Electrical Equipment and Machines:
Finite Element Analysis
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Lecture No 31
Permanent Magnets: Theory

Welcome to this lecture 31. In this lecture and in the next lecture, we will study theory of permanent magnets and corresponding FE formulation. As you are aware that the use of permanent magnets is increasing day by day in rotating machines and that is why it is important to understand permanent magnet theory and corresponding FE formulation.

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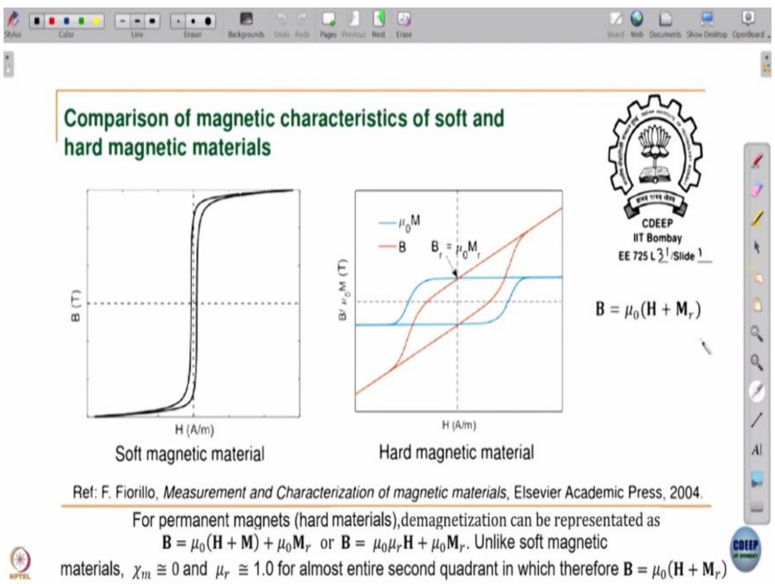
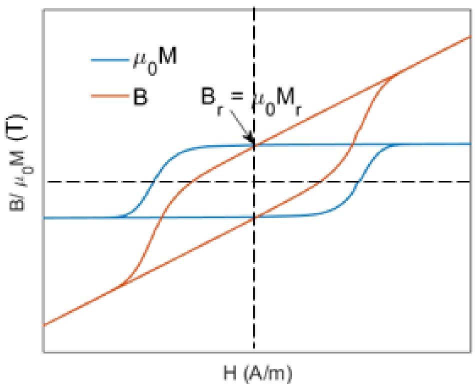


In this slide, we are comparing magnetic characteristics of soft and hard magnetic materials. If we recall the lecture on magnetic materials, there we discussed about three materials which are diamagnetic, paramagnetic, and ferromagnetic materials. There we did not discuss permanent magnets and at that time it was mentioned that we will study permanent magnets as part of the course later because it requires a more detailed understanding of the corresponding FE formulation. So, now we are at the right point to discuss permanent magnets. We all know that for soft magnetic materials, the BH loop is narrow and for hard magnetic materials, BH loop is wider as shown in the two figures in the above slide.

There are many interesting things to note here. First of all, we should know the following basic equation for permanent magnets.

$$B = \mu_0(H + M_r)$$

The M_r correspond to the y intercept as indicated in the following figure.



Another thing to note is that the magnetic characteristic is represented by M vs H characteristic and not B vs H characteristic.

It is the M vs H because there are millions of bound current loops in magnetic material and they are represented by one vector which is perpendicular to the area of current loop. The dipole vector is represented by **m** vector and when an external field is applied then **m** gets aligned with the external field. We have seen this concept in basics.

So, magnetic characteristics are M vs H when we say a material is magnetized Nd and then **m** will get aligned with H. But generally we talk about magnetic characteristic or hysteresis loop in terms of B and H. Then B vs H characteristic can be determined by using $B = \mu_0(H + M) + \mu_0 M_r$.

Now, in the above figure, we have plotted $\mu_0 M$ vs H and B vs H to match the units. Because $\mu_0 M$ will have the unit of B as the units of M and H are A/m. So, that is why we have plotted $\mu_0 M$ vs H in blue colour and B vs H in orange colour.

Now, along the +ve x axis if we increase the H values then there is no change in M that means the material is in saturated state. That means all the domains are aligned along the applied field and there is no further magnetization. So, when the material is saturated $\mu = \mu_0$. That is why B vs H characteristic will be sort of a straight line whose slope is equal to μ_0.

When the material comes out of the saturation, that means if we have taken some permanent magnet material which was not fully magnetized and apply external field H to take the material into saturation and if we increase the H value substantially in the direction opposite to magnetization then the value of B reduces to 0 in the second quadrant.

When H external is reduced to 0 in the first quadrant then the value of B is given by the equation $B_r = \mu_0 M_r$. It should be noted that M does not decrease in the first quadrant because it is in saturation state. When H is increased substantially in negative direction then only M will reduce. In the above figure, we can see that M starts reducing only when the value of H become much negative.

That is the main difference between a soft magnetic material and a hard magnetic material. The value of H_c is very high for hard magnetic materials. In the above figure, we can see that B starts reducing much before in the first quadrant itself. So, here another interesting thing to note is B goes to 0 before M goes to 0.

That means, when we go on applying –ve values of H, B reduces to 0 first because the two terms in the right hand side of the following equation get cancelled.

$$B = \mu_0(H + M_r)$$

In the above equation, $-\mu_0 H$ term will cancel $\mu_0 M_r$ at some point to reduce the value of B to 0. Then if we take the value of H to further negative then we are bringing those magnetic domains to start getting reversed and then the value of M reduces. So, when the value of M is reducing the permeability is different than μ_0 and it is substantially high.

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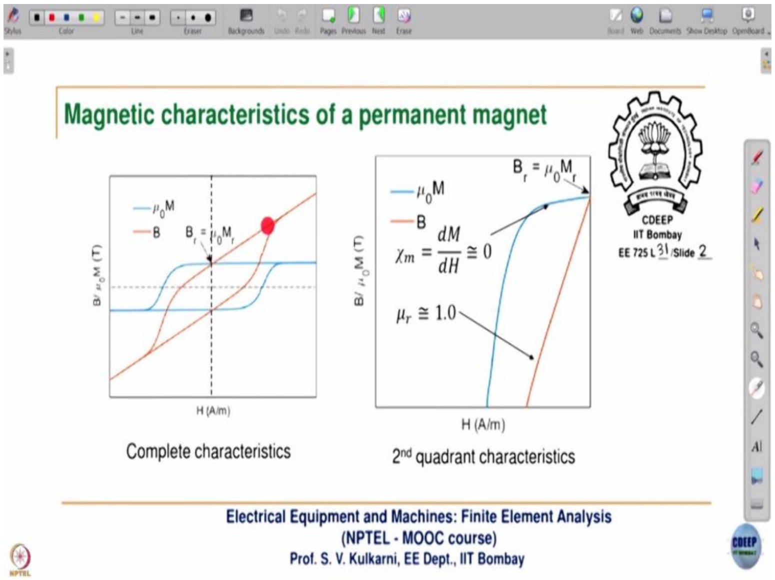

The characteristics have been plotted again in this slide. Now, if we zoom the curve in the second quadrant then part of the curve will be like in the figure on the right hand side.

