\pi R^2}$$

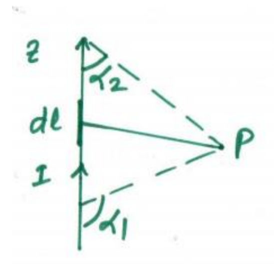
The above expression represents the well-known Biot-Savart law. Now the point to note here is that similar to line current you can have surface and volume currents as shown in the following figure.



In all the three cases given in the above figure, the overall unit of a current element is A-m . For example, the unit of $I d\mathbf{l}$ is A-m. In the above figure, for $\mathbf{K}dS$, $\mathbf{K}$ is the surface current density, when does surface current come into picture? When a high-frequency excitation is there and skin depth is very small compared to the thickness of the conductor, current tries to remain at the surface and then you have what is known as surface current density. We will discuss a little bit more about this concept later.

Surface current density **K** is associated with the surface shown in the above figure. And the corresponding unit of **K** is A/m because it flows through a very thin surface, so one dimension is not there. So the over all unit of $\mathbf{K}dS$ is A/m × m^2 and is again A-m. The third type of current which we commonly use in field calculations is J which is called as volume current density and its unit is A/m^2. The overall unit is again A-m. For all the three quantities, the overall unit is A-m . So always remember, I in Idl is a scalar quantity, $d\mathbf{l}$ is the vector, so $d\mathbf{l}$ is the one which gives the direction to the current element, whereas in the other two cases, **K** and **J** are vectors.

Next consider a current element, which is like an infinite line current as shown in the following figure.



The magnitude of the vector H at point P is given by the following expression

$$\bar{H} = \frac{I}{4\pi\rho}\left[\cos\alpha_2 - \cos\alpha_1\right]\hat{a}_\phi$$

In the above expression, α_2 and α_1 are the angles as shown in the above figure. And if it is an infinite line current, then α_2 and α_1 will tend to 0 and 180° respectively. Then the above expression reduces to the following well-known formula

$$\bar{H} = \frac{I}{2\pi\rho}\hat{a}_\phi$$

Always remember, E in electrostatic or electricity is equivalent to H in magnetic fields or magnetics.

We saw in electrostatics, $E \propto 1/R^2$. But here in the above expression of H for an infinite current $H \propto 1/\rho$ or $1/R$. But for a finite current element, H is always $\propto \frac{1}{R^2}$. The question is if it is an infinite charge distribution what is the relation between and E and R? If it is an infinite line charge, then E has a $1/R$ dependence.

Another point which I have also explained earlier is $\nabla \times \mathbf{H} = \mathbf{J}$ which is the point form of Ampere's law and the integral form of this equation is

$$\oint \bar{H}\cdot d\bar{l} = \underset{(NI)}{I}$$

Since $\nabla \times \mathbf{H}$ in general is non-zero, this field will be a non-conservative field. We have earlier seen what is the difference between conservative and non-conservative fields. So this is an example of non-conservative field.

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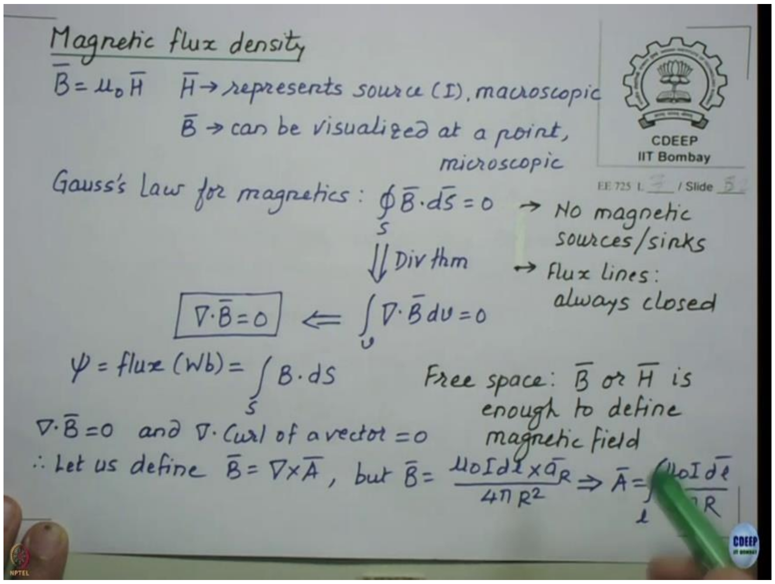

Now coming to magnetic flux density, we know $\mathbf{B} = \mu_0\mathbf{H}$ in case of free space. In materials, we will see later how this expression of B gets modified and how relative permeability (μ_r) comes into picture. Now there is often confusion between what H represents and what B represents. So one can say that H represents a current source and the unit of H is A/m. So you can say it is a macroscopic representation of magnetic field, whereas you can visualize B at a point and you cannot visualize H at a point. So you can say that H is representing the current source, which is producing it. So B definitely can be visualized at a point, we say flux density at this point is so much Tesla which represents Webers (flux) passing through a cross-sectional area.

