Electrical Equipment and Machines: Finite Element Analysis
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Lecture No. 38
Nonlinear FE Analysis

Welcome to lecture 38. Now, we will go to the next complexity in our FEM formulations. In this lecture, we will consider nonlinearity which we have not considered till now. In this course, first we saw simple static cases then we went to time harmonic cases and then we considered voltage fed or current fed coupled circuit field cases. We also saw the complexities like moving mesh in case of rotating machines. But, in all these problems we had considered essentially linearity.

Now, we will see how to handle nonlinearity of magnetic materials and then the formulation can be integrated into any of the formulations that we have seen earlier to solve any nonlinear problem in two dimensions. We will handle the nonlinearity by using the well-known Newton Raphson technique which is widely used in power systems and power system analysis because of its fast convergence.

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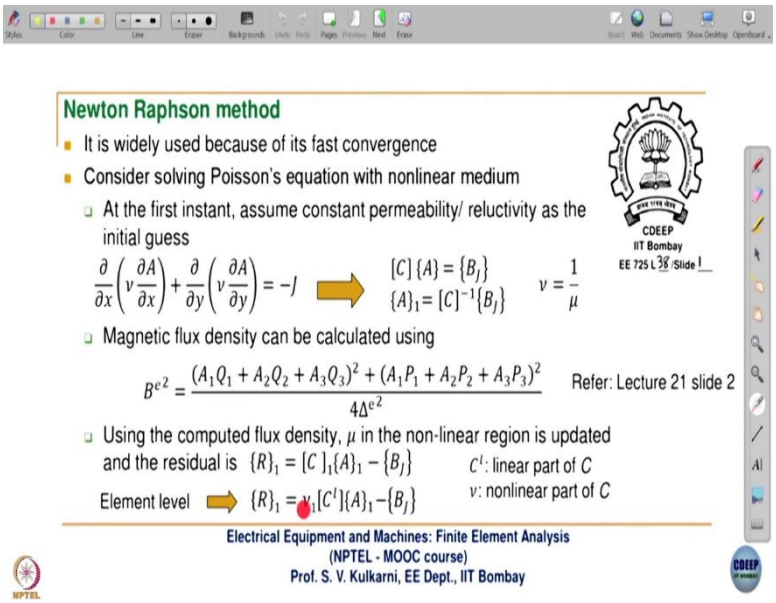

Let us solve the following Poisson's equation with nonlinearity involved.

$$\frac{\partial}{\partial x}\left(\nu\frac{\partial A}{\partial x}\right)+\frac{\partial}{\partial y}\left(\nu\frac{\partial A}{\partial y}\right)=-J$$

At the first instant, we will assume constant permeability in the magnetic medium and everywhere in the medium as the initial guess. So, ν in the above equation is $\frac{1}{\mu}$ and the equation gets reduced to the following equation after applying the FEM discretization procedure.

$$[C]\{A\} = \{B_J\}$$

So, we will get $\{A\}_1 = [C]^{-1}\{B_J\}$. We are calling this step as the first iteration and the $[C]$ matrix is calculated using the initial guess of permeability. Here we have to remember that the global coefficient matrix $[C]$ contains the information about geometry and material properties (μ). So, in the whole magnetic medium, we will be assuming constant permeability (μ) and then we will be calculating the entries of $[C]$.

The matrix $[C]$ is the initial guess as we are in the first iteration. Matrix $\{A\}_1\left(= [C]^{-1}\{B_J\}\right)$ has magnetic vector potentials at each and every point in the domain at the first iteration. $\{B_J\}$ comes from the source conditions $(-J)$ in the governing PDE. After calculating $\{A\}_1$ in the first iteration we can calculate magnetic flux density in all elements using the following standard equation which we have seen in the previous lecture.

$$B^{e2} = \frac{(A_1Q_1 + A_2Q_2 + A_3Q_3)^2 + (A_1P_1 + A_2P_2 + A_3P_3)^2}{4\Delta^{e2}}$$

The derivation of this equation can be found in lecture 21 slide number 2.

