Electrical Equipment and Machines: Finite Element Analysis
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Revisiting EM Concepts: Electrostatic Boundary Conditions and Shielding
Lecture 6

Welcome to the sixth lecture of this course. At the end of the fifth lecture, some students asked questions about polarization phenomena. I will explain it little bit more in detail.

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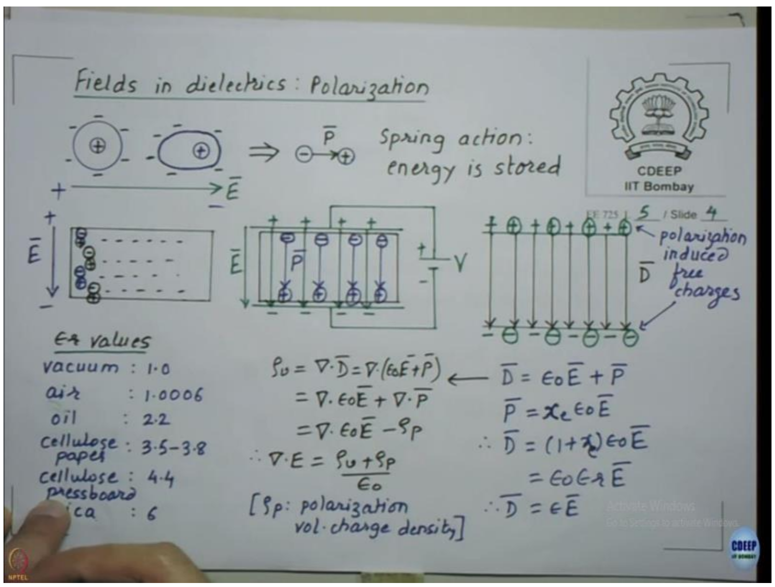

When we actually represent the polarization phenomena by additional induced positive and negative charges enclosed in circles on the capacitor plates as shown in the above slide, effectively, we have eliminated that dielectric; in the sense we do not have to consider polarization vector and all that. That is all now implicit into these additional positive and negative charges. So again, we don’t have to bring in that polarization P vector, once we have taken its effect into account by these additional positive and negative charges.

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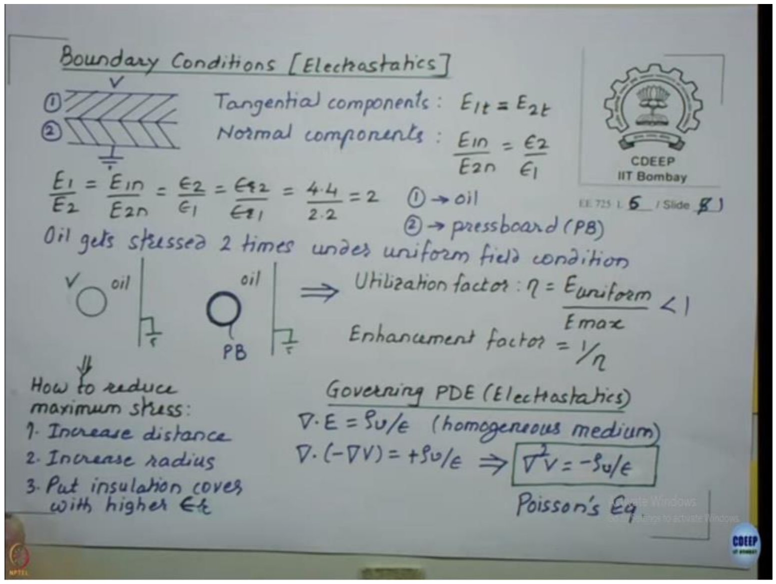
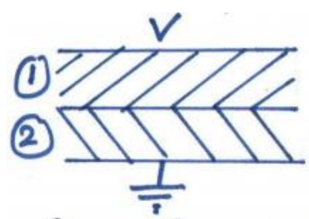


Now, coming to the sixth lecture, we will get into the electrostatics. Now if you take a composite dielectric with the two materials – 1 and 2 as shown in the following figure.



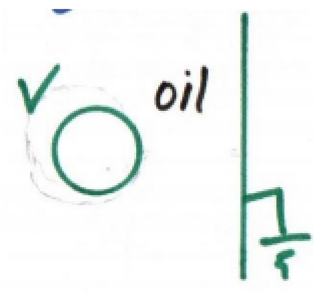
Here, 1 is oil and the other is the pressboard. Oil's dielectric constant is 2.2 and for pressboard it is 4.4. The tangential component is continuous, $E_{1t} = E_{2t}$. And the normal component is discontinuous in the inverse ratio of the two permittivities as given below.

$$\frac{E_{1n}}{E_{2n}} = \frac{\epsilon_2}{\epsilon_1}$$

So now, actually for this configuration, $\frac{E_1}{E_2} = \frac{E_{1n}}{E_{2n}} = \frac{\epsilon_2}{\epsilon_1} = \frac{\epsilon_{r2}}{\epsilon_{r1}} = \frac{4.4}{2.2} = 2$

because tangential component in this case is 0, is it not?