That is why B can be called as the microscopic representation of magnetic field. This is a very subtle difference between the two. And only one of them is good enough to represent magnetic fields if it is free space. Also, in free space, if one is known, other is automatically known. So Gauss's law of magnetics is represented as

$$\text{Gauss's law for magnetics} : \oint_S \bar{B} \cdot d\bar{s} = 0$$

If you apply divergence theorem to the above equation, then you will get the second Maxwell's equation in point form as given below

$$\boxed{\nabla\cdot\bar{B}=0} \Leftarrow \int_v \nabla\cdot\bar{B}\,dv=0$$

This equation tells that there are no magnetic sources or sinks and flux lines are always closed.

But we should always note that $\iint_S \mathbf{B}\cdot d\mathbf{S}$ gives you the flux crossing the surface S. Moment you make it closed surface integral, it becomes 0. Now, $\nabla . B = 0$ and we know that divergence of curl of a vector is always 0. So then we can define B vector as curl of a vector, say $\mathbf{B} = \nabla \times \mathbf{A}$. What is this **A**? we will see in the next slide. Actually by using $\mathbf{B} = \nabla \times \mathbf{A}$ and the following expression of B

$$\bar{B}=\frac{\mu_0 I\, d\bar{l}\times\bar{a}_R}{4\pi R^2}$$

you can then derive the expression for A as given below.

$$\bar{A}=\int_l \frac{\mu_0 I\, d\bar{l}}{4\pi R}$$

Again, as I mentioned to you in most of the equations where I appears, involving vectors, I will be accompanied by $d\mathbf{l}$ vector which gives the direction of current. If you want to represent the same equation in terms of volume current distribution, then this I$d\mathbf{l}$ will get replaced by $\mathbf{J}dv$ where **J** is the vector, dv is just a scalar.

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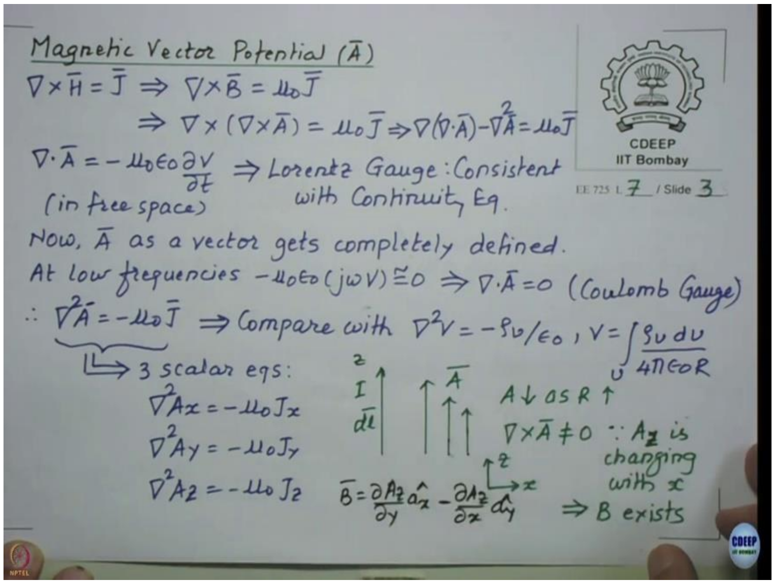
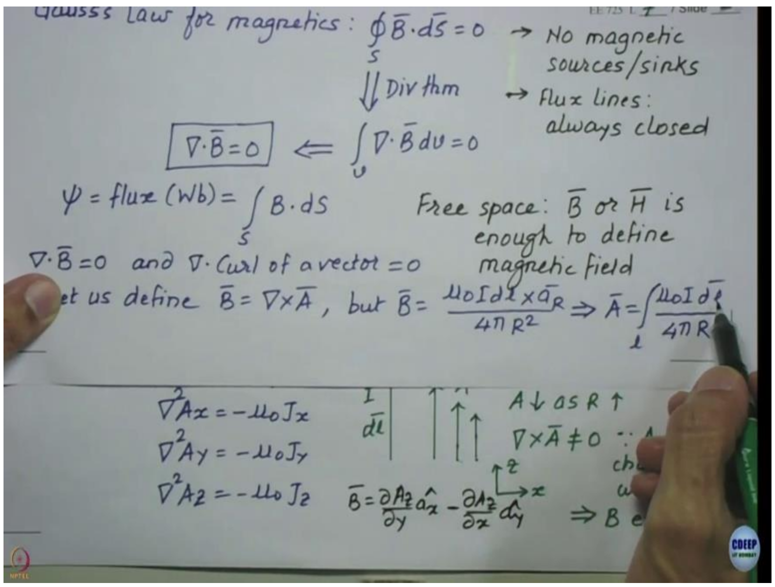