Using the computed flux density, μ in nonlinear (magnetic) region can be updated. To update the value of μ, we have to express μ as a function of B. We will see the procedure in a later slide. As μ is a function of B and the value of B is calculated in each and every element of the domain then μ in each element of the nonlinear region can be updated separately and the value of μ is different in different elements. Though, in the first guess we have assumed μ as constant in the entire nonlinear region.

In general, μ is different in different finite elements for the same magnetic domain because B would be different in different elements. Now, residue in element is calculated by using the following equation.

$$\{R\}_1 = [C]_1\{A\}_1 - \{B_J\}$$

Here, R stands for residue in the first iteration, matrix $[C]_1$ is updated with new values of μ and $\{A\}_1$ is magnetic vector potentials determined in the first iteration.

Please do not confuse $\{A\}_1$ with $A_1, A_2,$ and A_3 which are the magnetic vector potentials at the nodes of any element e. Whereas matrix $\{A\}_1$ is the column vector of magnetic vector potential at the 3 nodes in the element in the first iteration. Later on, in the global level matrix equation $\{A\}_1$ will be magnetic vector potentials at all the nodes in the problem domain in the first iteration.

Now, this residue is represented by the above equation. So, $[C]_1\{A\}_1 - \{B_J\}$ with the updated μ need not be necessarily 0. If the value of initial guess of μ is close to the calculated μ then only the value of residue will come to 0 in that particular element. But, in general, the value of residue will not be 0.

So, we need to refine the value of μ in every iteration to obtain a perfect solution residue equal to 0. At the element level, we are splitting matrix $[C]$ into 2 parts as $\nu[C^l]$. The value of ν is nothing but $\frac{1}{\mu}$ and we are assuming that the value of permeability over the whole element region is constant. So, for every element, μ is constant.

For different elements, μ will be different and it will be representing nonlinearity. Whereas $[C^l]$ is a linear part of C which is not changing with μ.

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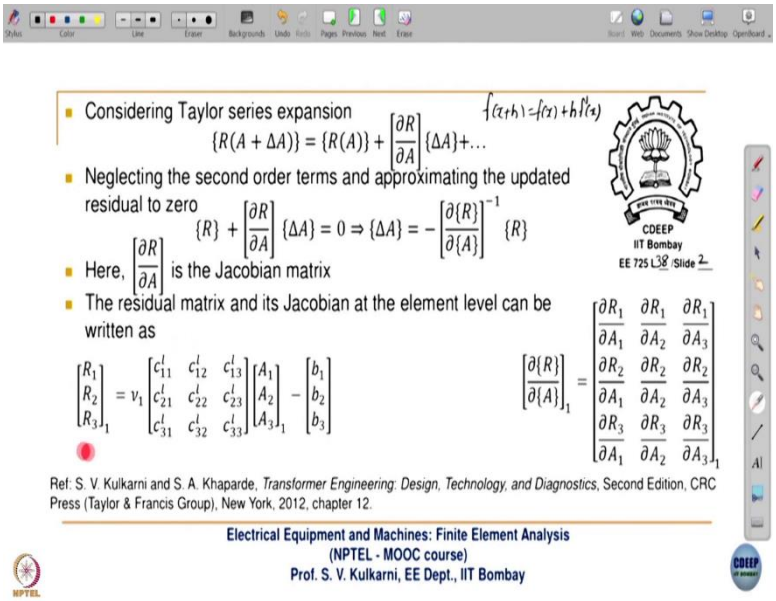

Now, we will consider the following Taylor series expansion of R.

$$\{R(A+\Delta A)\} = \{R(A)\} + \left[\frac{\partial R}{\partial A}\right]\{\Delta A\} + \ldots$$

In the above equation, higher order terms are neglected. The above expression, is similar to

$$f(x+h) = f(x) + hf'(x)$$

So, we are neglecting second order and higher order terms and approximating the updated residual to 0, because our objective is to make the value of residue as 0.