As already explained, $\mu_r \approx 1$ in the saturation region. When the material comes out of saturation then μ_r will not be equal to 1 and it will be high. Here up to the point where M is more or less constant, we can consider the region as saturated. That means in the saturation region the value of $\frac{dM}{dH} = 0$.

Because in the saturation region, there is not much change in M with the change in H, so $\frac{dM}{dH} = 0$ which means $\chi_m = 0$. Also, we know that $\mu_r = 1 + \chi_m$. So, if $\chi_m \approx 0$ then $\mu_r \approx 1$. So, $\mu_r \approx 1$ means $\mu = \mu_0$ in the saturation region. In the entire saturation region, the blue line or the curve that corresponds to M vs H characteristic is horizontal because $\chi_m = 0$. That is why the slope of B vs H is also μ_0.

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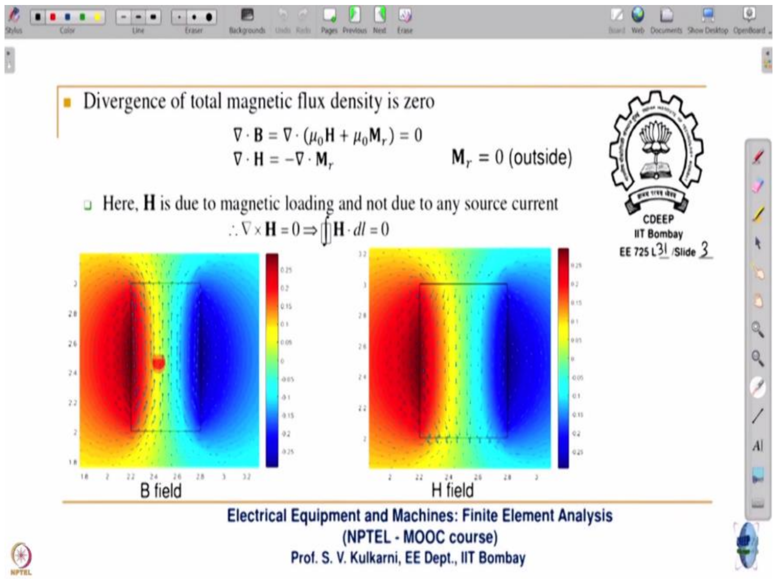

We generally know that always $\nabla\cdot\mathbf{B}=0$ because magnetic monopoles do not exist and all that theory we have seen earlier. Now, let us understand what happens to the values of $\nabla\cdot\mathbf{H}$ and $\nabla\cdot\mathbf{M}$. Now, the figures given in the above slide corresponds to a magnet which is kept in air. The figure on the right hand side corresponds to B vectors which are directed as shown in the figure.

Always remember that outside the magnet B vectors will go from North pole (N) to South pole (S) and inside the magnet vectors will go from S to N. The direction of the corresponding H field vectors is reverse inside the magnet and it is along the B vectors outside the magnet. This should happen to satisfy the following equation.

$$\nabla\cdot\mathbf{B}=\nabla\cdot(\mu\mathbf{H}+\mu_0\mathbf{M}_r)=0$$

In the above equation, μ is not exactly equal to μ_0 but it is almost equal to μ_0 because χ_m is almost equal to 0, that is why $\mu\approx\mu_0$. Generally, based on the practical properties of permanent magnets given in the literature the value of μ_r is something like 1.05 and it is close to 1. So, this confirms that the value of μ can be considered as μ_0.

So, then if we agree that we can write the permeability values in the above equation as μ_0 then μ_0 gets cancelled and we get $\nabla\cdot\mathbf{H}=-\nabla\cdot\mathbf{M}_r$. This also indicates that the permanent magnet operates in the second quadrant. Because the permanent magnet is magnetized and it is kept in air which acts as a load on this permanent magnet.

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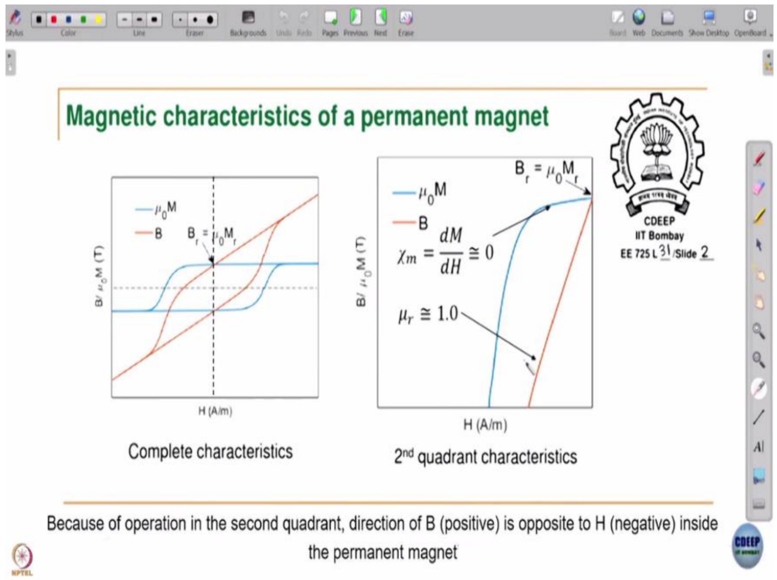

So, the permanent magnet is going to operate in the second quadrant as shown in the figure on the right hand side.

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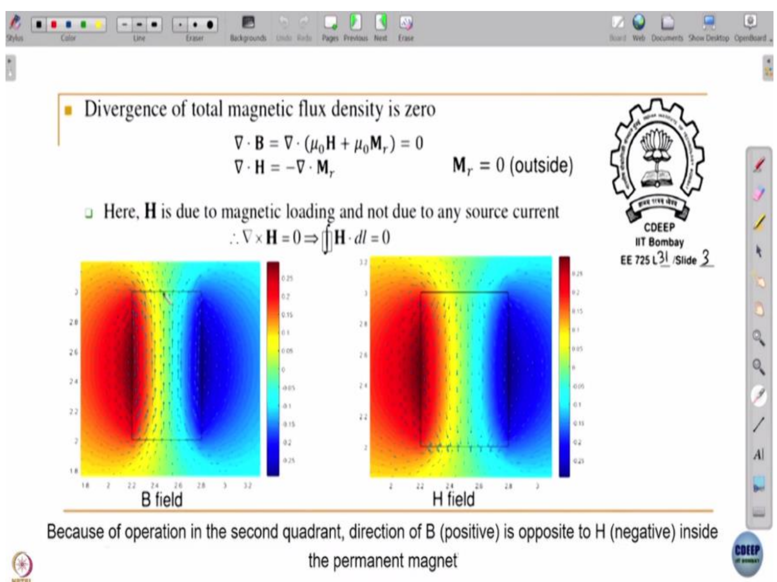

Even though the permanent magnet is kept in air without any physical load the air will act as a load. If we short circuit the permanent magnet then the value of H will be 0 and it will not act as a load. More about it we will discuss little later.