Now we come to a very important quantity in electromagnetics, which is called as magnetic vector potential (**A**). Because you have scalar potential V in electrostatics and then **E** can be calculated in terms of V as $\mathbf{E} = -\nabla V$, but in magnetics, you have only **B** and **H** fields and earlier there was no other quantity in terms of which you can calculate **B** and **H** fields. So using Maxwell's equations, researchers derived the magnetic vector potential (**A**) and later on, I will explain you this 'potential' is not a correct word , because potential means it is like a scalar.

But this **A** is a vector quantity. Maxwell originally called this magnetic vector potential as electro kinetic momentum vector. Actually, it should have been called as a vector, but this magnetic vector potential word has been very commonly used in literature, and most of us are used to it now. But we should remember that it is a vector and as Maxwell correctly termed it as electro kinetic momentum vector. When we discuss time-varying fields, we will see how this quantity is a momentum vector. We know $\nabla \times \mathbf{H} = \mathbf{J}$ and using $\mathbf{B} = \mu_0 \mathbf{H}$, this equation can be recast as $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ and then you can replace $\mathbf{B}$ with $\nabla \times \mathbf{A}$ and then applying the vector identity for $\nabla \times (\nabla \times \mathbf{A})$ results in the following equation

$$\nabla \times (\nabla \times \bar{A}) = \mu_0 \bar{J} \Rightarrow \nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A} = \mu_0 \bar{J}$$

Remember that we are still dealing with vacuum or free space and we are not describing magnetic materials yet. So we are in free space that is why μ_0 is appearing. Later on, we will bring in $\mu = \mu_0 \mu_r$ .Till now we have expressed $\mathbf{B}$ as curl of $\mathbf{A}$, by definition , a vector is completely defined if its both curl and divergence are defined.

Now we have defined only curl of $\mathbf{A}$ vector which is nothing but $\nabla \times \mathbf{A} = \mathbf{B}$. We have not yet defined divergence of $\mathbf{A}$. So we will now define $\nabla \cdot \mathbf{A} = -\mu_0 \epsilon_0 \frac{\partial V}{\partial t}$. Again, this is free space that is why here μ_0 and ϵ_0 are appearing. And the above definition of divergence of A is called as Lorentz gauge. Now, this is not an arbitrary definition. It can be proved that this Lorentz gauge is consistent with the continuity equation. This derivation is available in standard textbooks and is straightforward. So this Lorentz gauge is consistent with the continuity equation and now, this A gets completely defined because we have defined curl of A which is B and divergence of A as given by the following expression.

$$\nabla \cdot \mathbf{A} = -\mu_0 \epsilon_0 \frac{\partial V}{\partial t}$$

So A as a vector is completely defined. Now, at low frequencies the right hand side term of the above equation will be almost negligible because the frequency is small, the $\frac{\partial V}{\partial t}$ term in the above equation can be written as $j\omega V$ in the frequency domain. And since this $\mu_0 (= 4\pi \times 10^{-7})$ and $\epsilon_0 = (8.85 \times 10^{-12})$ are very small numbers, their product is also small and the frequency is small, the term $-j\omega\mu_0\epsilon_0 V$ is going to be a very small number. So that is why the right hand term of the Lorentz gauge is 0 at low frequencies, so $\nabla \cdot \mathbf{A}$ becomes 0 at low frequencies and this is called Coulomb gauge. That is why in low frequency electromagnetics, $\nabla \cdot \mathbf{A}$ is always taken as 0.

