Electrical Equipment and Machines: Finite Element Analysis
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Lecture No. 34
FE Analysis of Rotating Machines

Welcome to lecture 34. We will continue our discussion where we stopped in the previous lecture.

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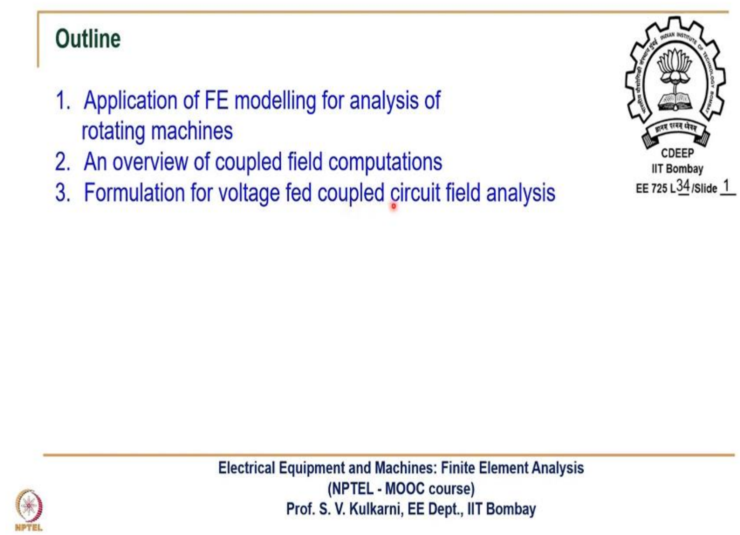

In this lecture, we will briefly review the application of FE modelling for analysis of rotating machines. We will take a permanent magnet machine as an example for the purpose. Next, we will see how various engineering fields like thermal, structural, fluid, etc., are coupled to electromagnetic fields in a typical electrical equipment or machine. External circuits are also coupled to electric and magnetic fields. In this course, we will study FE formulation for voltage and current fed devices. To start with, in this lecture, we will study FE formulations for voltage fed devices and in a later lecture we will see the corresponding formulation for current fed devices.

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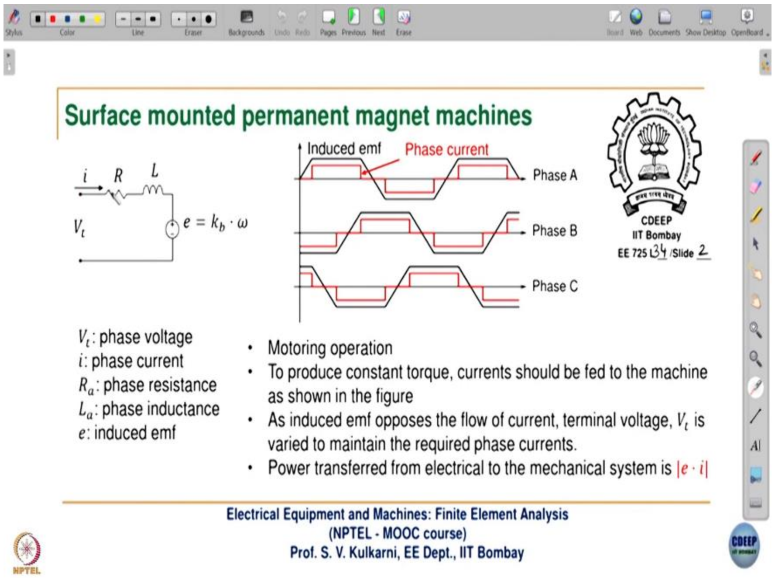
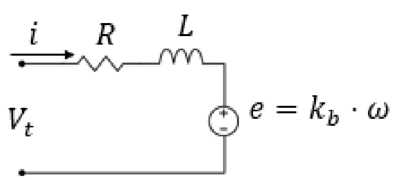
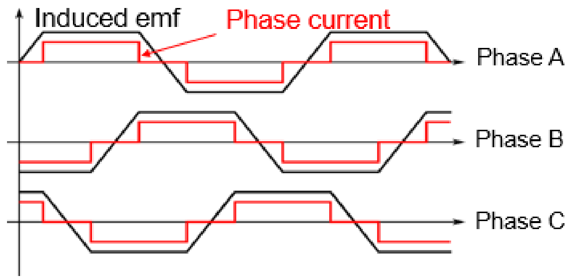


Any rotating machine can be modeled by using the following equivalent circuit.