The tangential component is 0 because we are assuming the configuration as a parallel plate capacitor and the field lines are all vertical. So there is only a normal component. Now, $\frac{E_{1n}}{E_{2n}}$ is equal to $\frac{\epsilon_2}{\epsilon_1}$ which is nothing but $\frac{\epsilon_{r2}}{\epsilon_{r1}}$ which is $\frac{4.4}{2.2}$ and that is 2. That means oil gets stressed 2 times more compared to pressboard under uniform field conditions. So what is this uniform field? Individually if you see, in oil and solid insulations, the field is uniform. Of course, there is a discontinuity of the field value across the interface but individually, the field is uniform in both the dielectric materials. Now, we define what is called as utilization factor which can be defined for the following case which we have seen earlier also.



Suppose you have oil in between the high voltage lead and ground as a dielectric medium, then you will have, uniform field when this configuration is replaced by a parallel plate capacitor. That means, instead of this lead, if you place a plate, then the field will be uniform with the same distance.

So with respect to that, the maximum stress will be at the point (corresponding to the minimum distance between the two electrodes) that we defined in the previous lecture. And its magnitude will be more compared to the uniform condition with two parallel plates. So, that’s why the unitization factor is defined as

$$\text{Utilization factor}: \eta = \frac{E_{uniform}}{E_{max}} < 1$$

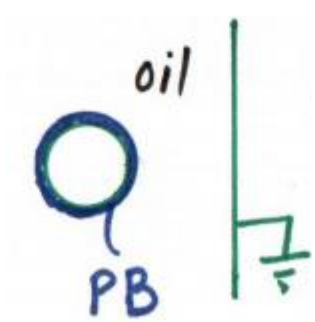
Here the value of the E_{max} is going to be higher as compared to $E_{uniform}$. So, the value of utilization factor is always less than 1. Whereas in some textbooks and some research papers, they talk of enhancement factor, wherein the enhancement factor is an indicative of how much field is getting enhanced due to non-uniform fields. So the enhancement factor will be defined as:

$$\text{Enhancement factor} = 1/\eta$$

The ideal case is that you should have the utilization factor close to 1. So, the utilization factor being close to 1 means what? Everywhere the insulating material is getting uniformly stressed, so all the regions of the material are getting optimally utilized. For an insulation system, the moment you have the utilization factor less than 1 or the enhancement factor greater than 1, somewhere the field stress is more, somewhere the field stress is less; that means the material is not optimally utilized in various regions. So when we do finite element analysis for insulation design, we do some kind of parametric studies to reduce the maximum stress and to make the utilization factor as good as possible.

One option is you can increase the distance between the lead and ground but that will be uneconomical. Because, if the distance or the clearance increases, the size of the system will go up. Second: increase the radius of this high voltage lead. That is a good option and that is generally done if the standard lead sizes with bigger radius are available and if the space constraint is not there. But that also may have some limitations, because you are going to put more copper because this lead is a metal.

The question asked by one of the students is when we increase the radius, whether the E_{max} will increase, what is done here is, the distance between the lead and ground is kept same and the radius will be increased. The clearance will be maintained and the radius will be increased. So the stress will come down here. The third option is, put insulation with higher dielectric constant (pressboard (PB)) over the high voltage lead as shown in the following figure.



What this is going to achieve? As per the theory that we just saw; moment you have a higher dielectric constant material, the maximum electric stress on the lead is going to reduce. So on the conductor surface, the stress is going to go down because now there is a solid insulation with higher dielectric constant. And inside the oil, the maximum stress is going to be just at the surface of the solid insulation. There also, the stress value is going to go down because you are going a little bit away from the high voltage electrode. So the conditions will be a little bit better as compared to the bare conductor case. The governing equation for electrostatics is starting with $\nabla \cdot E = \frac{\rho_v}{\epsilon}$.

Replace E by $-\nabla V$ and then you get this well-known Poisson's equation $\nabla^2 V = -\frac{\rho_v}{\epsilon}$. Now here, the assumption we are making is that the material is homogeneous. Otherwise, if it is not a homogeneous material, then this ϵ will become a part of the opearand as given below.

$$\nabla \cdot (-\epsilon \nabla V) = \rho_v$$

Because there are partial derivatives with respect to x, y, z, if the dielectric constant is a function of xyz, then that dielectric constant should be a part of operand.

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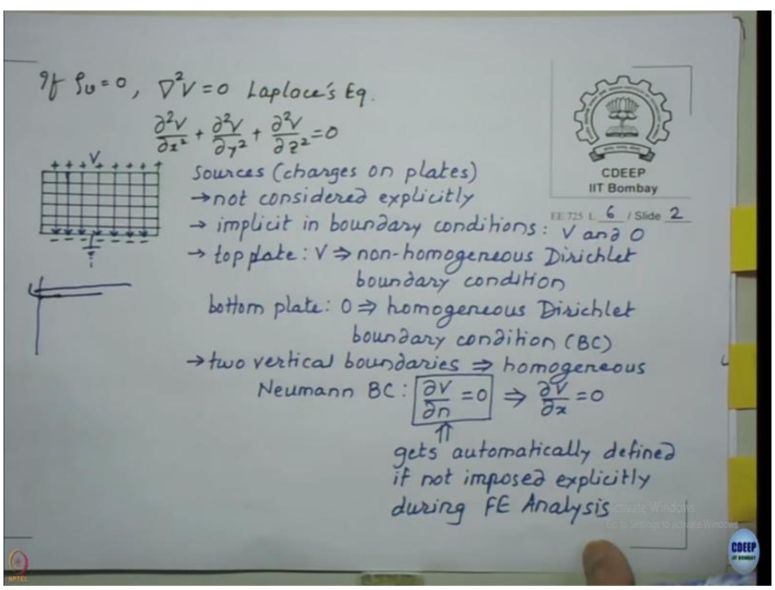
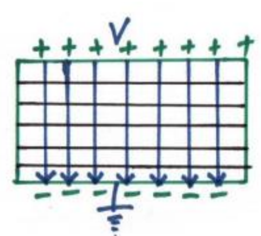


For the case shown in the above slide, if volume charge density ρ_v is 0, then we get the well-known Laplace's equation. And let us understand a little bit about this problem. When we expand Laplace's equation, that is the ∇^2 operator, you get the following equation.