When you are dealing with high frequency electromagnetics involving wave propagation, antennas and whatnot, $\nabla \cdot \mathbf{A}$ is taken as the expression of Lorentz gauge. This is a difference which differentiates low-frequency and high-frequency electromagnetic field computations. Now if we all agree that this is a consistent theory, then we will take $\nabla \cdot \mathbf{A} = 0$. And then we obtain

$$\nabla^2 \bar{A} = -\mu_0 \bar{J}$$

Now compare the above equation with Poisson's equation in electrostatics, which is $\nabla^2 V = -\rho_v/\epsilon_0$. Again this is free space and the corresponding expression for V is

$$V = \int_v \frac{\rho_v \, dv}{4\pi\epsilon_0 R}$$

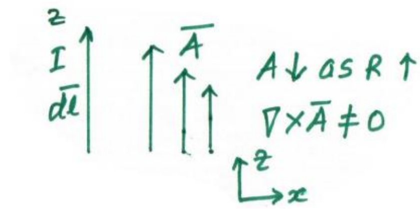
Now the expression for V and the expression for A that we saw in the previous slide, you can see the similarity in terms of μ_0 and ϵ_0. If one is in numerator, other will be in denominator. We will always find that for magnetic fields $\mu_0 I d\mathbf{l}$ appears, and for electric fields $\rho_v dv/\epsilon_0$ appears . For magnetic field quantities, μ_0 is in numerator, and for electric fields ϵ_0 is in denominator. So you can see, 4π is common and there is a similarity between the expressions of A and V. So now this $\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$ is a vector Poisson's equation that can be split into three scalar equations as given below.

$$\underbrace{\nabla^2 \bar{A} = -\mu_0 \bar{J}}$$

↳ 3 scalar eqs:

$$\nabla^2 A_x = -\mu_0 J_x$$
$$\nabla^2 A_y = -\mu_0 J_y$$
$$\nabla^2 A_z = -\mu_0 J_z$$

A little bit more understanding of magnetic vector potential we will do. Of course, we have understood its expression and genesis from the definition of its divergence and curl. But let us understand in space how it works.



In the above figure, I is the line current. If I is directed vertically along this $d\mathbf{l}$ vector, see again, $d\mathbf{l}$ has shown purposely as a vector and this defines the direction of current.

So we should be consistent whenever we show something, we should always remember that these small things make our understanding clear. For the current element in the above figure, magnetic vector potentials at any point in space are also directed in the direction of current because on the right hand side of the expression of **A** that we have seen in the previous slide, you have only one vector i.e, $d\mathbf{l}$, so the direction of $d\mathbf{l}$ will be the direction of **A**. So **A** is also vertically directed and the magnitude of **A** is reducing as you go away from the source because it is inversely proportional to the distance from the source.

Now here $\nabla \times \mathbf{A} \neq \mathbf{0}$, obviously because $\nabla \times \mathbf{A} = \mathbf{B}$. But how do we understand it? Here suppose we take the current in the above example as in z direction, then **A** is also in z direction and you can see that A_z is changing in space with x. If we apply $\nabla \times \mathbf{A}$ for this case, then **B** is given as

$$\bar{B} = \frac{\partial A_z}{\partial y}\hat{a}_x - \frac{\partial A_z}{\partial x}\hat{a}_y$$

Now here we are talking about two-dimensional approximations in which current is in z direction and the **B** field will be in xy plane. Here moment A_z is changing with x, that means at least one of the components of **B** is going to be non-zero, that means **B** exist.

So here in the above figure, A is changing with x in zx plane. After 90 degrees rotation of this zx plane in ϕ direction , you will have A_z varying with y and at any intermediate plane between the above-discussed planes A_z will vary with both x and y. So that clearly helps us to understand the magnetic vector potential and its relation with **B** and how the **A** vector varies with distance from the source.

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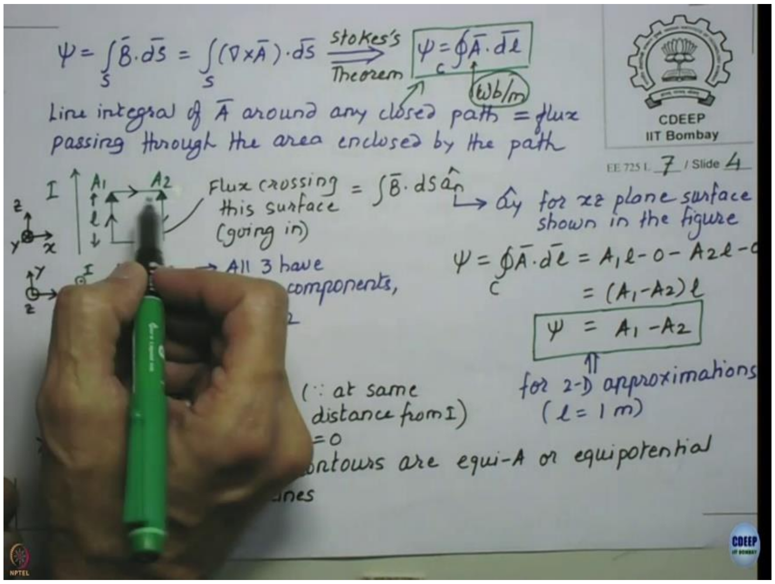