Equating the above equation of R to 0 modifies the equation as

$$\{R\} + \left[\frac{\partial R}{\partial A}\right]\{\Delta A\} = 0 \Rightarrow \{\Delta A\} = -\left[\frac{\partial \{R\}}{\partial \{A\}}\right]^{-1}\{R\}$$

Again remember, this R and A are written in curly brackets because they are column vectors. If we are at the element level the dimensions of R and A matrices will be 3×1. The entries of A matrix will be A_1, A_2, and A_3. Similarly, the entries of R will be R_1, R_2, and R_3 which represent residue at each node of the element. The entries of $\frac{\partial\{R\}}{\partial\{A\}}$ (Jacobian matrix) are given below.

$$\left[\frac{\partial \{R\}}{\partial \{A\}}\right]_1 = \begin{bmatrix} \frac{\partial R_1}{\partial A_1} & \frac{\partial R_1}{\partial A_2} & \frac{\partial R_1}{\partial A_3} \\ \frac{\partial R_2}{\partial A_1} & \frac{\partial R_2}{\partial A_2} & \frac{\partial R_2}{\partial A_3} \\ \frac{\partial R_3}{\partial A_1} & \frac{\partial R_3}{\partial A_2} & \frac{\partial R_3}{\partial A_3} \end{bmatrix}_1$$

The above matrix is the element level Jacobian matrix.

$$\begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}_1 = \nu_1 \begin{bmatrix} c_{11}^l & c_{12}^l & c_{13}^l \\ c_{21}^l & c_{22}^l & c_{23}^l \\ c_{31}^l & c_{32}^l & c_{33}^l \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix}_1 - \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Now, this residual matrix will have three components, because this R is a column matrix with R_1, R_2, and R_3 in the iteration 1. $\nu_1 \left(= \frac{1}{\mu_1}\right)$ is the material property in the first iteration.

The entries of the matrix $[C^l]$ will have C_{11}^l, C_{12}^l, C_{13}^l up to C_{33}^l and the superscript l represents linear part which is not a function of μ. So, if we are solving a static problem then there is only one value of ν_1, $\{R\}_1$ and $\{A\}_1$. If we are solving a transient nonlinear problem then in every iteration of a time instant, ν_1, $\{R\}_1$and $\{A\}_1$ will be constant.

When we go to the next time instant and do the iterative procedure again then of course the values of ν_1, $\{R\}_1$ and $\{A\}_1$ will change. So, that is why subscripts are only for these variables. The matrix $\left[\frac{\partial\{R\}}{\partial\{A\}}\right]_1$ for the first iteration is given in the previous equation. The size of this matrix is 3×3 because the dimensions of $\{R\}_1$ and $\{A\}_1$are 3×1.

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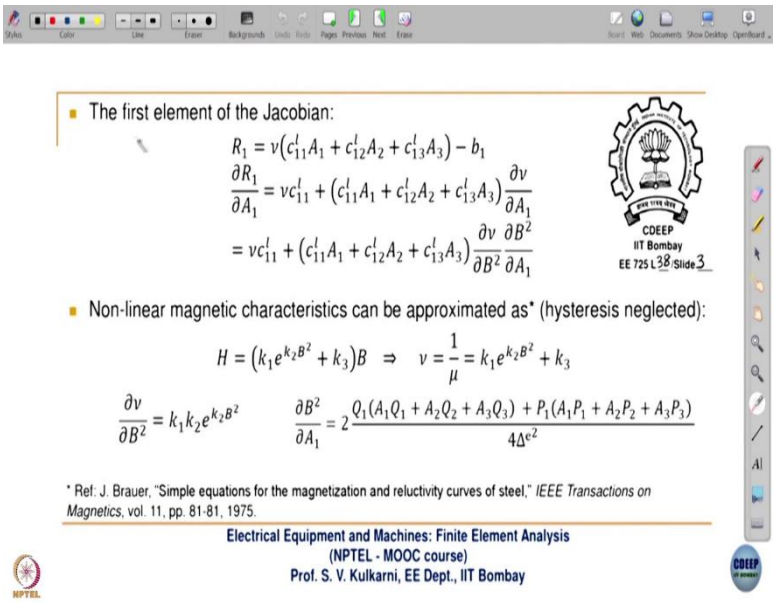