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

Now again, consider a simple parallel plate capacitor problem given in the above slide. Although it looks very simple, a lot of concepts are there to understand. Now, here the equipotential lines are horizontal and E field lines are vertical and they are orthogonal to each other. Now the first question is when we actually solve Laplace's equation for this capacitor problem and of course this is trivial, since fringing of fields is neglected, you do not have to solve the problem using a numerical technique. The equipotential lines (solution for this problem) are shown in the above slide.

But for the sake of argument, suppose we have to model this and solve using FEM, then we have to solve this Laplace's equation. Then we are imposing the voltage difference in terms of boundary conditions. But here now, you have charges as shown below; positive and negative charges and the corresponding voltages are also given in the following figure.



So when you solve this using FEM, you do not worry about these charges (source). What is a source? In electromagnetics, there are only 2 sources; one is charge and the other is the current. Actually, if you see, the source is only charge because current is only a manifestation of moving charges.

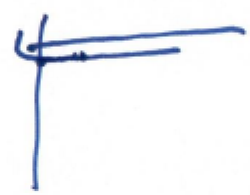
But for the sake of discussion, we consider 2 sources – one is charge and the other is current. So here, basically, the charge distribution shown in the above figure is giving this potential V. So now but when we do FE analysis of this problem, since in finite element method we solve it as a boundary value problem, we only define the boundary potentials. We do not worry about the charges because charges are implicit in the voltage definition, that is the boundary condition, is it clear?

The question asked by one student is – what is this implicitness? Basically, this charge distribution is what giving the voltage V. Now these charges have come from some battery that is connected to charge this capacitor. The charge distribution would be such that it will give the corresponding voltage V. A little bit more about this, when we consider the end effects. Since we are not considering the end effects here, you are getting uniform field and at edges the field is uniform. So now before going to those end effects, let us discuss boundary conditions.

So for the top plate, you are defining the voltage V and it is called as non-homogeneous Dirichlet boundary condition. Whenever you specify voltages, then it is called as Dirichlet boundary condition. And if the voltage is non-zero it is called as non-homogeneous, if the voltage is 0 it is called as homogeneous Dirichlet condition. So this terminology you will quite often see in the textbooks on FEM and Electromagnetics. Bottom plate voltage is 0. Now on two vertical boundaries, what effectively is happening? There, we have defined homogeneous Neumann condition. Neumann condition means the derivative of the field with respect to the distance. Now, what is this n? n is the unit normal. The normal to the vertical boundaries will be in x direction.

That is why it is $\frac{\partial V}{\partial n} = 0$, effectively $\frac{\partial V}{\partial x} = 0$. Because we have defined homogeneous Neumann condition on the vertical boundaries, and effectively the field has become uniform. Suppose if you want to analyze the end effects, what you have to do, we will see in the next slide. Another point to note is, those who have used FEM and solved some electrostatic problems, they will realize that they never define this $\frac{\partial V}{\partial n} = 0$. They just define the top plate and bottom plate potentials. These vertical homogeneous Neumann conditions are not explicitly defined. They get automatically defined during the FEM analysis. How? We will see when we understand the FEM theory. There I will explain how they get automatically imposed even if you do not define them even in commercial software. Now the point to be noted here is that, we have defined homogeneous Neumann condition here, on these vertical boundaries. That means what? Here suppose you take the vertical boundary on the right hand side and the boundary condition is $\frac{\partial V}{\partial x} = 0$, that means the voltage at a point on the right hand side of the boundary is same as the voltage at the corresponding point on the left hand side of the boundary. That means you are making the horizontal lines shown in the figure as equipotential lines.

If homogenous Neumann conditions are not imposed, then at the corner point of the top plate the equipotential lines are going to turn sharply like the way shown in the following figure.



So in fact, $\frac{\partial V}{\partial x} \neq 0$ if you want to consider the end effects. By imposing homogenous Neumann conditions and not considering the outer boundary we have ensured that the field is uniform at the ends. The question asked by one of the students here is, when we do not consider the end effects, what will the potential distribution even above and beyond the plates?