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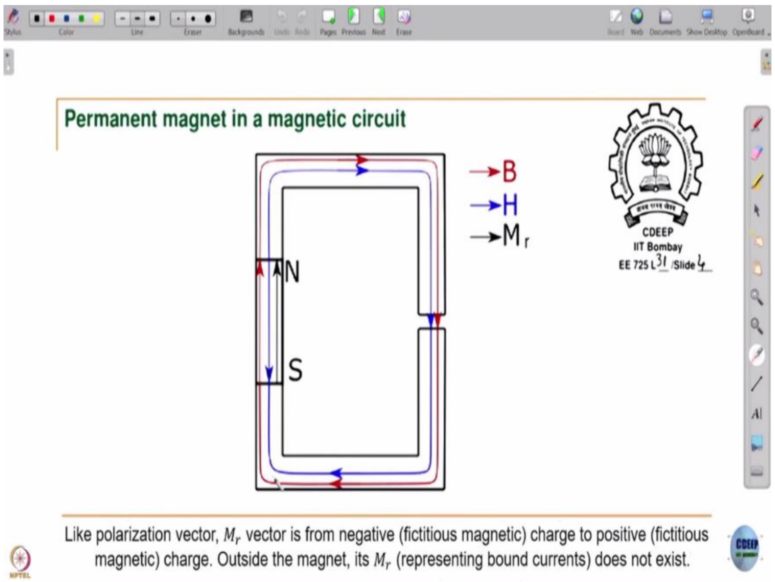

In this slide, we will analyze a magnetic circuit to understand those flux plots that we have seen in the previous slide. In the magnetic circuit there is a permanent magnet with north pole and south pole and it is connected to a core material whose permeability is almost tending to infinity. Along with the core material there is an air gap in the magnetic circuit.

In the figure given in the above slide, B is continuous and is indicated with red vectors and $\nabla \cdot \mathbf{B} = 0$. M and $\mathbf{M}_r$ vectors are directed from south to north. The H vectors are given in blue colour and remember the $\mathbf{M}_r$ is the residual magnetization and it is directed opposite to the H vectors because the magnet is operating in the second quadrant and the magnet is still in the saturated condition. Demagnetization has not happened by any means.

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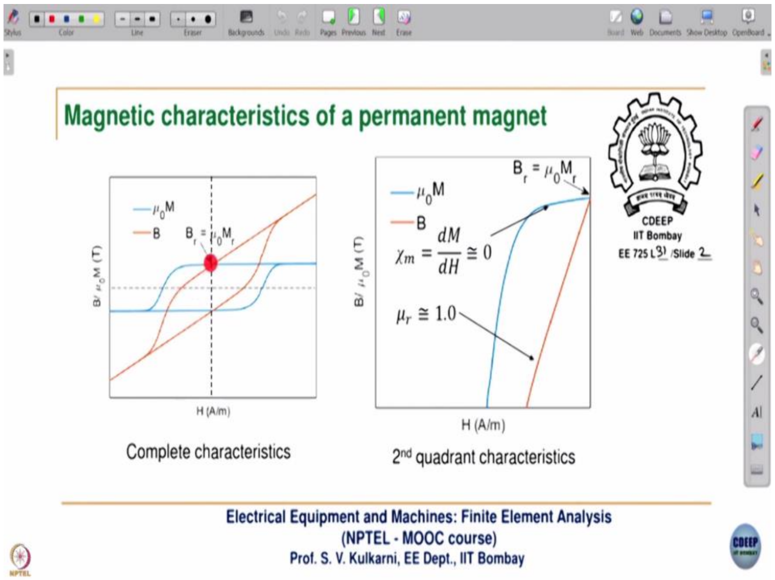

The residual magnetization M_r indicated in the figures given in the above slide represents DC bias. Remember that here we are discussing about the saturation region in second quadrant where BH characteristic is linear.

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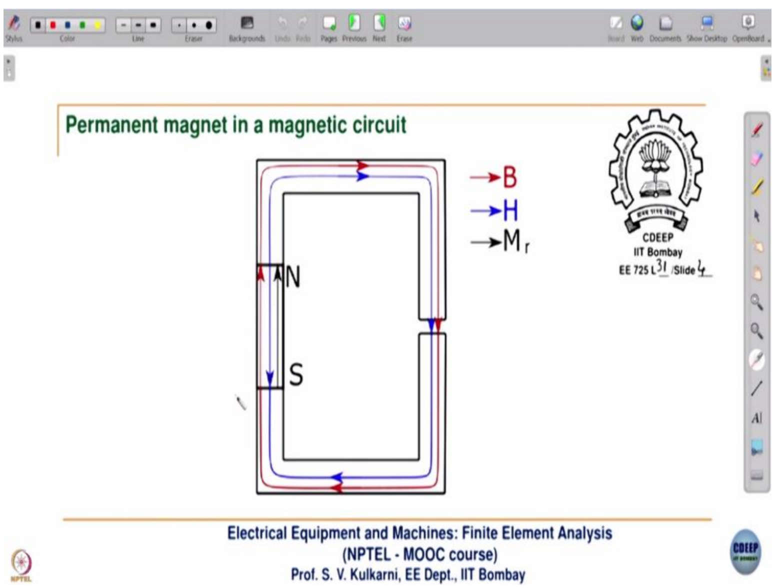

Here, if we consider the permeability of the core material as a high value which is the case of a ferromagnetic material then in the core region also M will be there. But if we are assuming the material as linear then M will be proportional to H and that is why we will get $B = \mu H$ in the core region. Also at each point of the core region, the value of μ is constant which implies that $\nabla \cdot \mathbf{B} = 0$.

Only if we consider the material as non linear then the value μ will change with space and then $\nabla \cdot \mathbf{H} \neq 0$ and its value will be related to the corresponding change in μ with space. But here we are assuming that the high permeable core material is linear then the values of $\nabla \cdot \mathbf{H}$ as well as $\nabla \cdot \mathbf{B}$ is equal to 0.

