Electrical Equipment and Machines:
Finite Element Analysis
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Lecture No. 35
Voltage Fed Coupled Circuit Field Analysis

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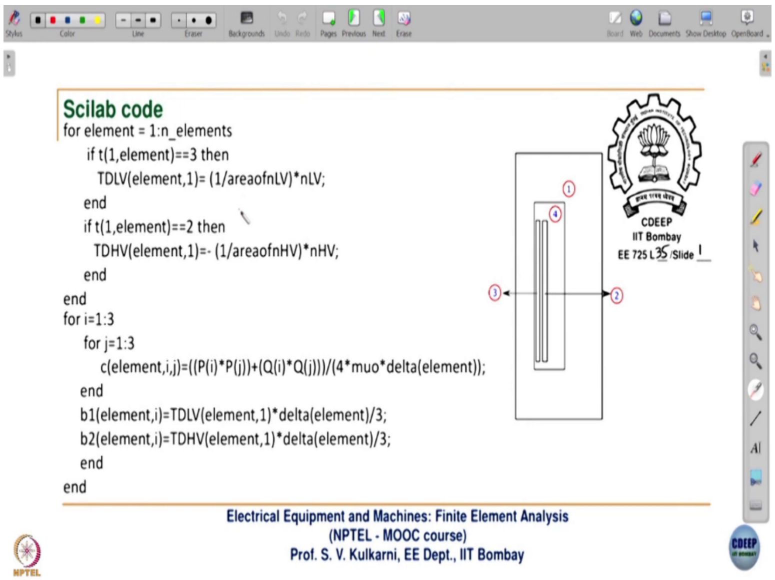

Welcome to lecture 35. In the previous lecture, we saw voltage fed coupled circuit field analysis. We will now see an example and develop a code as we have been doing earlier. Also, in this lecture, we will describe only the new part of the code and rest of the code will be similar to what we have seen in earlier lectures.

Let us take a transformer core window with LV and HV coils, which is shown in the figure given in the above slide. Now, we will only model the window and we have to remember that there will be LV and HV coils on the other side of the window.

So, what are the new things that are to be added to this code? First of all, earlier when we developed a code to calculate leakage reactance of a transformer we knew the value of current in both windings. In fact, we ensured that $N_1 I_1 = -N_2 I_2$ so that we have a perfect balance of ampere turns.

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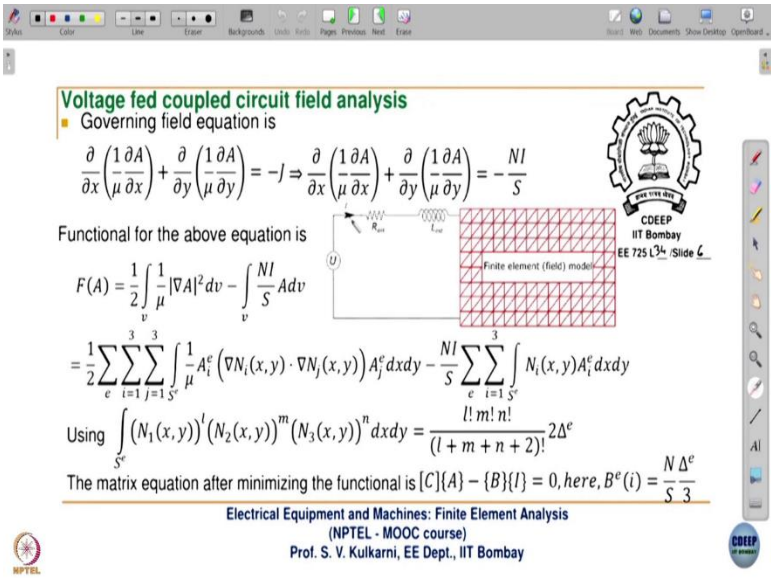

But here in this formulation, current is unknown, because in general the current in the primary winding of a transformer is fed by a voltage source connected to the winding. So, voltage is known and current is unknown.