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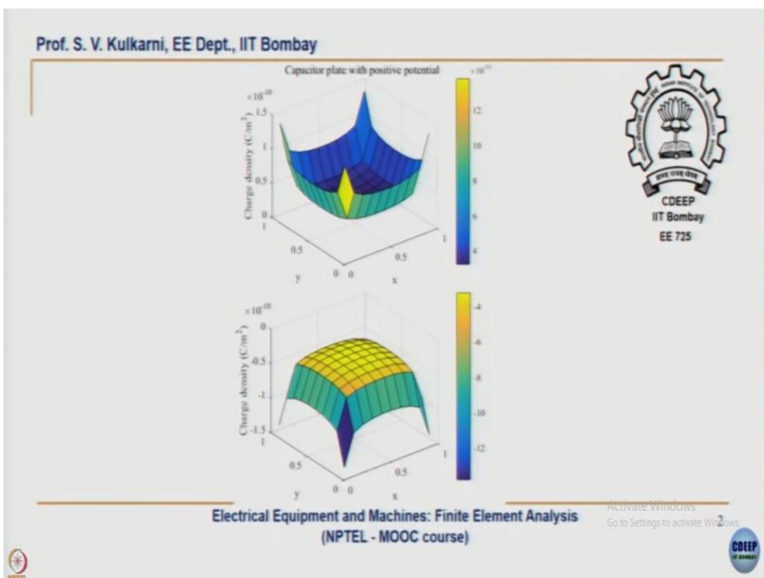

We will understand the answer for this question using the above slide which shows charge density distribution on top and bottom plates of a capacitor and this is a three dimensional distribution and with all end effects considered. So you can see at all the four corners of the top and bottom plates, you have a sudden and sharp rise in charge density distribution: positive charge density on the top plate and negative charge density on the bottom plate.

Because of the sudden rise in charge density at the four ends of both these plates, you have a sudden increase in electric field intensity because of sharp corners. And therefore those increased values of electric field intensity have to be supported by corresponding higher charge densities. So it is very clear from this the three dimensional distribution of charge densities, the corresponding end effects and the high rise of electric field intensity when end effects are considered.

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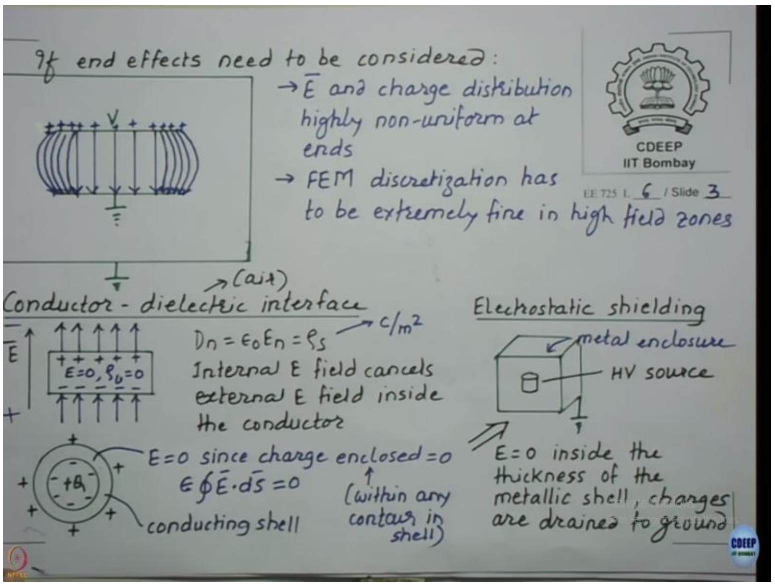
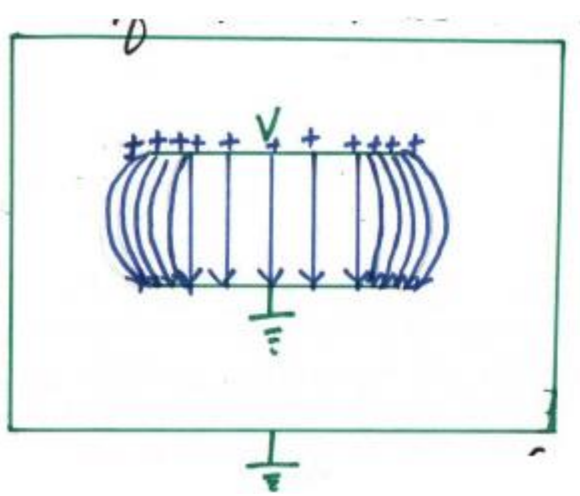


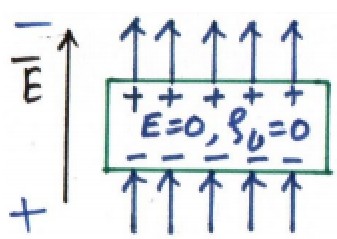
Now the same 3D configuration that we saw in the previous slide is shown in 2D in the following figure, because here on the paper, I can show in 2D only.



So now what I have shown? The charges in the middle, they are sparsely distributed and charges at the end, they are highly concentrated because the electric field intensity is high. That is what we saw in the previous slide. So to reproduce exactly those high fields and high charges at the ends, you need to have very fine FEM discretization or mesh at the ends. And how to achieve this we will see when we start looking into FEM.

Also, remember that when we want to take the end effects into account, we have enclosed the capacitor configuration in a bigger box as shown in the above figure. So the outer box is like a bigger rectangle and in 3D, it will be a cuboid. And these distances from the capacitor plates to the box should be sufficiently high so that the boundary conditions do not affect the field distribution. So that is one thing. Also remember that the field distribution will not just be inside the capacitor as shown in the figure but there will be some field lines that originate from the positive charges on the outer surface on the top plate and terminate on the outer surface of the bottom plate. If you actually do the FEM analysis, you will see that.

The field is everywhere inside the box, although the magnitude of field will be much less in the outer part. So we will take one more this last point and then we will go to the next topic. Now we will see the conductor dielectric interface which is very important and there are some very interesting applications. So first is, let us have an isolated conductor in electric field as shown in the following figure.