The first element of the $\{R\}_1$matix is given below

$$R_1 = \nu\left(c_{11}^l A_1 + c_{12}^l A_2 + c_{13}^l A_3\right) - b_1$$

This expression is obtained by expanding the first row of previous matrix equation. The $\frac{\partial R_1}{\partial A_1}$ is evaluated as given below.

$$\frac{\partial R_1}{\partial A_1} = \nu c_{11}^l + \left(c_{11}^l A_1 + c_{12}^l A_2 + c_{13}^l A_3\right)\frac{\partial \nu}{\partial A_1}$$

In the above equation, R_1 and A_1 are not in curly brackets because this represents the residue of only node 1. So, $\frac{\partial R_1}{\partial A_1}$ will simply have 2 terms which are obtained by following uv rule of

differentiation. Now, we are writing $\frac{\partial v}{\partial A_1}$ as $\frac{\partial v}{\partial B^2}\frac{\partial B^2}{\partial A_1}$ and the above equation is rewritten as given below.

$$vc_{11}^l + \left(c_{11}^l A_1 + c_{12}^l A_2 + c_{13}^l A_3\right)\frac{\partial v}{\partial B^2}\frac{\partial B^2}{\partial A_1}$$

The nonlinear magnetic characteristic with hysteresis neglected can be approximated as

$$H = \left(k_1 e^{k_2 B^2} + k_3\right)B$$

In the above equation, H is expressed as some nonlinear function of B. The whole term in the bracket is representing ν as

$$\nu = \frac{1}{\mu} = k_1 e^{k_2 B^2} + k_3$$

Because $H = \nu B = \frac{B}{\mu}$ or $\frac{B}{H} = \nu$.

So, ν or μ is a function of B^2 and we are taking it as a function of B^2 because in the FEM formulation we directly get value of B^2. Now, $\frac{\partial \nu}{\partial B^2}$ will be simply represented as

$$\frac{\partial v}{\partial B^2} = k_1 k_2 e^{k_2 B^2}$$

Then, $\frac{\partial B^2}{\partial A_1}$ will be

$$\frac{\partial B^2}{\partial A_1} = 2\frac{Q_1(A_1Q_1 + A_2Q_2 + A_3Q_3) + P_1(A_1P_1 + A_2P_2 + A_3P_3)}{4\Delta^{e2}}$$

The above equation is obtained by evaluating the expression of B^2 that we have seen in the previous slides.

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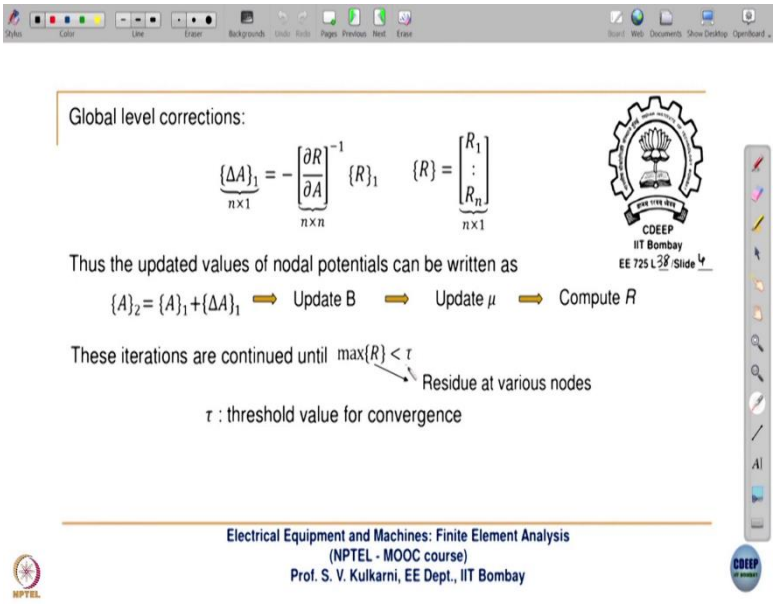