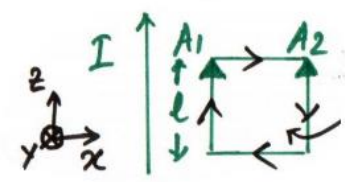
Now, the above slide is very important for 2D FEM calculations that we are going to see throughout this course. Our course will be dominated by 2D magnetic field computations and that is why this slide it is quite important. Here ψ denotes flux, which is calculated by

$$\psi = \int_S \bar{B} \cdot d\bar{s}$$

Remember again, I am repeating $\oiint_S \mathbf{B} \cdot d\mathbf{S} = 0$. $\iint_S \mathbf{B} \cdot d\mathbf{S}$ is equal to the flux crossing that surface. Now you replace $\mathbf{B}$ in the above equation by $\nabla \times \mathbf{A}$ and then you apply Stokes' theorem. You will get the expression of ψ as given below

$$\psi = \int_S \bar{B} \cdot d\bar{s} = \int_S (\nabla \times \bar{A}) \cdot d\bar{s} \xrightarrow[\text{Theorem}]{\text{Stokes's}} \boxed{\psi = \oint_C \bar{A} \cdot d\bar{l}} \quad (\text{Wb/m})$$

Now if the flux ψ is in Webers and unit of $d\mathbf{l}$ is in meters, so the unit of $\mathbf{A}$ becomes Weber per meter (Wb/m). The contour integral of A around any closed path is equal to the flux passing through the area enclosed by the path. This is what is described by the above equation. Now again let us understand what is happening in space.

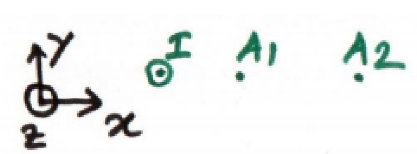


In the above figure, you have current *I* again in z direction. Here the corresponding coordinate axis is also marked, in which *z* is vertically up, *x* is in the horizontal direction, and *y* is going into the paper. The directions can be obtained by placing the fingers of the right hand along *x*, and you turn them around from the *x* to *y*, you will get the z direction.

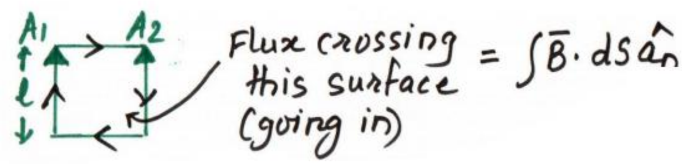
Again, you have the current source directed along the z direction as shown in the above figure. So you have A_1 and A_2 at two points, which again will be in z direction as shown in the figure. Then we are just taking a contour as indicated in the figure . If you take the top view of the system in the *xy* plane, you will see the current source just as a dot. If the current is dot and it is coming out, then the direction of field vectors are as per the right hand rule, which defines that thumb is pointing in the direction of the current and the direction of field will be given by the fingers. So the field will be as shown in the following figure.

In the above figure, the plane of the paper is *xy* and z comes out of the plane. So it is basically governed by the right hand rule.

So now consider the above figure with A1 and A2 is represented in 2D as shown in the following figure.



In the figure, I have shown the current just by a dot. For the same 2D representation in the previous figure, I have shown the field. Consider magnetic vector potential at any two points in the *xy* plane as A_1 and A_2. You should remember the flux is always associated with the corresponding area through which it flows. Now, the area through which the flux (shown in the previous figure) is flowing, has one dimension along the z axis. So here, flux really does not flow through this paper surface. You have to always visualize that flux is crossing some surface. So the surface for this example is along zx plane as shown in the following figure.



So suppose current is in z direction, then flux will cross zx plane or the corresponding surface parallel to this plane or if you turn the surface by 90 degrees in ϕ direction, then it will be zy. So always it is crossing some surface. And the flux crossing the surface is given by the above equation and $\hat{\mathbf{a}}_n$ in the equation is unit normal to the surface.