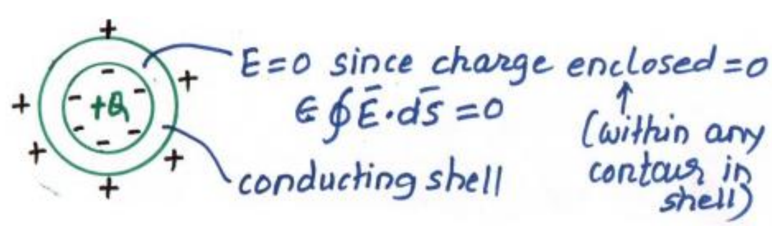


Now the electric field is the vertical direction, and you are keeping an isolated conductor. The moment you keep it, the E field is everywhere in the vicinity of the conductor. The positive charges will get pulled up and the negative charges will go down. The top surface of the conductor is positive and the bottom surface is negative.

The electric field lines inside the conductor will be from the top surface to bottom surface (vertically downward). There is no volume charge inside the conductor because all the charges got displaced towards the surfaces. So there is no volume charge and there is only surface charge. That means in the first Maxwell's equation $\nabla \cdot D = \rho_v$ should be modified because there is no volume charge. But now since there is no volume charge, the corresponding equation gets sort of modified as $D_n = \epsilon E_n = \rho_s$ where ρ_s is the corresponding surface charge density. This is a manifestation of the first Maxwell's equation written in a different form because now instead of volume charge density, you have surface charge density.

It makes sense because ρ_s has the unit of C/m^2. So that matches with the unit of D which is also C/m^2. That is why $D_n = \rho_s$. The internal E field also has to be 0, because inside this conductor there are no charges within the volume. In electrostatics, when currents are not flowing, there are no charges, you take any Gaussian surface inside the conductor and calculate $\oiint \epsilon \mathbf{E} \cdot d\mathbf{S}$, you will get identically 0 because the charge enclosed is 0. And if you calculate E that also will be equal to 0, because there are no charges inside. But the electric field intensity is only outside the plate. And why electric field is 0 that can be also understood by noting that internal E field gets cancelled because the E field due to surface charges is vertically down. That will exactly cancel this external (applied) field. That ensures this condition that E is 0 inside the conductor, because there are no volume charges. So both things are in sync with each other. Now there is one very interesting example of electrostatic shielding and Faraday cage. So what is that? Let us understand.

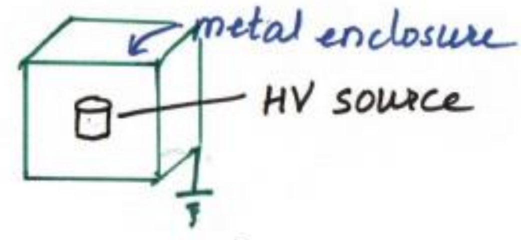
Suppose you have a conducting shell, maybe you can assume a spherical conducting shell which has some space inside and there is some charge inside that spherical shell. That is plus Q as shown in the following figure.



Now that plus Q is going to induce negative charges. When I say induce, that means it is going to pull the negative charges by attraction. And the inner surface will get negatively charged as shown in the above figure and the outer surface of the shell will get positively charged. So now if you take any point in the conducting shell, E is equal to 0 by the same logic that we just now saw and that can be reconfirmed by taking a Gaussian surface. And now the net charge enclosed is 0 by any surface inside the conductor because the positive Q and negative Q will cancel each other.

The net charge enclosed will be 0 in the Gaussian surface taken inside the conducting shell. So that means what you have ensured is E is 0. But when you take a practical application, you cannot have these positive charges remaining on the outer surface of the shell. You have to drain them to ground. Otherwise, if you put it like the way shown in the above figure, the outer surface charges can be quite high depending on the value of plus Q inside the shell and corresponding voltage will be high. So that is why you need to ground this outer surface and then it becomes a perfect shielding.

That is the principle behind this electrostatic shielding. Inside a Faraday cage, which is a metallic enclosure, there is a high voltage source as shown in the following figure. And then we ground that Faraday cage. And then what happens?



It is the same thing, the only difference is the conducting shell is a cube as shown in the above figure with some finite metal thickness and now if we ground the outer surface of the cube, all these positive charges will go to ground and within the metal thickness, the E is going to be 0 here because of the same phenomena that we discussed. If the charge of the HV source is plus Q, there will be minus Q on the inner surface of this metallic enclosure and if you take any Gaussian surface within the thickness of that Faraday cage, the total charge enclosed will be 0. That is why E everywhere will be 0 inside that.

You have effectively isolated the inside and outside environments because there is no E field in between. So there is some region which is separating inside and outside regions wherein E is 0. That means there will not be effects of outside field on the inside field and vice versa. Is this point clear? So that is the principle behind this electrostatic shielding. I will just repeat what we just now said about this electrostatic shielding.

You have some high voltage source which represents this positive charge Q. And around it you have this conducting shell and then you have negative charges induced as shown on the inner surface of the shell and positive charges charges on the outer surface of the shell. And since there is no charge inside this conducting shell, E is 0 and then that is how you can say that the 2 inner and outer regions of the shell got sort of electrically isolated because E is 0.