Now, the permanent magnet can be modelled using two approaches. One such approach is Amperian model wherein a permanent magnet is represented by a single current loop.

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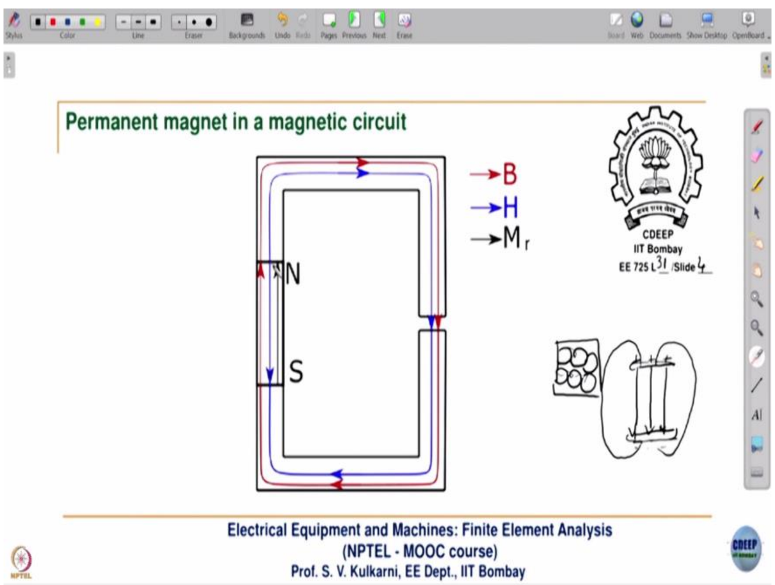
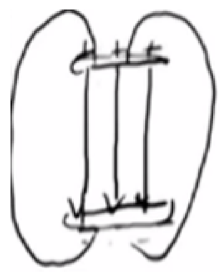


Suppose, if we take a cross section of permanent magnet then there are number of bound current loops as shown in the following figure

The currents in the inner current loops will get cancelled. So, only the branches of outer current loops will remain and internal currents will get cancelled and that is why we will get one equivalent of current loop.

Suppose we take a cross section along the vertical height, then there will be so many single current loops and they will add to M. So, this is called as Amperian model. There is another model which helps us to explain $\nabla \cdot \mathbf{M}$ and $\nabla \cdot \mathbf{H}$ not being equal to 0 at these boundaries or poles and that model is based on magnetic charges.

Although isolated magnetic charges do not exist, they are of use to us to understand the concept. Here, we are placing the magnetic charges on the poles of a magnet as shown in the following figure.



We are placing positive charges at the north pole and negative charges on the south pole. Whenever in magnetics, if we have to represent any field by using magnetic charges, they cannot appear in isolation and they will always appear in pairs.

So, the moment positive charge is placed then there should be a negative charge because isolated north and south poles do not appear. So, the magnetic charges have to appear in pair. Inside the magnet, there are no magnetic charges.

Now, the electric and magnetic fields become analogous. We know in case of parallel plate capacitor, electric field intensity is directed from positive to negative charges. Similarly, magnetic field H indicated in blue colour in the figure given in the above slide is directed from positive to negative charges inside the magnet. So, perfect analogy is established between electric and magnetic fields. Sometimes the magnetic charge model is helpful to understand the field distribution inside and outside the permanent magnet.

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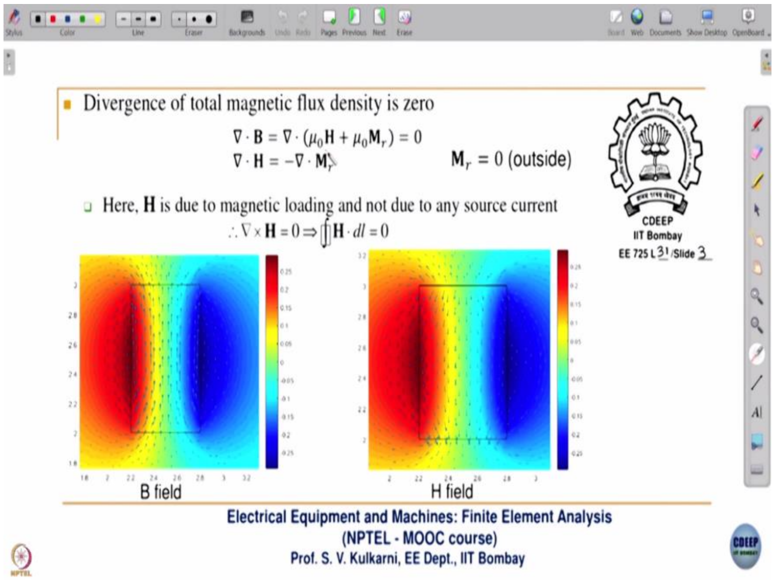
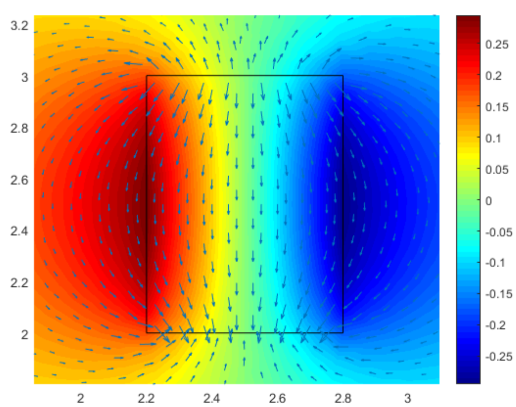


Going further, as we have discussed B field is continuous all along the region surrounded by the magnet and the direction of H field is reversing as shown in the figures given in the above slide. We can also observe that in the middle region of the magnet H field is perfectly vertical and at the ends it becomes little slanted and merges with the H field vectors which are outside the permanent magnet as shown in the following figure. This will avoid the discontinuity.



So, there is no sudden change in H field at the interface between air and permanent magnet. At the poles of the permanent magnet, we can observe that there is a sudden change of H field vectors because there are equivalent magnetic charges. So, there has to be a sudden change in the value of field like seen in a capacitor example. In capacitor also field lines originate or terminate on charges.

So, in magnets also on the surfaces of the poles (horizontal boundaries) there is a certain discontinuity and sudden change in the direction of H field.

But along the vertical boundaries of the above figure there is a gradual change in H field vectors and outside the magnet, the H field vectors merge with the corresponding B field. Inside the magnet H field opposes the M field which is vertically up. Always remember that the directions of field vectors correspond to the magnet operating in the second quadrant. That means the magnet is in saturated condition and it is placed in air region. Moment we say M_r then M is not proportional to H because it is constant. Only when the magnet comes out of saturation the proportionality will hold.