In this circuit, we have terminal voltage (V_t) and induced voltage (e) and the current i. The direction of current will be decided based upon whether we are analyzing a motor or a generator. Now, based on the direction of the current shown in the above figure, the circuit is representing a motoring operation. So, in the previous lecture, we are discussing surface mounted permanent magnet machines that means on the rotor surface we have permanents magnets mounted.

I am assuming that those who are listening to this course have some background of these machines and they know some basic operational and design or constructional details of these machines. So, they can appreciate what is being said here. Here, now the above circuit is representing a motoring operation and then we have current and induced voltage waveforms as shown in the following figure.



So, to produce constant torque, the current should be fed into the machine as shown in the equivalent circuit. As the induced voltage is of trapezoidal form to have a constant torque which is proportional to product of induced emf and the current then we have to have current waveform which is of rectangular pulses as shown in the above figure. This current waveform can be controlled by using power electronic converters.

As long as we maintain the constancy of induced emf and current, we will get constant torque. So, as induced emf opposes the flow of current, terminal voltage V_t is varied to maintain the required phase current. So, the moment the rotating machine starts rotating we have induced emf. The induced emf is going to oppose the current which is flowing into the motor. To maintain this current as constant, V_t has to be adjusted by the source. Effectively, the source is going to supply additional ei power because when the speed was 0, $e = 0$. When the speed is non zero the corresponding induced voltage ei will be there in opposition to the source effectively.

So, the source has to supply the power ei, here, i is constant. The ei power is converted into the mechanical torque at the shaft. The power or energy transferred from electrical to the mechanical system is ei. So, now this is a motoring operation.