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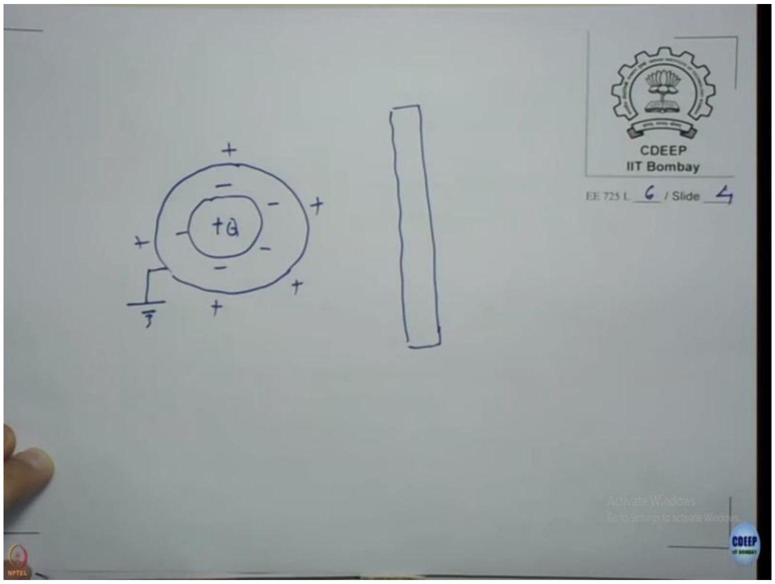
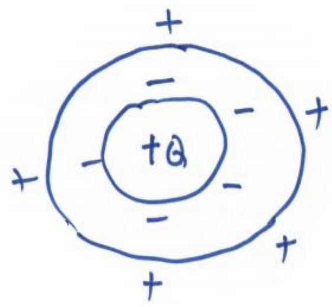
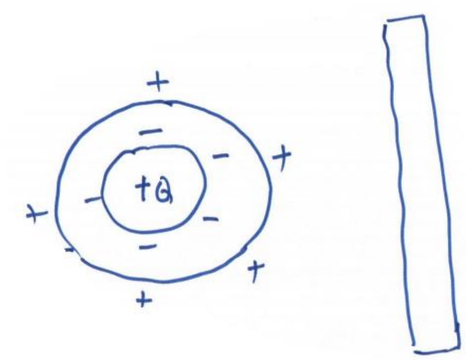


Now, we will just understand further this electrical isolation of the +Q inside the conduction shell using the following figure.



Now suppose if the plus Q charge which is shown concentric with the spherical shell in the above figure is shifted to the right hand side, what will happen? These negative charges on the right hand side of the inner surface will be more. And negative charge density on the other side of the inner surface will be less. But what will happen to these positive charges on the outer surface? They will remain as they are because there is no connection between the positive charges on the outer surface and the negative charges on the inner surface, because E is 0 in between the two surfaces.

But suppose if I bring a ground plate in the vicinity of the above-discussed system as shown in the following figure,



what is going to happen? The positive charges will get concentrated on the surface which is facing the ground plate. So when we initially discussed the electrical isolation system, we assumed that there are no other charged objects in its vicinity. And then that is why these positive charges are independent of the movement of this high voltage source or plus Q source which is within this innermost part of the spherical shell. If you have understood this, then as I mentioned in the previous slides, we should ground the outer surface, so that the positive charges on this surface will get drained to the ground and then this will be electrically safe even to touch.

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Niow, I will show you one interesting application and practical demonstration of this electric isolation or electrostatic shielding concept. In the above slide, what you see is a high voltage source, this source is a high voltage transformer for which 230 V is the input and 100 kV is the output. You can see all these electrodes which are at high voltage are smooth and round electrodes because otherwise they will lead to high voltage stresses and would themselves become sources of corona or partial discharge.

The high voltage equipments is enclosed in the aluminum cage as shown in the above slide. And here you can see there is a copper strip which runs at the bottom level of this Faraday cage from inside. And this copper strip connects positively all these individual panels of the aluminum to ground through an earth pit.

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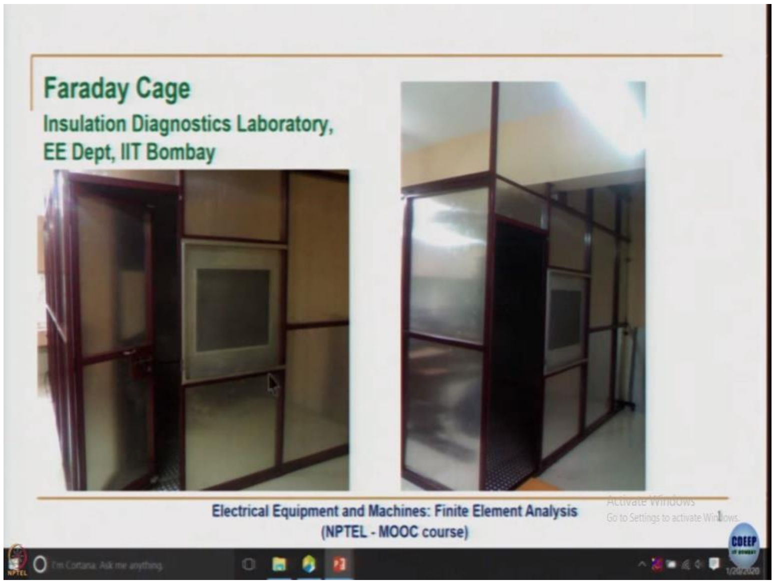

High voltage source is inside this cage and there is a mesh as shown in the figure on the left hand side of the above slide. So this cage is grounded through an earthing pit and becomes a case of electrostatic shielding. And here, the thickness of this aluminum cage is not so much important from the point of view of electrostatic shielding. You can use whatever thickness mechanically sufficient and commercially available. But later on, we will see when we are interested in electromagnetic shielding the thickness of the metal is important. What we just saw is electrostatic shielding from the point of view of high voltage.