Again, I mentioned in one of the previous lectures that the direction of this $\hat{\mathbf{a}}_n$ will be decided by the corresponding contour integral, because this is an open surface, which has two possibilities for $\hat{\mathbf{a}}_n$, for the given case it could be either $\hat{\mathbf{a}}_y$or $-\hat{\mathbf{a}}_y$. But now we are taking some contour integral, so the direction of the contour integral decides the direction of the unit normal for that open surface. So I am again repeating some of these points as they are very important for the general vector calculus point of view and visualizing electromagnetic field distribution in space.

Now in this case, it will be $\hat{\mathbf{a}}_y$ for the xz plane surface shown in the above figure. So the expression of flux is

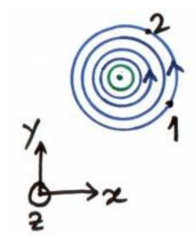
$$\Psi = \oint_C \bar{A} \cdot d\bar{l}$$

Now if you evaluate the integral for the contour shown in the above figure, the result would be

$$\Psi = \oint_C \bar{A} \cdot d\bar{l} = A_1 l - 0 - A_2 l - 0$$
$$= (A_1 - A_2) l$$

where, *l* is the length of the segment of the contour directed in z direction. The other two segments in the x direction will not contribute because those lengths are perpendicular to A. So $\mathbf{A} \cdot d\mathbf{l}$ will be 0. And in one of the two segments along z, A and dl are oppositely directed. So that is why you have got a minus sign.

So then you have got $\psi = (A_1 - A_2)l$ for 2D approximation. Again in one of the previous lectures we discussed that whenever we do 2D approximation, we take 1 m depth in z direction and that is why, if we do that, then the value of l in the above expressionwill be 1 m in z direction. And then the expression of ψ is will just reduce to $A_1 - A_2$. Now coming to the following figure again,



if you take any 2 points on one of the contours as shown in the above figure, these points 1 and 2 are at same distance from the current source. So then $A_{1z} - A_{2z}$ will be equal to 0, because no flux is crossing the surface. That is why these contours are called as equi-A or equipotential contours or flux contours. Because the magnitudes of A_1 and A_2 are same, no flux crosses the surface formed by lines along z direction passing through these two points, in fact flux is going tangentially at these points.

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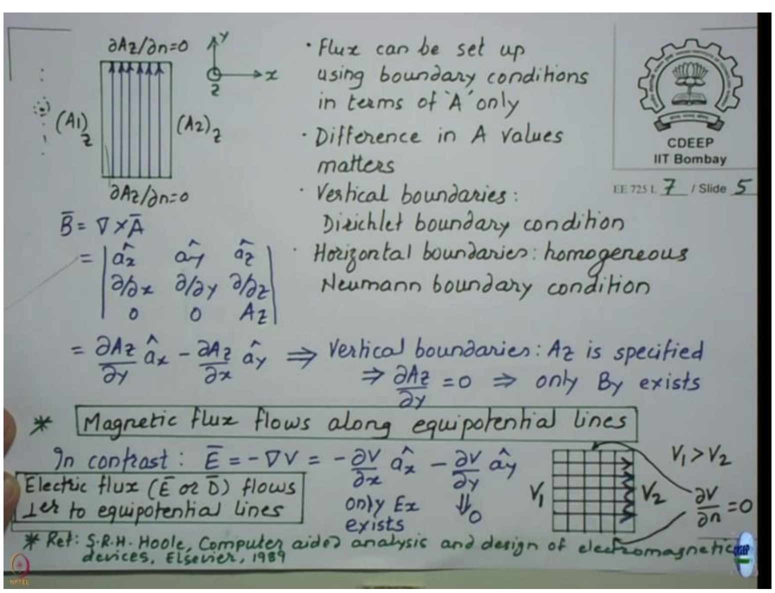
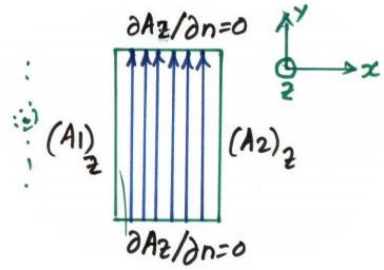


Another important point is that flux can be set up by using only boundary conditions in terms of **A**. Now for example consider the following rectangular geometry.



This could be a magnetic material. And suppose you want to set up the flux which is going vertically upwards as shown in the above figure. You can easily do that by imposing the boundary conditions as indicated in the figure. Like on the vertical edge on the left hand side there is A_1, now remember A_1 is z directed. So the plane of the paper is *xy* plane and **A** all along the edge is in z direction . Here on the other vertical edge, A_2 directed in z direction is defined.