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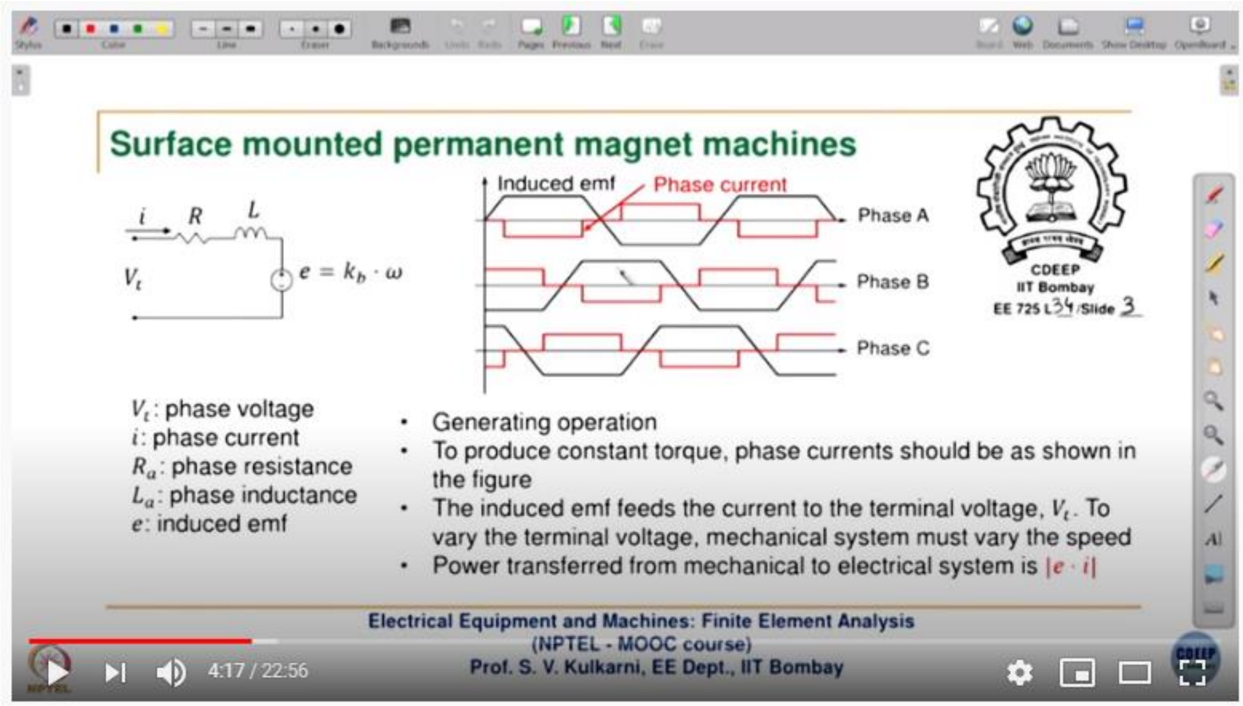
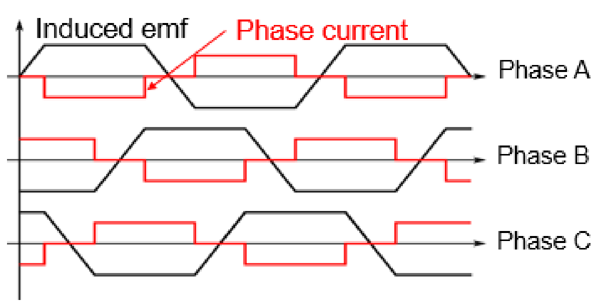


The same thing is true for generating mode with some changes. Here again the same equivalent circuit can be used with the current direction kept as same. But actually when the machine is acting as a generator the direction of current will reverse. For generating operation, the current actually will go from e to V_t. So, as the the current direction is shown opposite, the sign of current is changed (shown in the following figure) as compared to the previous slide.



But if we reverse the direction of current in the equivalent circuit which represents the actual generating operation then the current waveform will be like the way we saw in the previous slide and the same logic applies.

Here, again to produce a constant torque, phase current should be in phase with the induced emf. The induced emf feeds the current to the terminal voltage and to vary the terminal voltage, the speed of the mechanical system must be varied and the corresponding power transferred from mechanical to the electrical system is again ei. So, the direction of power flow is just reversed in this case.

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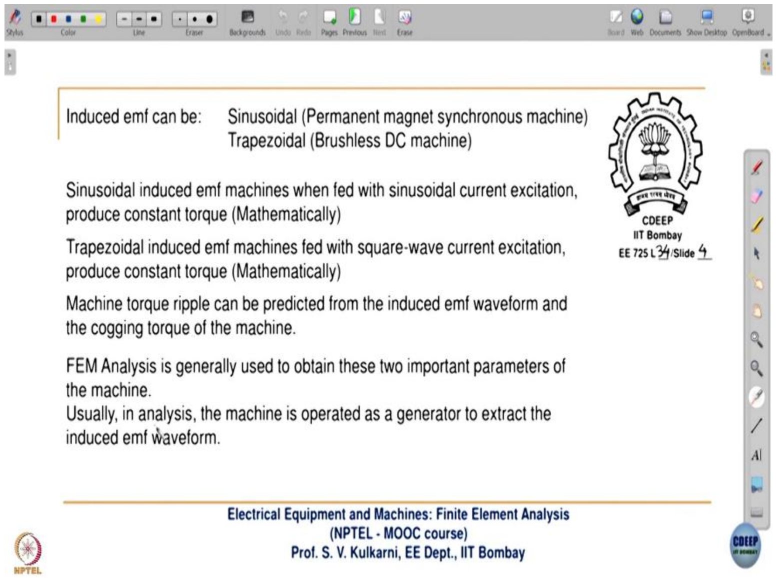

The induced emf is generally sinusoidal in case of permanent magnet synchronous machines and that can be achieved by using distributed windings in space. It can be also achieved by using concentrated winding with some changes such as pole shaping. Whereas in case of brushless DC machines we generally have trapezoidal emf distribution in the space and that can be achieved by using concentrated windings.