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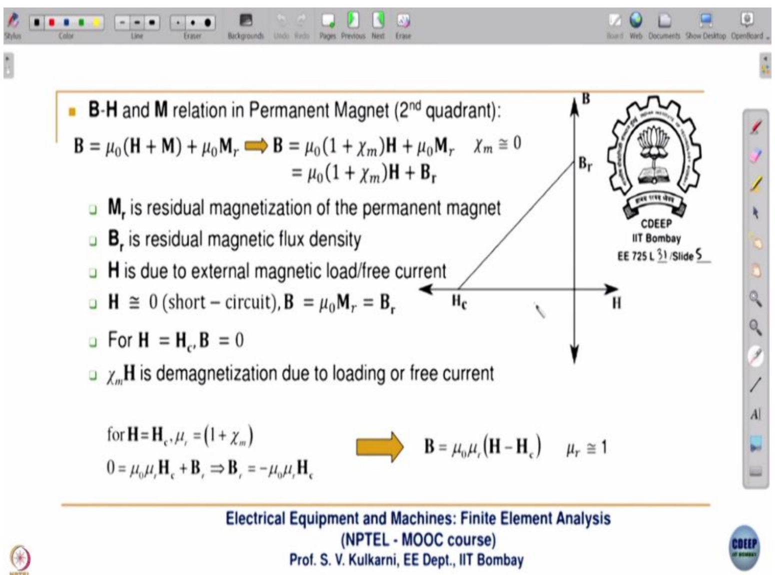

So, again we have explained that we are discussing about a permanent magnet operating in the second quadrant and we are considering B versus H characteristic which are shown in the figure given in the above slide.

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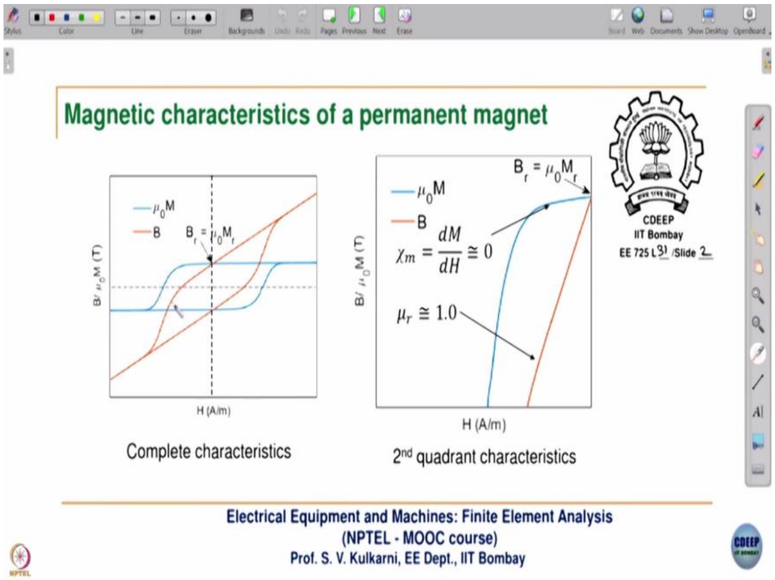

The B vs H characteristics of a magnet are essentially linear except at the end portion indicated in the left hand side figure in the above slide. This portion corresponds to the region where the value of M starts dropping. Because in this region there is a corresponding reduction in M when the magnet comes out of saturation. So, only at the end of the second quadrant, there is a significant reduction in B and M.

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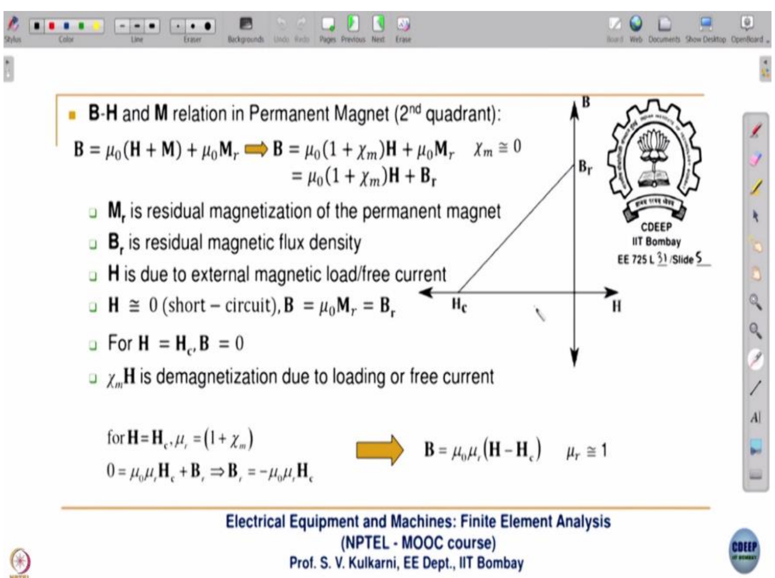

We are not considering the region where M changes considerably in this analysis. This point will never be reached. This region will represent perfect open circuit which is not possible in case of a permanent magnet. More about it after some time.

Here, we need to remember that we are modeling a permanent magnet with a straight line shown in the following figure given in the above slide without any nonlinearity at the end of the second quadrant. Now, $\boldsymbol{B} = \mu_0\mathbf{H} + \mu_0\mathbf{M} + \mu_0\mathbf{M}_r$. The $\mu_0\mathbf{M}_r$ is residual magnetization that we have seen earlier. $\mu_0\mathbf{M}$ is due to the inherent property of ferromagnetic material. The value of μ_r is not exactly equal to 1 and it is something like 1.05 or 1.06 for a permanent magnet.

μ_r not being equal to 1 is being represented by some scope for moments to get magnetized. The value of μ_r being equal to 1.05 represents that the material is not fully magnetized and there are still some magnetic domains which should get further aligned. This phenomenon has to be represented by M whereas M_r is representing all the domains which are already aligned and in saturation condition.

So, we can write $\boldsymbol{B} = \mu_0(1 + \chi_m)\boldsymbol{H} + \mu_0\boldsymbol{M}_r$. Then as we have discussed χ_m is almost approximately equal to 0 that means the value of μ_r is approximately equal to 1. If μ_r is exactly equal to 1 then this M will not appear. So now, M_r is the residual magnetization of a permanent magnet. B_r is the residual flux density and H is due to external magnetic load or free current. So it is again important to understand that if a permanent magnet is kept in the air then the surrounding air is acting as load. That is why H is due to loading by the surrounding air.

The second source of H is due to free current. If we pass the current in the direction of the magnetic field due to permanent magnet then we can take the magnet to saturation. If we demagnetize the magnet, then H is opposite to field of the permanent magnet. So by imposing the field from an external source we can take the magnet either in the first quadrant or in the third quadrant.