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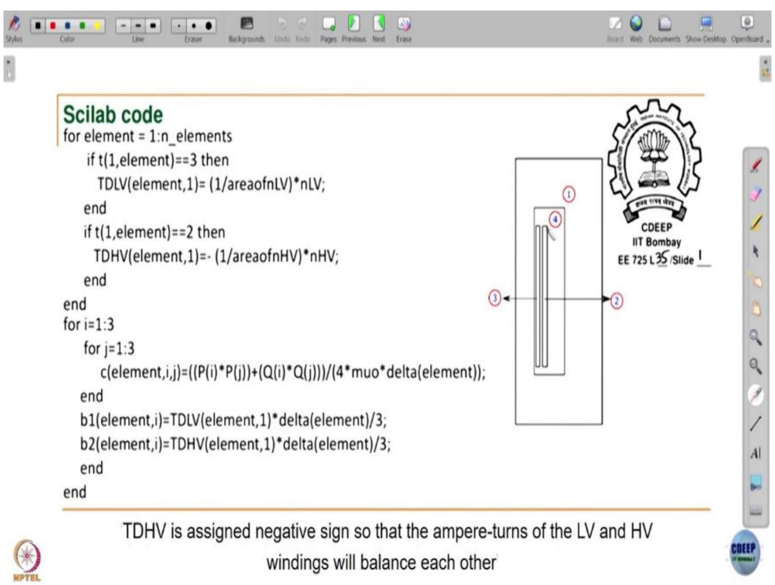

The current in the primary winding as well as the current in the short circuited secondary winding is unknown. So, in this problem the unknowns are the two current variables.

Here, subdomains 3 and 2 correspond to primary and secondary windings. To be more general, subdomain number 3 is low voltage winding because we do not know which is the primary winding and which is the secondary winding. So, we always consider LV and HV windings.

So, if the element is in LV (subdomain 3) then turn density (TDLV) is defined as

```
if t(1,element)==3 then
   TDLV(element,1)= (1/areaofnLV)*nLV;
end
```

Here, D stands for density. In the earlier problem we defined ampere turn density because current in the coil was known. So ampere turns density was calculated by dividing ampere turns by area. But in this problem, current is not known so we are defining turn density only. That is why the number of turns of LV (nLV) is divided by the cross sectional area of LV (areaofnLV). So, in the above part of the code turn density is defined by dividing number of LV turns by cross sectional area.

Similarly, for all the elements in HV windings the corresponding first entry in the t matrix will be two because the subdomain number of HV is two. So, turn density for HV will be number of HV turns (nHV) divided by cross sectional area of HV winding (areaofnHV) as given in the following part of the code.

```
if t(1,element)==2 then
   TDHV(element,1)=- (1/areaofnHV)*nHV;
end
```

So, for all the elements in LV and HV windings we would have defined the turn density when we execute the following for loop.

```
for element = 1:n_elements
     if t(1,element)==3 then
        TDLV(element,1)= (1/areaofnLV)*nLV;
     end
     if t(1,element)==2 then
        TDHV(element,1)=- (1/areaofnHV)*nHV;
     end
end
```

Then the following code to form element level coefficient matrix will be exactly identical to the codes which we saw in the previous lectures.

```
for j=1:3
    c(element,i,j)=((P(i)*P(j))+(Q(i)*Q(j)))/(4*muo*delta(element));
end
```

The next change is in the formation of b matrices. At the element level , b1 for LV and b2 for HV are formed by using the following code.

```
b1(element,i)=TDLV(element,1)*delta(element)/3;
b2(element,i)=TDHV(element,1)*delta(element)/3;
```

The above part of the code is changed and again here the change is instead of ampere turn density we have defined turn density of LV and turn density of HV by multiplying them with $\frac{\Delta}{3}$. Because in the previous lecture we have seen that the entries of element level b matrices are defined by using the following equation.