Sinusoidal induced emf machines when fed with sinusoidal current excitation produce constant torque. Similarly, trapezoidal induced emf machines with square wave current excitation (as we have seen in the previous slides) produce constant torque. The machine's torque ripple and cogging torque can be predicted from induced voltage waveforms.

So, if we know the induced emf waveforms we can predict the performance parameters of a machine accordingly. This is the importance of analysis that we have seen in the previous lecture. In such FE analysis, even though we may have a motor it is generally simulated as a generator to extract induced emf waveform. This is done for convenience only.

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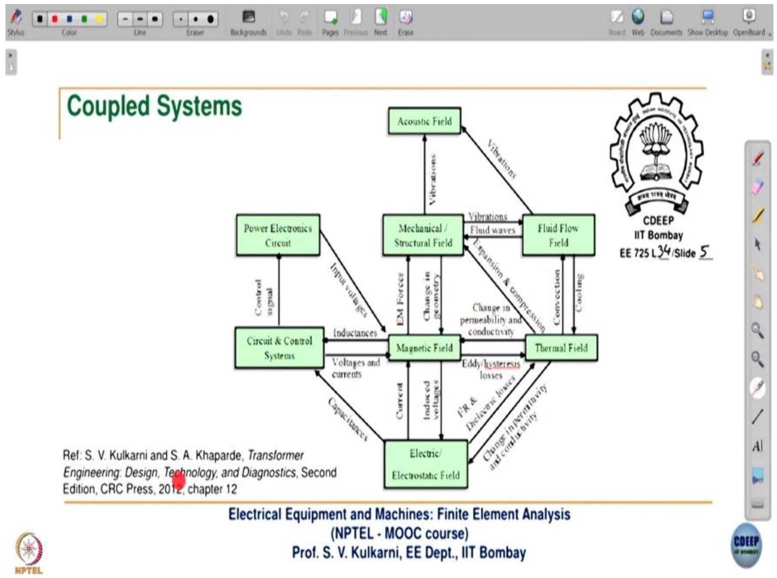

Now, we will see a new topic, coupled field systems. In the second lecture, we had explained about the diagram shown in the above slide. In this figure, couple fields in a transformer are explained. So, in any electromagnetic device like transformer, motor, generator we have so many physical fields. For example in a transformer, we have magnetic field coupled with electric field, thermal field, fluid field, acoustic field and circuits. Then there are coupling parameters between any of these two fields or systems. For example, between magnetic field and thermal field, the coupling parameters are eddy and hysteresis losses and temperature dependency of electromagnetic properties. Basically, theses losses are produced due to alternating magnetic field and they lead to temperature rise which in turn may change the permeability and conductivity. This change in electromagnetic properties would change the magnetic field. So, the coupling parameters are losses and temperature dependency of electromagnetic properties.

In this course we are going to see coupled circuit field computations. We are not going to formulate thermal and mechanical fields coupled to electromagnetic fields. But a detailed discussion on how to couple thermal and mechanical fields to electromagnetic fields can be found in the reference given in the above slide and some of the many other good books and papers on the subject are mentioned in the reference given in the above slide. So now, we will start our discussion on coupled circuit field computations.