So, now $H = 0$ represent short circuit condition. It will be $B = \mu_0 M_r$ and that is equal to B_r. But generally perfect short circuit is not possible because the value of H will not be exactly equal to 0 as there will be a finite value of μ_r for a ferromagnetic material even if we take the magnetic circuit as short circuited.

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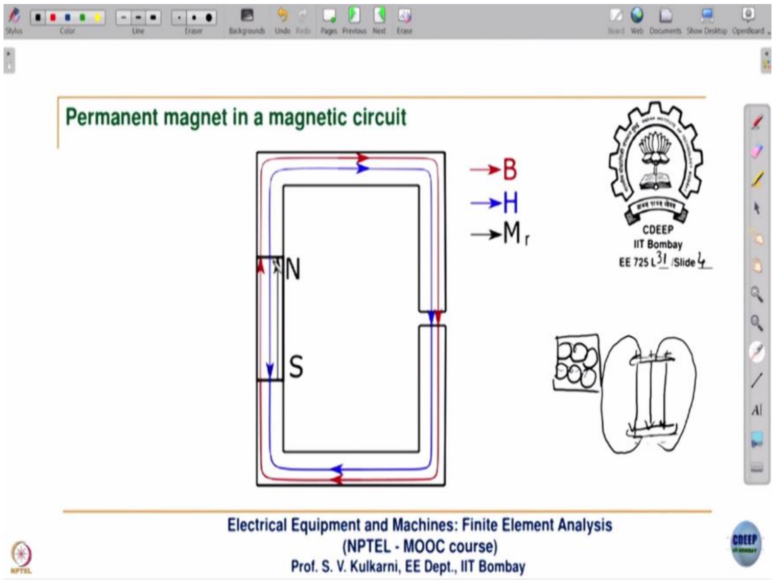

If we consider the complete magnetic circuit as a completely short circuited without air gap which is in the figure given in the above slide. Although we take the value of μ_r as infinite but actually, it is not infinite. So the of μ_r is high but not infinite. That is why we will not have a perfect short circuit practically.

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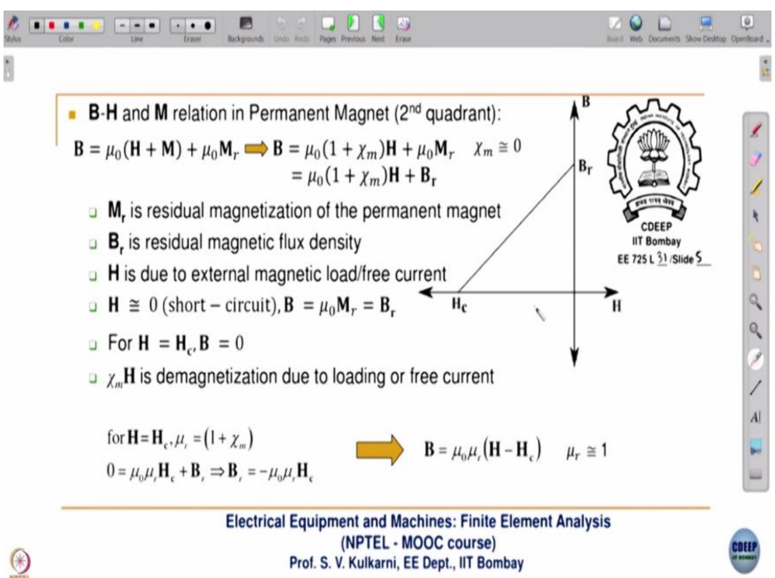
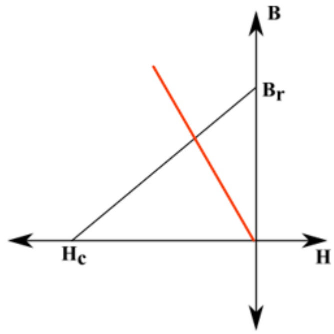


So, then the load line will be something like the red colour line in the following figure if we short the permanent magnet by a high permeable ferromagnetic material.



As explained earlier, $\chi_m H$ is a demagnetization due to loading or free current. If we have a free current or exciting the permanent magnet by some free current whose field is opposite to the direction of permanent magnet field then the permanent magnet material will get demagnetized.

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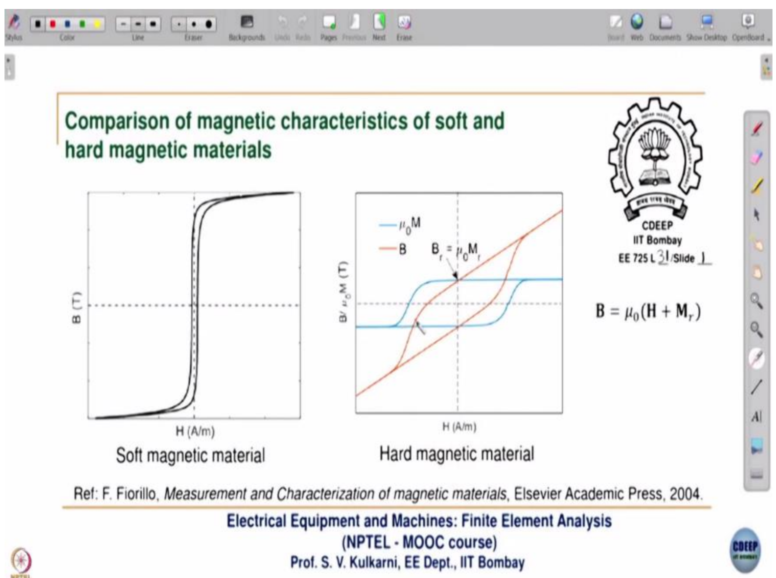

As we have seen earlier, the characteristic of a permanent magnet is with a wide loop that is why the value of H_c is very high as indicated in the figure given in the above slide. We have assumed the characteristics are linear at the end portion of BH curve. We are assuming it for simplicity.

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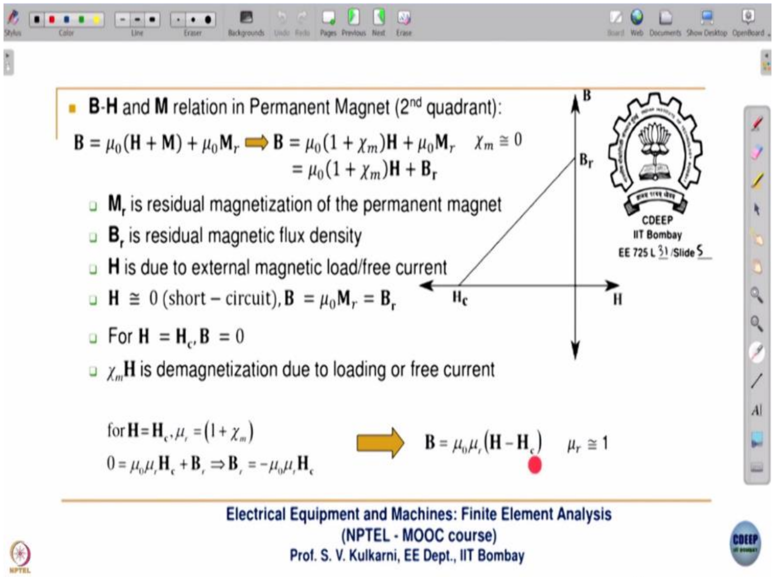

So, for $H = H_c$, B= 0. As explained, B reduces to 0 first and then the value of M becomes 0. So, if we substitute that and then $B = B_r$ then $B_r = -\mu_0\mu_r H_c$. That is why the original expression for B can be modified as $\boldsymbol{B} = \mu_0\mu_r(\boldsymbol{H} - \boldsymbol{H_c})$.