Now, we will go to the following global level equation.

$$\underset{n\times 1}{\{\Delta A\}_1} = -\underbrace{\left[\frac{\partial R}{\partial A}\right]^{-1}}_{n\times n}\{R\}_1$$

The variables or matrices in the above equation are global level quantities which are obtained by combining element level quantities as we did earlier. So, the size of $\{\Delta A\}_1$ matrix will be $n \times 1$ and it is a column matrix. The size of Jacobian inverse will be $n \times n$ because the product of Jacobian inverse matrix and residue column matrix at iteration 1 will give $\{\Delta A\}_1$ matrix whose size is $n \times 1$. The entries of $\{R\}_1$ are

$$\underset{n\times 1}{\{R\}} = \begin{bmatrix} R_1 \\ \vdots \\ R_n \end{bmatrix}$$

So, the nodal potentials for the second iteration can be written as

$$\{A\}_2 = \{A\}_1 + \{\Delta A\}_1$$

By adding $\{\Delta A\}_1$ to $\{A\}_1$ we will get magnetic vector potential values for the second iteration. Then, we have to go to the next iteration.

So, by using the above equation we would have got A in the second iteration. Using the corrected A we can update B^2 which is a function of nodal potentials for a finite element. From B, we calculate ν or μ which has been expressed as a function of B^2. Then using the calculated ν again we calculate element level and global level residual matrices. The rest of the procedure will be repeated till we meet the convergence criterion.

If there are say 1000 nodes then there will be 1000 residues. If the maximum value of those 1000 residues is less than some predefined threshold number which can be a very small number then we can stop saying that the convergence has reached and we have got the solution at that instant.

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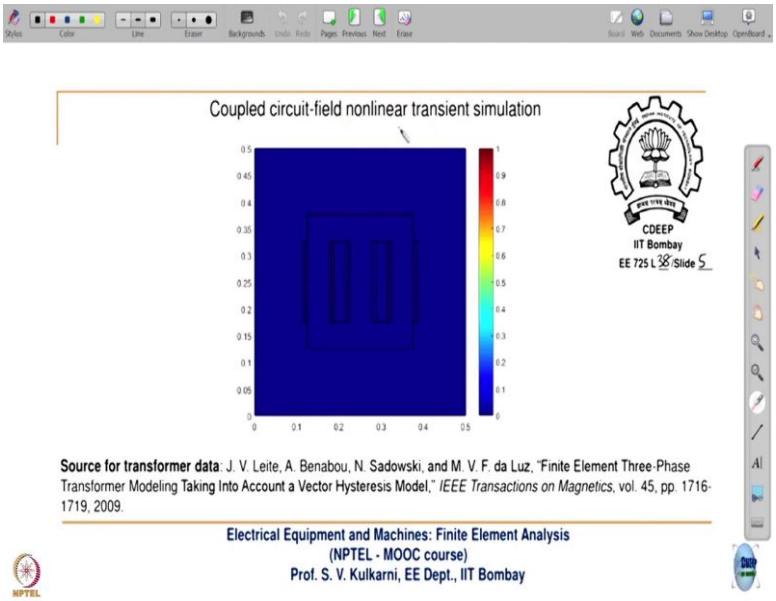

Now, we will take one case study as we have been doing after developing every FE formulation. This problem is a coupled circuit field nonlinear transient simulation. Here, there are three complications. In the previous lectures, we had seen coupled circuit field and transient formulation and for those formulation we are including nonlinearity in this lecture. So, in this case study, we are combining all these 3 formulations together. In the above slide there is a 3 phase transformer with 3 primary windings which are excited by a voltage source and the transformer will draw inrush currents. Now, we will study the inrush currents in all the 3 phases. So, the primary windings of the 3 phases are driven by a coupled circuit which is the time-varying voltage source. That is why the formulation is transient and the nonlinearity of the magnetic material is also modelled. Now, we will see the simulation in time.