$$B^e(i) = \frac{N}{S}\frac{\Delta}{3}$$

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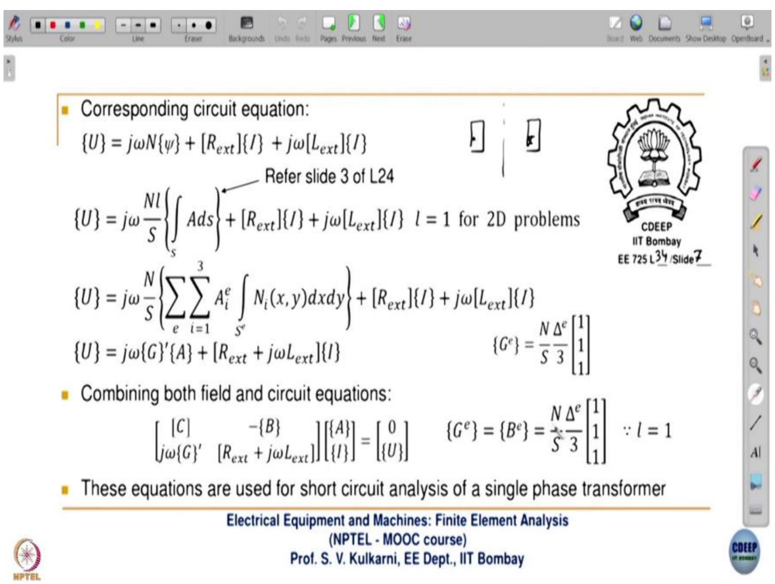

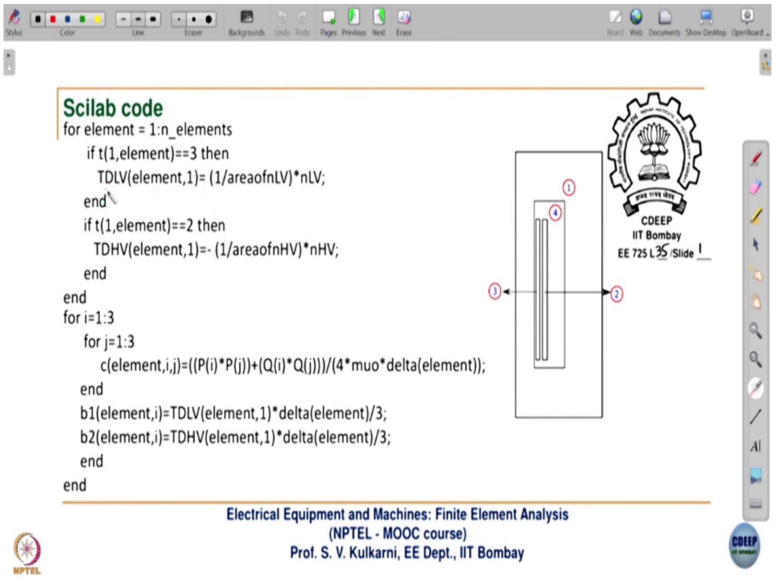

So, here the question asked by one student is why we are taking this minus sign with the turn density of HV? We are taking minus sign because we know that ampere turns are equal and opposite. If we take minus sign for one winding then the corresponding induced current will be positive. So, the HV current with minus sign will balance the LV ampere turns.