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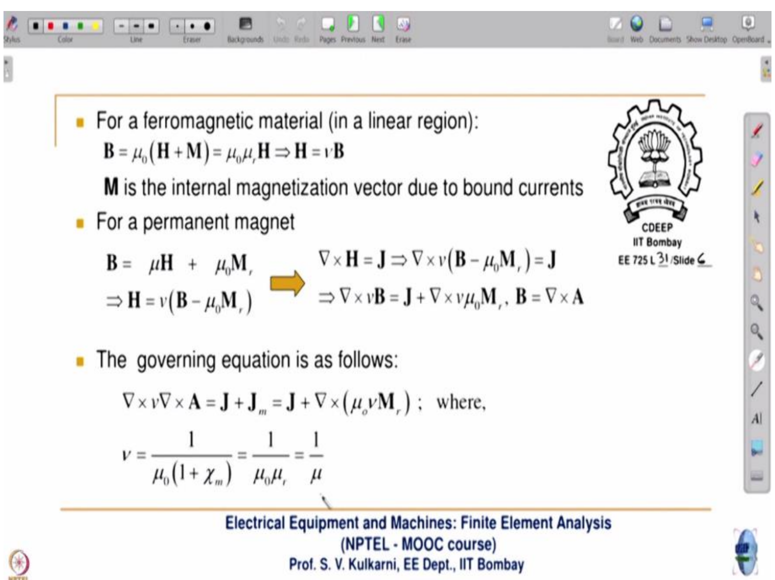

For a ferromagnetic material, in general, $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$. So, the moment we say $B = \mu_0\mu_r\mathbf{H}$ we are assuming that the material is linear that means $M \propto H$.

Then we can write $\mathbf{H} = \nu\mathbf{B}$, $\nu = \frac{1}{\mu}$. For a permanent magnet, we have $\mathbf{B} = \mu\mathbf{H} + \mu_0\mathbf{M}_r$. After rearranging the term we get $\mathbf{H} = \nu(\mathbf{B} - \mu_0\mathbf{M}_r)$ where ν is reluctivity. The $\mathbf{H}$ in $\nabla \times \mathbf{H} = \mathbf{J}$ can be replaced by $\nu(\mathbf{B} - \mu_0\mathbf{M}_r)$ and after rearranging the curl equation we get,

$$\nabla \times \nu\mathbf{B} = \mathbf{J} + \nabla \times \nu\mu_0\mathbf{M}_r$$

After substituting $\mathbf{B} = \nabla \times \mathbf{A}$, we get

$$\nabla \times \nu\nabla \times \mathbf{A} = \mathbf{J} + \mathbf{J}_m = \mathbf{J} + \nabla \times (\mu_o\nu\mathbf{M}_r)\ ; \quad \text{where,}$$
$$\nu = \frac{1}{\mu_0(1+\chi_m)} = \frac{1}{\mu_0\mu_r} = \frac{1}{\mu}$$

We are calling $\mathbf{J}_m = \nabla \times (\mu_o\nu\mathbf{M}_r)$ which is due to the permanent magnet.

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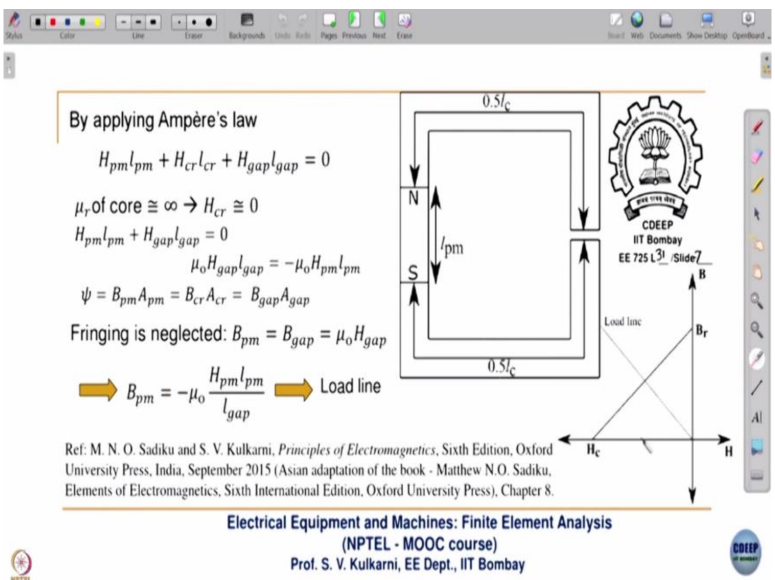


Before going into FEM formulation for a permanent magnet, let us understand the concept of load line carefully. Again we take the magnetic circuit with a permanent magnet, a high permeable core material, and an air gap. So, the load in this circuit is high permeable material with a small gap because the permeability of the core material can be assumed as infinity. So, now applying Ampere's law for this magnetic circuit (or loop) we get the following equation.

$$H_{pm}l_{pm} + H_{cr}l_{cr} + H_{gap}l_{gap} = 0$$

Here, pm stands for permanent magnet. The above equation represents that all the ampere turns or MMFs are added to 0. We are considering the value of μ_r as infinity. So, the value of H_{cr} is close to 0. So, the above equation is modified as given below

$$H_{pm}l_{pm} + H_{gap}l_{gap} = 0$$

By multiplying both sides of the above equation by μ_0 and taking one of the terms to the right hand side of the above equation we get

$$\mu_o H_{gap}l_{gap} = -\mu_o H_{pm}l_{pm} \rightarrow H_{gap} = -\frac{H_{pm}l_{pm}}{l_{gap}}$$

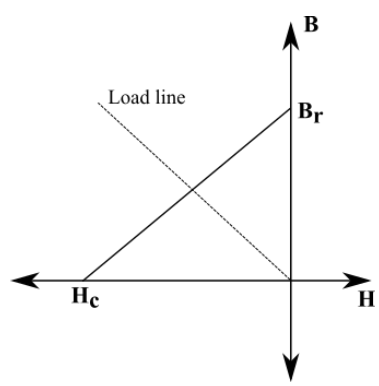
Now, by neglecting the fringing at the gap we get

$$\psi = B_{pm}A_{pm} = B_{cr}A_{cr} = B_{gap}A_{gap}$$

Remember that flux density multiplied by corresponding cross sectional area is equal to flux. So, then $B_{pm} = B_{gap} = \mu_0 H_{gap}$. In this equation, we are substituting the expression of H_{gap} and we get

$$B_{pm} = -\mu_o \frac{H_{pm}l_{pm}}{l_{gap}}$$

So, this equation represents the equation of the load line for the magnetic circuit under consideration. The load line for the magnetic circuit is shown in the following figure.