And on the two horizontal lines of the rectangle, you have homogeneous Neumann condition. I have explained in one of the previous lectures, homogeneous Neumann condition, that means the normal derivative of the field variable is 0. So now for the top horizontal line, the unit normal will be $\hat{\mathbf{a}}_y$. See, why it is normal? Because normal is always associated with a surface, so then you may be wondering, where is the surface in the above figure. Always remember practical things are 3D. So here also the rectangular block shown in the figure is a 2D approximation of a 3D cuboid.

The third dimension is into the paper and it is 1 meter depth. So that means the length $\times$ 1 meter and the corresponding surface formed by the 1 m depth and the top edge will have the unit normal. So the unit normal is always associated to a surface. Always you have to imagine that this is a 2D approximation. The actual 3D figure and the corresponding surface here is formed by the edges in the figure line and 1 meter depth segment.

The unit normal will be outward for each surface since it is a closed volume. The difference in boundary conditions in terms of A matters, that means $A_1 - A_2$ will create flux in this block. So you take the difference accordingly to set up a given flux and flux density, it does not mean that there is no current source anywhere. The current source is there and that current source could be distributed current source as shown with dotted lines in the above figure. I have shown the direction of the current purposely with a dot here because that only will produce the flux in this direction.

There is a distributed current source which will give this condition that A is coming out of the plane of the paper because the current is also coming out. Why I am saying here distributed current source? If it is a point source, you will not get constant A values on the vertical edges of the above rectangle. Then, the flux contours become circular, the values will be constant only on the circle. So that is why I am saying some distributed current source only will give you constant **A** values on the vertical edges. Now on vertical boundaries we have Dirichlet boundary conditions and on horizontal boundaries we have homogenous Neumann boundary conditions.

Now these Dirichlet boundary conditions are not homogeneous because potentials defined on these edges will have some finite value. Suppose if the value of A_2 is defined as 0 then the value of A_1 should be changed. Because only the difference of A matters to set up flux. So if A_2 is defined as 0 and then A_1 correspondingly should have some number which sets up a given flux. Then this A_2 will become homogenous Dirichlet boundary condition. Whenever the potential specified is having some finite value, it is non-homogenous. If the potential specified is 0, it is called as a homogeneous Dirichlet boundary condition. Now consider the following expression of **B** in terms of **A** (for 2D) which we have seen earlier.

$$\bar{B} = \nabla \times \bar{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 0 & A_z \end{vmatrix} = \frac{\partial A_z}{\partial y}\hat{a}_x - \frac{\partial A_z}{\partial x}\hat{a}_y$$

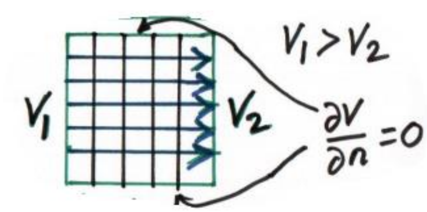
Here the above expression is valid for 2D because there is only A_z component and A_x and A_y are 0, because current is directed in z direction. So A is directed only in z direction. So A_x and A_y components are 0. That is why you get a simplified expression for **B** in the two dimensional case. Now let us discuss a very important point which is not described in many textbooks of electromagnetics or even in books of numerical techniques on electromagnetics. But this actually explains the magnetic field lines and electric field lines clearly and the corresponding difference between them.

So now on the two vertical boundaries here, we have defined A_z, that means $\frac{\partial A_z}{\partial y} = 0$ because A_z is constant on those vertical boundaries. So $\frac{\partial A_z}{\partial y} = 0$, that means the y component of **B** becomes 0 and only x component exists. So in this case, the flux lines or the equi-A lines and the B vectors, they flow together because both are in the same direction. In this example, it is y direction. So that is why the magnetic flux flows along equipotential line in magnetics. Now let us see the corresponding thing in electric field lines case.

In contrast, $\mathbf{E} = -\nabla V$ and then you have got the following expression for $-\nabla V$.

$$\bar{E} = -\nabla V = -\frac{\partial V}{\partial x}\hat{a_x} - \frac{\partial V}{\partial y}\hat{a_y}$$

And now let us take again a simple parallel plate capacitor case shown in the following figure.



Now here, I am taking the capacitor plates which are vertically oriented. So you have V_1 and V_2 specified as shown in the above figure. Now we know that the equipotential lines will be vertical and the E vector will be horizontal and orthogonal to equipotential lines. Let us verify that they indeed are orthogonal through the equation $\mathbf{E} = -\nabla V$.