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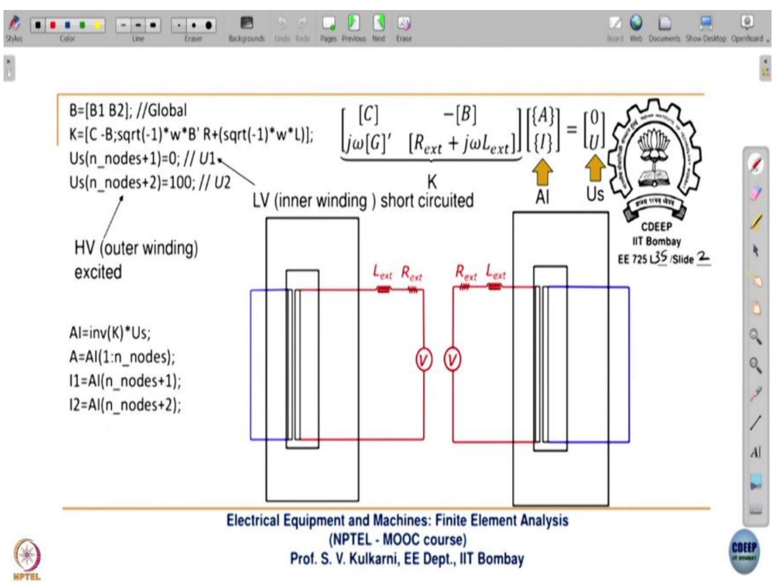

The procedure to form global level matrices using element level matrices is same as that we saw in the previous lectures. The global B matrix is augmentation of B1 and B2 matrices. In the previous lecture, we have seen the following expression of K matrix.

$$\underbrace{\begin{bmatrix} [C] & -[B] \\ j\omega[G]' & [R_{ext} + j\omega L_{ext}] \end{bmatrix}}_{\text{K}} \underbrace{\begin{bmatrix} \{A\} \\ \{I\} \end{bmatrix}}_{\text{AI}} = \underbrace{\begin{bmatrix} 0 \\ U \end{bmatrix}}_{\text{Us}}$$

K matrix is coded as given below.

```
K=[C -B;sqrt(-1)*w*B' R+(sqrt(-1)*w*L)];
```

The K matrix consists of element coefficient matrix C that corresponds to $\frac{1}{\mu}\nabla^2$term in the PDE and $-B$ representing turn density. In the second row $\sqrt{-1}\omega$ is $j\omega$. Here, both G and B matrices are same because we are considering per meter depth calculations. So B$'$ is nothing but G$'$.

Remember that the dimensions of the K matrix and other matrices are number of nodes + 2 $\times$ number of nodes + 2. Suppose if there are 1000 nodes then n_nodes will be 1000 and there will be additional two entries, of which one term corresponds to the LV terminal and the second term corresponds to the HV terminal. So, the right hand side matrix of the above equation will be 0 and its size is the number of nodes and then the U matrix will have two entries U1 and U2. That is why the dimension of the Us will be 1002×1.

Here the LV (inner winding) is short circuited and it is executed by using the following command.

```
Us(n_nodes+1)=0; // U1
```

The voltage across the LV winding is made 0 and the voltage across HV winding (100 V) is defined by the following command

```
Us(n_nodes+2)=100; // U2
```

So, this completes our coupled circuit field formulation with voltage applied to one of the windings and the other winding short circuited. The currents in LV (I1) and HV (I2) windings are unknown. So the {I} matrix in the above formulation will be I1 and I2 and {A} will be unknown magnetic vector potential. If there are 1000 nodes in the field model then the size of {A} will be 1000×1. Since there are two windings the size of $\begin{bmatrix} \{A\} \\ \{I\} \end{bmatrix}$ $is\ 1002 \times 1$.

Then we are calling $\begin{bmatrix} \{A\} \\ \{I\} \end{bmatrix}$ matrix as a combination of $\{A\}$ and $\{I\}$ matrices and the right hand side matrix as U and it has two terms corresponding to the voltages across the two windings and the matrix representing all the nodes in the field model.

The $\begin{bmatrix} \{A\} \\ \{I\} \end{bmatrix}$ matrix contains the unknown variable and it can be calculated by using the following code.

```
AI=inv(K)*Us;
```

Then, the entries of first 1 to n_nodes will give us A, and I1 and I2 will be the corresponding entries at the positions n_nodes+1 and n_nodes+2. So, after getting the solution by the above command, using the following three commands we will get field solution (A), I1 and I2.

```
A=AI(1:n_nodes);
I1=AI(n_nodes+1);
I2=AI(n_nodes+2);
```

Now, we will see examples of implementation of the coupled circuit field computation.