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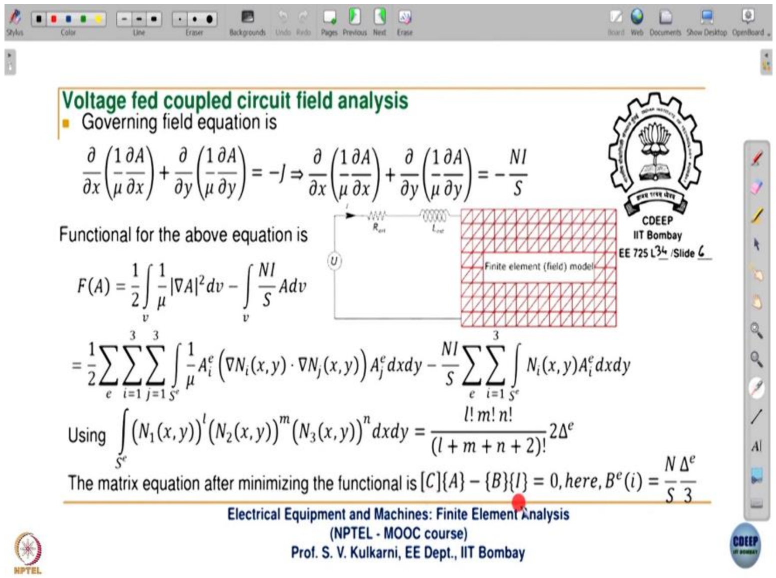

First, we will see voltage fed coupled circuit field analysis which is quite common in most of the devices. In fact all electrical devices and equipment are typically voltage fed. For example, we always see some voltage source feeding a transformer or a motor or a generator. So, the governing field equation is given below

$$\frac{\partial}{\partial x}\left(\frac{1}{\mu}\frac{\partial A}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{1}{\mu}\frac{\partial A}{\partial y}\right)=-J$$

We are now quite familiar with the above equation. The J in the above equation is now written as NI/S as given in the following equation. Here, S is the cross sectional area of coil, through which the current is following.

$$\frac{\partial}{\partial x}\left(\frac{1}{\mu}\frac{\partial A}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{1}{\mu}\frac{\partial A}{\partial y}\right)=-\frac{NI}{S}$$

The functional for the above PDE is given in the following equation.

$$F(A)=\frac{1}{2}\int_v \frac{1}{\mu}|\nabla A|^2 dv-\int_v \frac{NI}{S}A dv$$

We have seen this equation earlier. In the above functional expression, instead of $JAdv$ we have $\frac{NI}{S}Adv$. The dv will become $dS(=dxdy)$ when we consider 2D approximation. After applying the FEM procedure, the first integral in the above equation will become

$$\frac{1}{2}\sum_{e}\sum_{i=1}^{3}\sum_{j=1}^{3}\int_{Se}\frac{1}{\mu}A_i^e\left(\nabla N_i(x,y)\cdot\nabla N_j(x,y)\right)A_j^e\,dxdy$$

We have seen the derivation for this expression in the earlier lectures. So, for each element we will have 9 terms and then the summation over all the elements will add up to get the total energy. The above expression corresponds to one energy term. The second energy term is given below which is on account of the source current J $\left(=\frac{NI}{S}\right)$ in this formulation.

$$\frac{NI}{S}\sum_{e}\sum_{i=1}^{3}\int_{Se}N_i(x,y)A_i^e\,dxdy$$

In the above expression, the inner term leads to 3 terms that correspond to 3 nodes of a triangular element. We have seen that earlier J (or NI/S) gets apportioned to $\frac{J\Delta}{3}$ or $\frac{NI}{S}\frac{\Delta}{3}$ to the 3 nodes of the corresponding element. Also, remember that when the functional gets minimized with respect to the unknown variables A_is then one A_i in the above two terms will be cancelled. That is the reason why only $\frac{NI}{S}\frac{\Delta}{3}$ corresponds to the above term. Because $\frac{1}{S}\iint NIdxdy$ is nothing but $\frac{N}{S}\frac{\Delta}{3}$. The variable I needs to be determined so it comes in the following matrix equation as $\{I\}$.

$$[C]\{A\}-\{B\}\{I\}=0$$

The entries of element level B^e matrix is $\frac{N}{S}\frac{\Delta}{3}$ and when we form global matrix equation the source matrix will become B. So the terms in the above matrix equation are global level matrices.