But suppose you are conducting some high voltage test like corona or partial discharge studies inside this setup, these tests generate high-frequency electromagnetic waves, those waves are shielded from going out and affecting some electronic products. Also, if there are some discharges happening outside, those discharges should not affect the tests that are being conducted inside. So either ways, you want electromagnetic shielding. Now, this is high-frequency shielding. So for that, later on when we study eddy currents, we will see that you need to have a finite thickness of this shield depending upon the frequency of interest. That we will study later. What we just saw was only the electrostatic shielding. The electromagnetic shielding, we will discuss little bit later.

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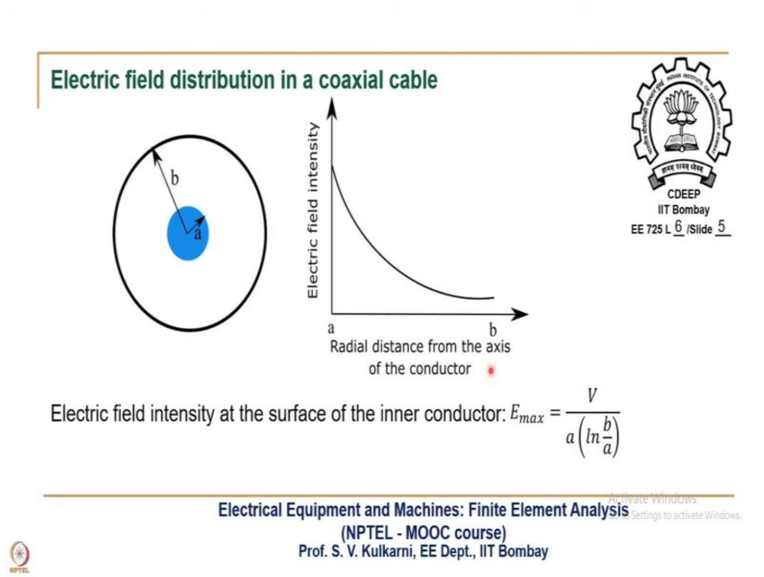

Before ending the discussion on electrostatics, let us discuss one practical example of condenser bushings in which capacitive grading is done to improve E field distribution. We know for a coaxial cable configuration with two cylindrical electrodes as shown in the above slide, the E field is non-uniform as given by the curve shown in the slide. The electric field intensity is high on the inner conductor surface. And as we go away from this inner surface in this insulation, the region near the outer electrode is less stressed.

Stress is higher at the inner conductor and it is lower at the outer conductor and therefore, that is not a good insulation design. E_{max} is given by the following formula,

$$E_{max} = \frac{V}{a\left(\ln\frac{b}{a}\right)}$$

How do we make this electric field intensity inside the insulation more or less uniform? We cannot make it absolutely uniform. But how do we make it close to uniform distribution? There are different ways in which one can achieve this. One of the common ways, we will discuss in the next slide.

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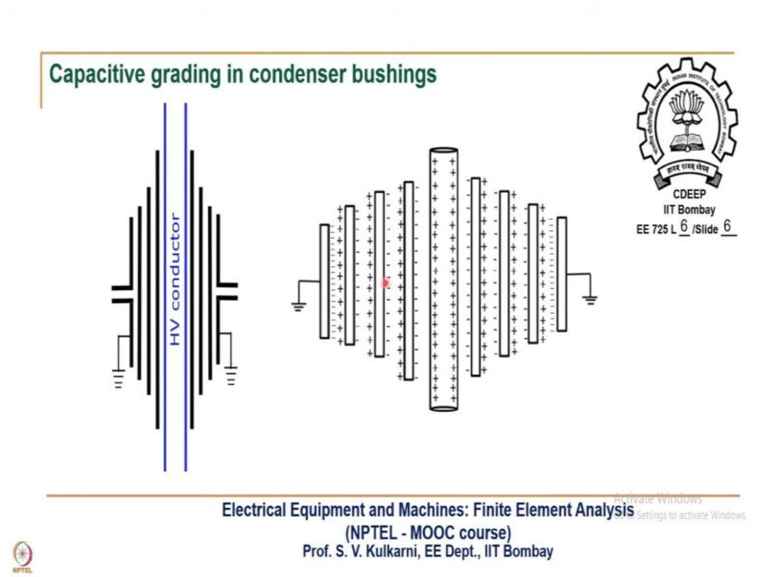

Let us understand how capacitive grading in condenser bushings achieves more or less uniform E field distribution. What are condenser bushings? They connect transformer's high voltage terminals to transmission lines. A condenser bushing consists of a central HV conductor which on one side it connects to a transformer winding, on the other side it connects to a transmission line. So what is done here is, the HV conductor is taken out of the transformer tank through the opening nd the black colour lines shown in the figure on the left hand side of the slide are the metal flanges which are at ground potential. So this HV conductor has to be properly insulated from the ground.