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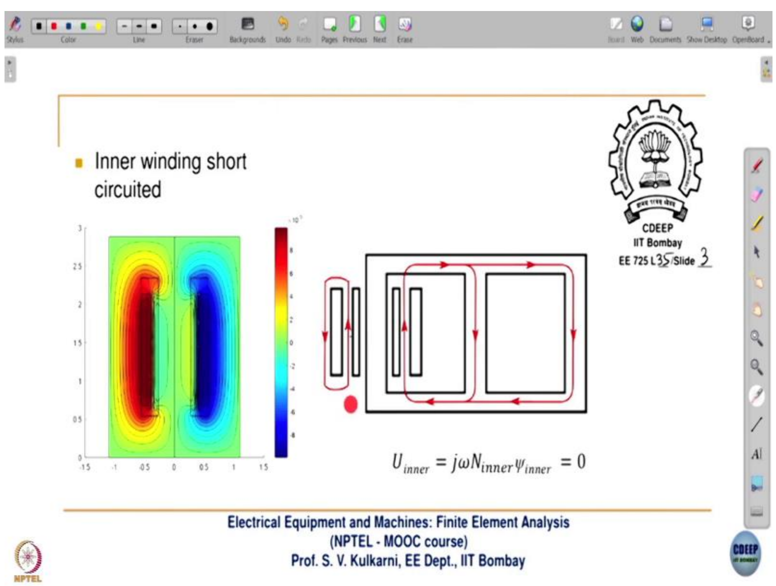
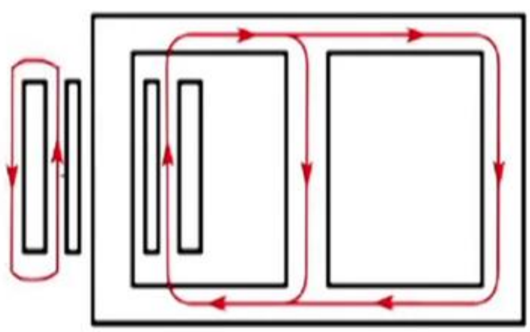


Let us see a case where we have inner winding short circuited that is what we had coded in the previous slide. Here, the inner winding is LV and the HV winding is supplied with a voltage source. Now, if we apply KVL to the inner winding then it will be

$$\{U_{inner}\} = j\omega N_{inner}\{\psi_{inner}\} = 0$$

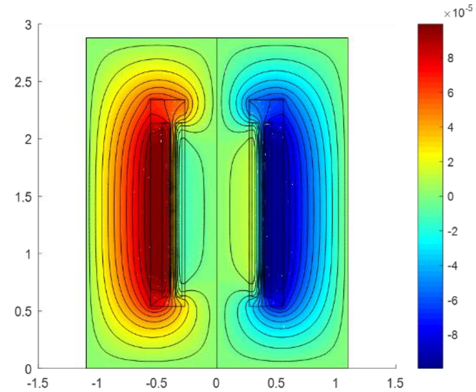
The induced voltage in the LV winding is $\frac{d}{dt}(N\psi)$. $N\psi$ is the total flux linkage and we are neglecting IR drop because we had neglected conductivity of the winding in the FE analysis. We are considering the winding as lossless. So the above equation represents that the total induced voltage will be equated to 0 when the inner winding is short circuited.

So, the total flux linkage associated with the winding is 0. The following is a representative figure for a three phase transformer.



In the present problem, we are modeling one of the three phases. The leakage flux will complete the path as indicated in the above figure.

In the above figure, we can see that the total flux linkage of LV winding is 0. Also, there is no single flux contour which is enclosed by the inner winding. So, the total flux linked by the inner winding is 0. The same thing can be observed here in the following FE plot.