The entries of local level element coefficient matrix C will be

$$\int_{Se}\frac{1}{2\mu}\left(\nabla N_i(x,y)\cdot\nabla N_j(x,y)\right)dxdy$$

When we actually evaluate the sum over all the elements, the element level matrices will combine to form global coefficient matrix C. $\{A\}$ is the unknown magnetic vector potentials at all the nodes in the domain.

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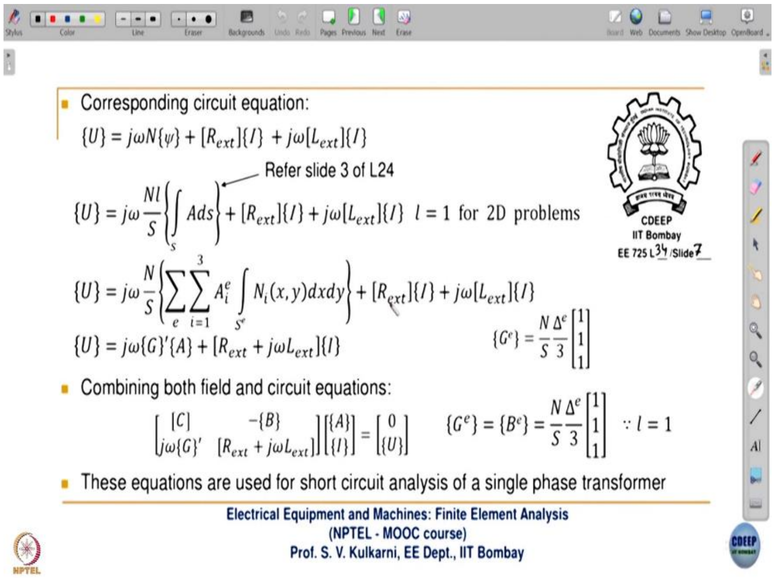
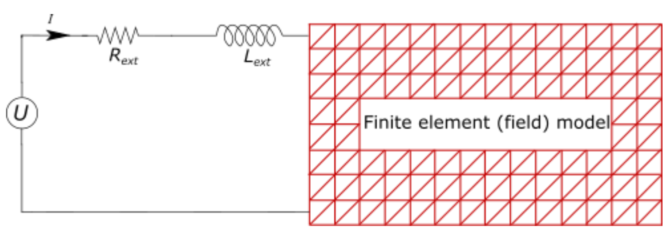


In this formulation, the new part is the following circuit equation.

$$\{U\} = j\omega N\{\psi\} + [R_{ext}]\{I\} + j\omega[L_{ext}]\{I\}$$

Consider a device which is meshed as given in the following figure



The corresponding finite element mesh is shown with red triangles in the above figure. This field model is fed by an external circuit with voltage source U, R_{ext}, and L_{ext}. The field model or the electromagnetic device is represented by the induced voltage at its terminal. So, this induced voltage is the corresponding field parameter in the circuit equation.

The induced voltage at the terminals of the electromagnetic device is given by $N\frac{d\psi}{dt}$. Then $\frac{d}{dt}$ is replaced by $j\omega$. So, $j\omega N\psi$ is the corresponding terminal induced voltage at this device. Now, we are forming the following circuit equation by writing the KVL for the loop formed by the external circuit and the field model.

$$\{U\} = j\omega N\{\psi\} + [R_{ext}]\{I\} + j\omega[L_{ext}]\{I\}$$

The above equation means that the external source voltage is equal to the corresponding terminal induced voltage at the device ($j\omega N\{\psi\}$) plus $[R_{ext}]\{I\}$ and $j\omega[L_{ext}]\{I\}$.