Here on the vertical boundaries of the above figure we are specifying V_1 and V_2. So along the vertical boundaries $\frac{\partial V}{\partial y} = 0$. Therefore only E_x exists. So it is clear that only E_x exists and that is what we have got, that E field vectors exist in x direction only. E vectors and V lines are orthogonal to each other. Remember it is E or D that flows perpendicular to equipotential lines because you are specifying V on the vertical boundaries. V is independent of y. So $\frac{\partial V}{\partial y} = 0$.

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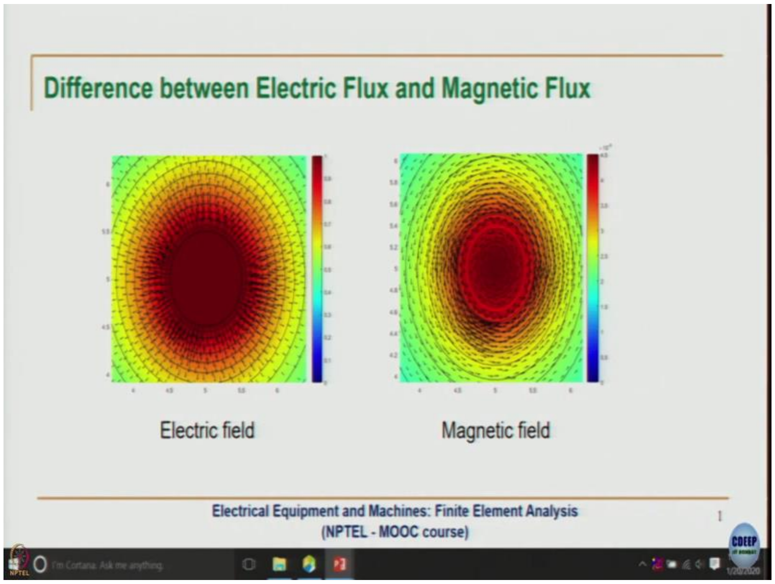

The above slide shows clearly the same difference between electric flux and magnetic flux. So here in this slide, you have two plots, one for electric field, and the other for magnetic field. In case of electric field, you have a high voltage conductor in the vicinity of some ground conductor which also could be concentric in this case because the equi-potential lines are symmetrical and concentric about the circular conductor. So here you have equipotential lines circular and E fields are crossing the equipotential lines orthogonally. And as you see in the above slide , there is no electric field inside this conductor on the left hand side, because the conductor is assumed to have very high conductivity which is tending to infinity, and that is the main difference between the electric and magnetic fields.

In magnetic field case, you have a current-carrying conductor shown on the right hand side of the slide, and there is internal as well as external field. So the field is there inside as well outside the conductor and you can see there are no orthogonal field vectors crossing the equi-potential lines. So that confirms the statement that we made that in case of magnetics, field vectors flow along the equipotential lines. In the case of electric fields, the E field flows orthogonally to the equipotential lines. So that brings out the difference between the electric and magnetic fields. With this, we end the seventh lecture. From tomorrow's class, we will further see magnetics and later on, we will see time-varying fields.

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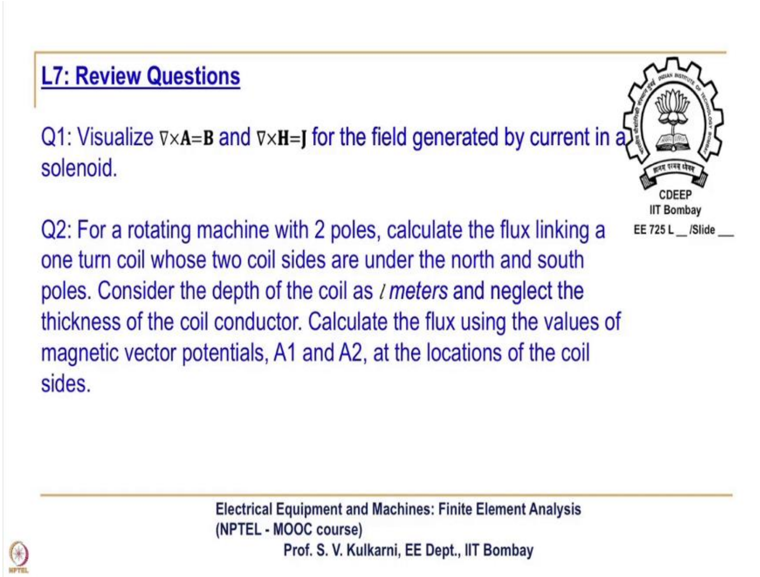

Thank you.