Also, we have to remember that we had modeled only half of the (one window) part of the above figure so it was a single phase case. The solution in the other half is just a mirror image of the solution and that can be obtained by taking the opposite sign of the solution of the modeled window. Suppose if the magnetic vector potential in the window region that we modelled is $+A$ then the potential at the corresponding mirror point will be $-A$ because the currents in the two regions are opposite.

So, just by taking negative of the magnetic vector potential in the half that we considered we can plot field solution for the second half. That is how we just plotted the total field distribution which is shown in the above figure. The middle line in the figure is just a symmetry line and it is not a flux line. In the above figure, we can see that all the flux which is there in the LV and HV windings and the gap is returning through the core. Also, we can neglect the single contour linking the inner winding because the value of flux or magnetic vector potential is very close to 0. That is why we can see that the flux of HV winding is returning through the core. If it is a three phase transformer which is enclosed in a tank then the flux will return through the outside air.

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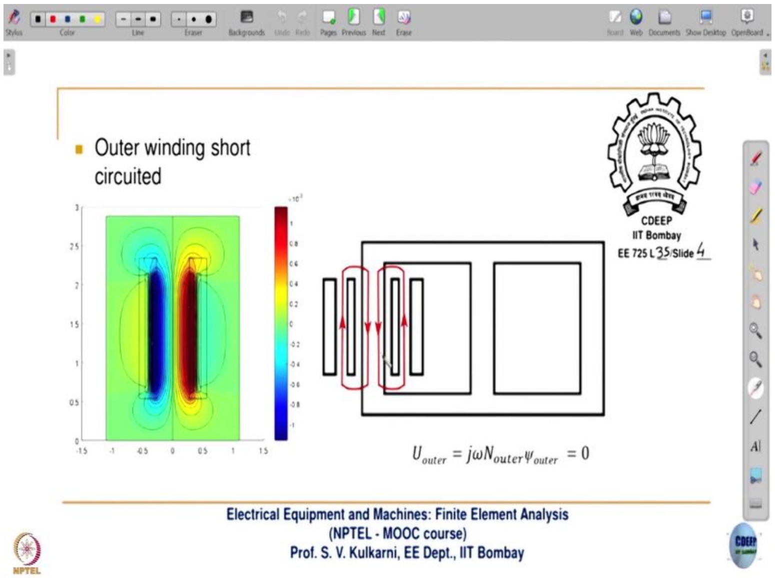
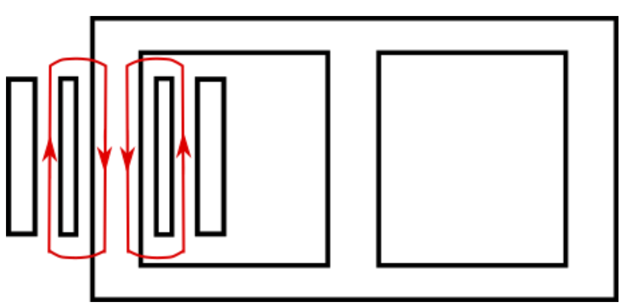


Now, we will see the case in which the outer winding is short circuited. So, for this case we have to make Us(n_nodes+2) as 0 and the corresponding excitation to the LV winding will typically go with the turns ratio. If the turns ratio is 10 then the short circuit voltage for the LV winding will be 10 V and voltage across HV will be 0.

Here, we have to only make these two changes in the code that we saw in the previous slide. Then we will get the result as shown in the above slide. In case of outer winding short circuited the following equation that we saw in the previous slide can be used.

$$\{U_{outer}\} = j\omega N_{outer}\{\psi_{outer}\} = 0$$

So, the total induced voltage in the HV winding will be equal to 0 and hence the total flux linkage associated with HV winding is 0. As we see in the following figure, the total flux linkage of the two halves of the HV winding is zero.



Here, all the flux in HV winding, gap and LV winding is returning through the core. Again, we can neglect the single contour whose value is anyway close to 0.