Now, in the above equation, U and I are in curly brackets meaning they are column vectors. Then, we replace this $\{\psi\}$ term in the above equation by the term $\frac{l}{S}\left\{\int_s A ds\right\}$ by invoking the theory that we saw in lecture 24. In fact in the previous topic also we have invoked this theory when we are calculating the flux passing through any two points in case of a two dimensional problem.

Here, we are assuming that we are modelling both sides of the coil. Suppose the field domain in the above figure is a coil with current going in from one terminal and current coming out from the other terminal. So, when we are taking A in the circuit equation then it will be average value of A. So, using the term $\frac{l}{S}\left\{\int_s A ds\right\}$ average value of A is calculated over the coil with so many elements.

We are calculating average value of A by evaluating $\int_s A ds$ over each element and then it is divided by the corresponding cross sectional area of the coil as we have seen in the previous lecture. Suppose if we are considering +A for one coil side and the direction of current is cross and for the other coil side if we are considering –A then the current will be dot.

So, their difference will represent the flux passing through the area formed by the two coil sides and the corresponding voltage induced in the coil. That is what is formulated in the above equation. When we evaluate $l\int_s A ds$, the integration will be over the entire coil area.

The term $\int_s A ds$ will be the total magnetic potential difference between the two points and when that term is multiplied by Nl will give the flux linkage with the coil. Then, the flux linkages multiplied by $j\omega$ will give the induced voltage. The value of l will be eventually taken as one because we are going to do per meter depth calculations. When we substitute A by the approximate potential expression we get,

$$\psi = \sum_{e}\sum_{i=1}^{3} A_i^e \int_{Se} N_i(x,y)dxdy$$

Then, we replace $\int_{Se} N_i(x,y)dxdy$ by the matrix G' which is given by the following expression

$$\{Ge\} = \frac{N}{S}\frac{\Delta}{3}\begin{bmatrix}1\\1\\1\end{bmatrix}$$

The entries of the matrix are again calculated by using the following formula.

$$\int_{Se} \left(N_1(x,y)\right)^l \left(N_2(x,y)\right)^m \left(N_3(x,y)\right)^n dxdy = \frac{l!\,m!\,n!}{(l+m+n+2)!} 2\Delta^e$$

By the above formula, we will get the expressions of the entries as $\frac{N}{S}\frac{\Delta}{3}$. The $\frac{\Delta}{3}$ term will be given by the above integral. On the left hand side of the circuit equation the size of the source matrix $\{U\}$ is 1×1 because in this problem we have only one voltage.

So, now we are ready to combine both field and circuit equations. First we will see the following circuit equation.

$$\{U\} = j\omega\{G\}'\{A\} + [R_{ext} + j\omega L_{ext}]\{I\}$$

Then we have the following field equation,

$$[C]\{A\} - \{B\}\{I\} = 0$$

So, now the coupled circuit field equation is as given below

$$\begin{bmatrix}[C] & -\{B\}\\ j\omega\{G\}' & [R_{ext} + j\omega L_{ext}]\end{bmatrix}\begin{bmatrix}\{A\}\\ \{I\}\end{bmatrix} = \begin{bmatrix}0\\ U\end{bmatrix}$$

This matrix equation is formed by combining the field and circuit equations. Here, matrices B and G are incidentally equal because $l = 1$.

We have to remember that in the expression for matrix B there was no l and the entries of the matrix is simply $\frac{N}{S}\frac{\Delta}{3}$. Whereas in the expression of G, we have l. If $l = 1$ then we get $G = B$. But if we are doing calculations for $l = 5$ m depth then $G \neq B$ because matrix G will get multiplied by 5. Since $l = 1$ in this case, matrices G and B will become equal.

So, when we use this formulation for coding a particular problem then we can do various other analyses, particularly transformers and rotating machines excited by external circuits. In the next lecture, we will see how to do short circuit analysis of a single-phase transformer. Thank you.

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