So, this equation is B vs H and the slope of this line is $-\mu_o \frac{l_{pm}}{l_{gap}}$. The line is in the second quadrant because H is negative and B is positive.

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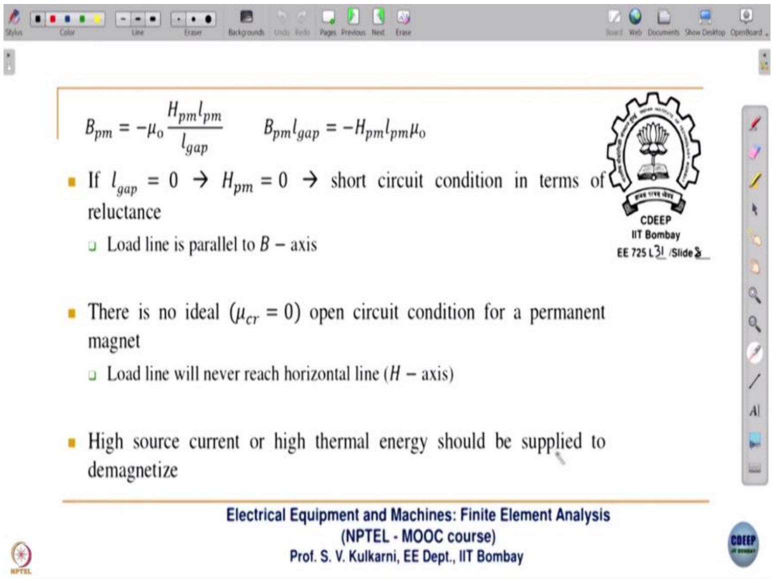

Let us understand the two extreme load conditions, open circuit and short circuit. Now, the expression for B_{pm} is rewritten below.

$$B_{pm} = -\mu_0 \frac{H_{pm} l_{pm}}{l_{gap}}$$

So, we can write $B_{pm} l_{gap} = -H_{pm} l_{pm} \mu_0$. Now, if the value of $l_{gap} = 0$ then the permanent magnet is short circuited and H_{pm} will be 0.

This short circuit condition is in terms of reluctance and then load line is parallel to B–axis. But this condition is not generally possible because we basically assumed that the value of μ for the core material is infinite. This assumption is not valid. If this assumption is valid then you will get the load line parallel to B-axis. Also, there is no ideal open circuit condition for a permanent magnet. Also, a high source current or high thermal energy is required to demagnetize a permanent magnet.

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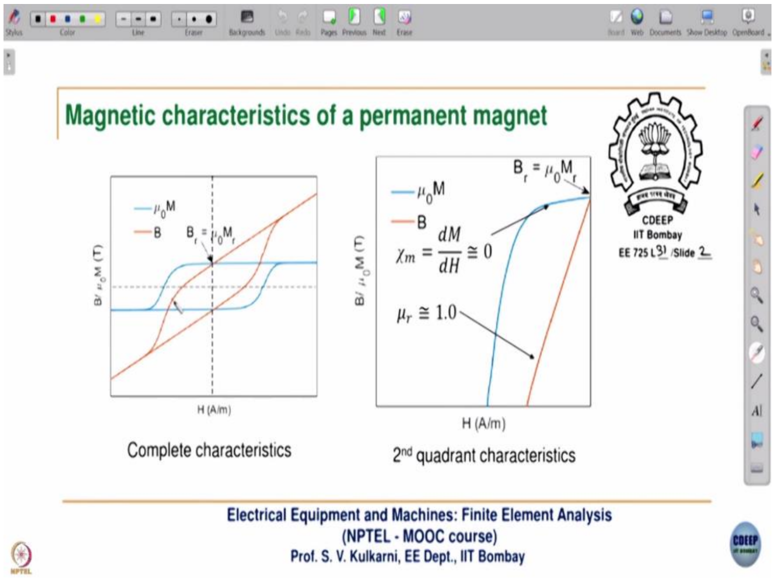

If we want to take the permanent magnet into demagnetization state, that means, if we are again bringing the magnet out of the saturation then either we have to supply much higher current greater than corresponding H_c or we have to increase its operating temperature which can increase the thermal energy. We have seen in the basics of electromagnetics that for ferromagnetic materials if we increase temperature too much like 700°C or 800°C then the randomization of domains becomes significant due to thermal agitation. The thermal energy will make all these domains disoriented in different directions and this is how we can demagnetize a permanent magnet.

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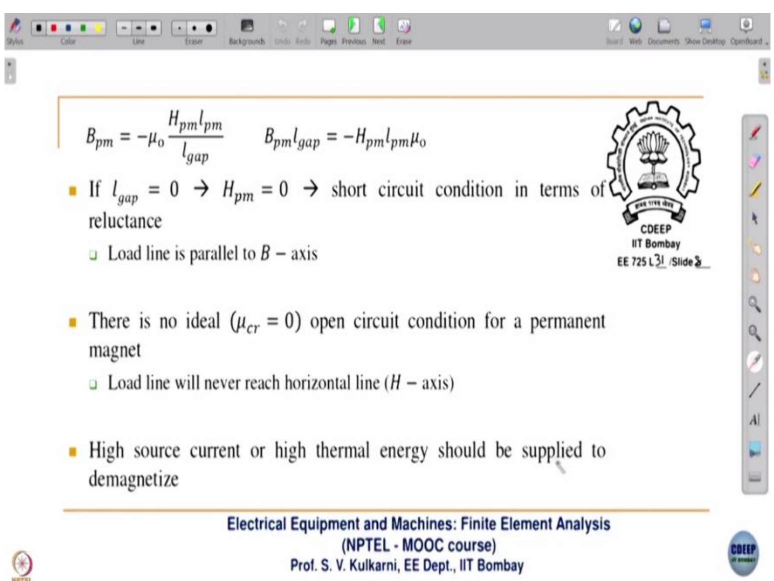

So, we will stop at this point in lecture 31. In the next lecture we will see the FE formulation for permanent magnets

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