So, here we have seen these two examples which correspond to simulating coils excited by voltages and how to find unknown currents. From the simulation by this formuation along with the flux plots we also got the currents in the two windings. Using the calculated currents we can easily calculate the actual impedance by evaluating V/I of the corresponding coils.

The evaluation of V/I will give the total leakage impedance of the transformer. So, voltage excitation to the inner winding divided by the corresponding current (I1) will give the leakage impedance of the transformer referred to the LV side.

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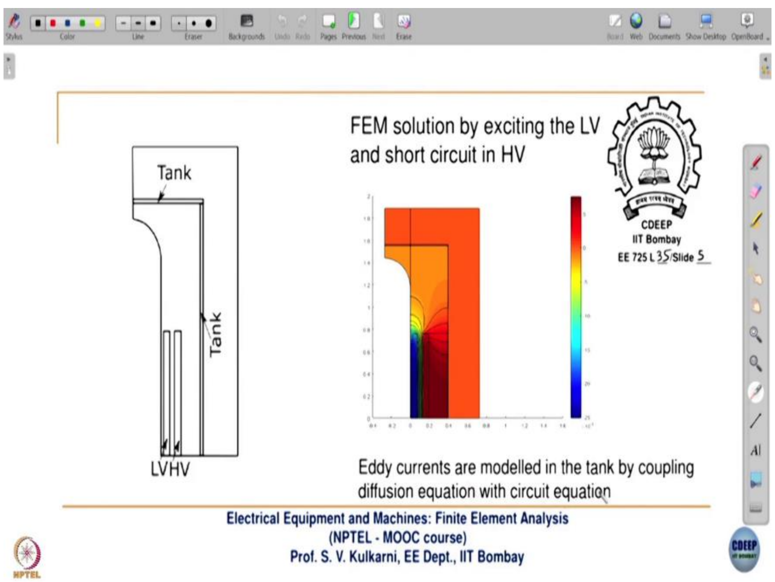
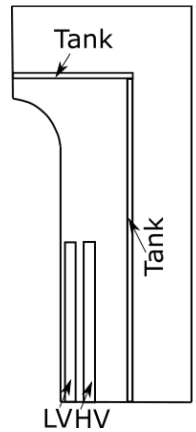


Now, we will see another example wherein the currents are unknown and eddy currents are also there. In this problem also we are exciting one of the LV and HV windings. There is a tank as shown in the following figure, in which eddy currents are induced due to the leakage field.



Eddy currents are induced in the tank because the time varying leakage field is incident on the tank which is made up of mild steel or any other conducting material.

Under short circuit condition, if we want to estimate the losses and induced currents in this tank then we have to solve diffusion equation. In the previous examples, we have solved only Poisson's equation. There we have not modelled eddy currents because we have neglected the conductivity. But if we have to calculate eddy currents induced in any part of the geometry in the field model then we have to model the corresponding diffusion term $\left(\mu \frac{\partial A}{\partial t}\right)$ which represents the induced eddy currents. Now, here we have to couple the diffusion equation with the circuit equation.

So, with this formulation, we will also get the eddy currents induced in the tank. In the above figure, we can see that we have modeled the external part because if the thickness of the tank is small then the flux will go out. If the thickness of the tank is much more than the skin depth then we can terminate our boundary at the outer thickness of the tank.

If we are interested to estimate the effects of variation in the tank thickness or material type on the eddy current losses then we have to model the outer air region and then impose A=0 on the outermost boundary. The above model can stimulate any flux that is coming out of the tank and we can find the corresponding effects.

Also, remember that the eddy currents induced in the tank show the effect on the leakage impedance of the transformer, because the eddy currents will affect the leakage field pattern and the AC part of the effective resistance of the transformer. The tank losses will get reflected in the effective AC resistance. So, that is why all such eddy current losses on account of leakage field will influence the leakage impedance of the transformer. Such analysis can be done by using the formulation in which we are solving the diffusion equation along with the circuit equation.

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