If we put simply paper insulation throughout on the conductor, then it will be non-uniformly stressed as explained in the previous slide. So, we insert here electrically floating cylindrical conductors to make the stress uniform and they get potentials based on their position. And in between these conductors, of course, there is paper insulation. Now, this actual configuration can be equivalently represented by this schematic, which is not to the scale and central conductor is assumed to be at some positive potential at the instant being considered. So there will be some positive charge on that central conductor. So now these floating metallic coils will get charged to negative and positive values as shown in the figure on the right hand side of the slide and that too alternately.

So now consider a foil which is just adjacent to the HV conductor, we will have negative charge on the surface that is facing the conductor and on the other surface of the foil you will have positive charge as shown in the above slide. Again negative and positive charge on the next cylindrical foil conductor and so on. So thing to note here is this total negative charge on the inner surface of this first foil which is adjacent to the conductor will be numerically equal to the positive charge on the HV conductor. Of course the positive charge on the other surface of the foil will be again equal to this negative charge in terms of magnitude. If we have understood this charge distribution, let us then go further and understand how uniform E field distribution is achieved.

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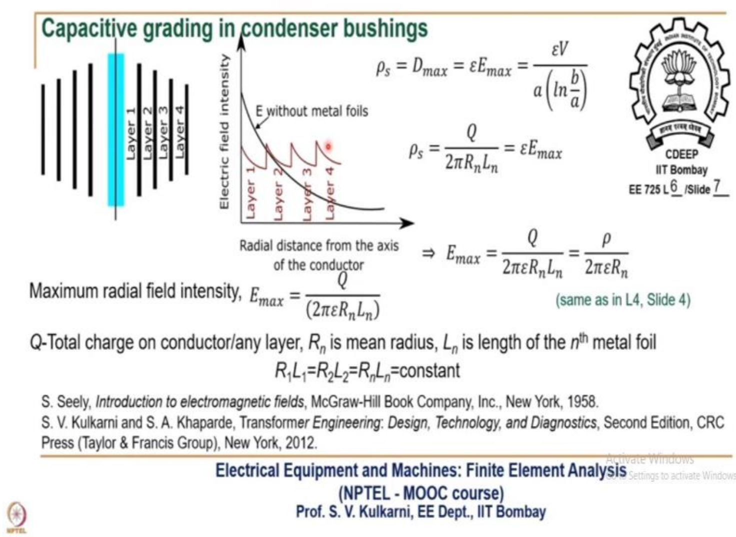

In 2D cross-section, these floating metallic cylindrical foils will appear as line charges. And for line charges on an n^{th} metal foil, E_{max} is given by the following standard formula

$$E_{max} = \frac{Q}{2\pi\varepsilon R_n L_n} = \frac{\rho}{2\pi\varepsilon R_n}$$

where R_n is mean radius and L_n is length of the nth metal foil. So let us derive the above formula from fundamentals. If you consider insulation between any 2 conductors, for example let us take the first two conductor foils which are floating at some potential. ρ_S will be given by the following expression and and D_{max} will be normal to the conductor; so it is in the horizontal direction.

$$\rho_S = D_{max} = \epsilon E_{max} = \frac{V}{a\left(\ln\frac{b}{a}\right)}$$

And what is ρ_S? ρ_S is the surface charge as we saw in the previous slide. So $\rho_S = D_{max} = \epsilon E_{max}$. And what is E max? E max is $\frac{V}{a\left(\ln\frac{b}{a}\right)}$ as seen on the fifth slide. So going further, ρ_S can also be written as Q upon surface area. And surface area is $2\pi RL$ because $2\pi R$ is circumferential length and L is length or height of the conductor. Therefore, $\rho_S = \frac{Q}{2\pi\epsilon RL}$. Using the above relation of ρ_S and E_{max}, we get

$$E_{max} = \frac{Q}{2\pi\epsilon RL}$$

Now if you replace $\frac{Q}{L_n}$ as ρ which is charge per unit length in C/m, then we get the following formula which was given in lecture four, slide four.

$$E_{max} = \frac{Q}{2\pi\epsilon R_n L_n} = \frac{\rho}{2\pi\epsilon R_n}$$

So now coming back to how do we get improved E field distribution in this formula, if we maintain E_{max} constant in each of these capacitors, now each set of two foils forms a capacitor. So in each of these capacitors, we can maintain E_{max} as constant by keeping this $R_n L_n$ product as constant. Q is anyway same for all these conductors as was explained on the previous slide.

So as the mean radius (R_n) is going up as you go away from the central conductor, what you do is, you have to reduce the height or length of this floating metallic conductors and by virtue of that, you can maintain E_{max} as constant and because of that, more or less uniform E field distribution is obtained. Now let us understand this more intuitively. When we are reducing height of these layers, what actually we are doing is, we are reducing the effective capacitance between these conductors? Why? Because as per the standard capacitance formula $\frac{\epsilon A}{d}$, the capacitance will be decreased as the surface area is reduced .

So if the surface area (A) reduces, the distance between the plates (d) being constant, the capacitance is reduced but the charge is same, so V will increase. Increase in the potential of the foil conductor reduces the electric field intensity between these two conductors. So this is how E_{max} comes down. And then E_{max} can be maintained constant by keeping the $R_n L_n$ product as constant. It should be remembered that here we have done some approximations, for example, we have neglected fringing at the ends. If we have to consider fringing, then of course, we need to do finite element analysis. And then how we do that for a typical high voltage insulation system, we will do that kind of analysis in one of the lectures later. So with this, we conclude lecture 6 and we have finished electrostatics part of our course and in the next lecture we will start with magnetic fields. Thank you.

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