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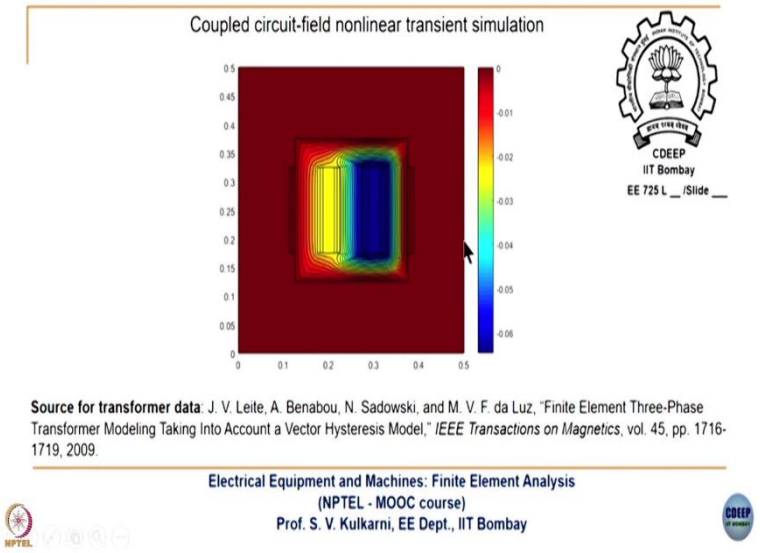

In this slide, we can see animated FEM solution obtained using the devoloped coupled circuit field nonlinear transient formulation which we saw in the present lecture and in the previous two lectures. We saw all the three formulations and if we write the corresponding code by considering all the complexities then we can get such time domain simulation results.

We have to model the material properties as a function of B because when the transformer is switched-on the flux density in the core can be quite high and can drive the transformer into deep saturation depending upon the residual magnetism at the instant of time of switching. In the simulation, we can also see that at some instants of time the flux is going out of the core because of the saturation. So, the flux coming out of the core depends on the saturation level and that can be easily studied using such simulations.

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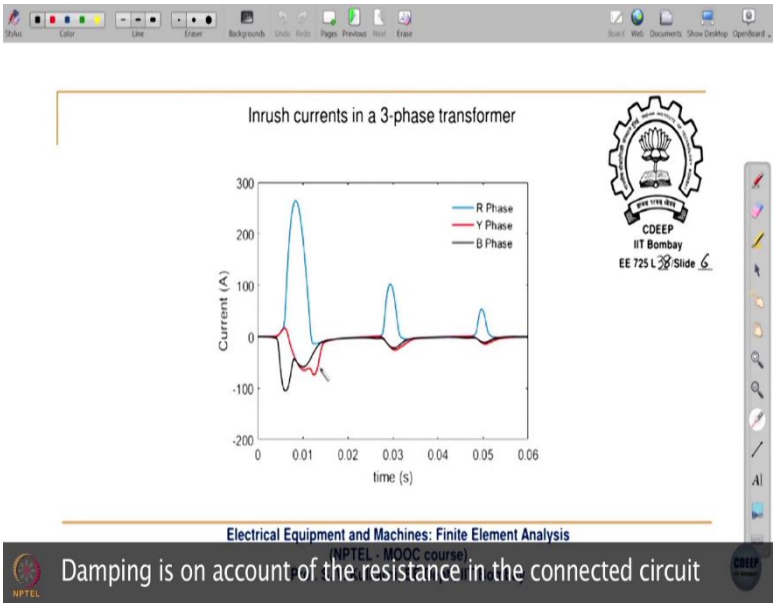

Now, we will see the inrush currents drawn by the transformer in time domain. The currents in the above slide are functions of time. In the figure, R phase, Y phase, and B phase currents are marked with different colours. The R phase current is maximum because in the simulation we excited all the 3 windings at an instant which corresponds to R phase voltage being 0. That is why this switching instant is worse for the R phase and it is not bad for Y and B phases. We know that an RL circuit draws maximum current if it is exticted with zero voltage. We are considering this as a lossless case and we are not considering resistances of the circuit. So, like this, we can do transient nonlinear coupled analysis for any electromagnetic device. Thank you.

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