Electrical Equipment and Machines: Finite Element Analysis
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Lecture 29
Eddy Losses in Transformer Windings

Welcome to 29th lecture, wherein we will see more about applications of non-homogenous Neumann conditions whose formulation is derived in lecture 28. In this lecture, we will see, how do we find out eddy current losses in windings of a power transformer by using FE analysis.

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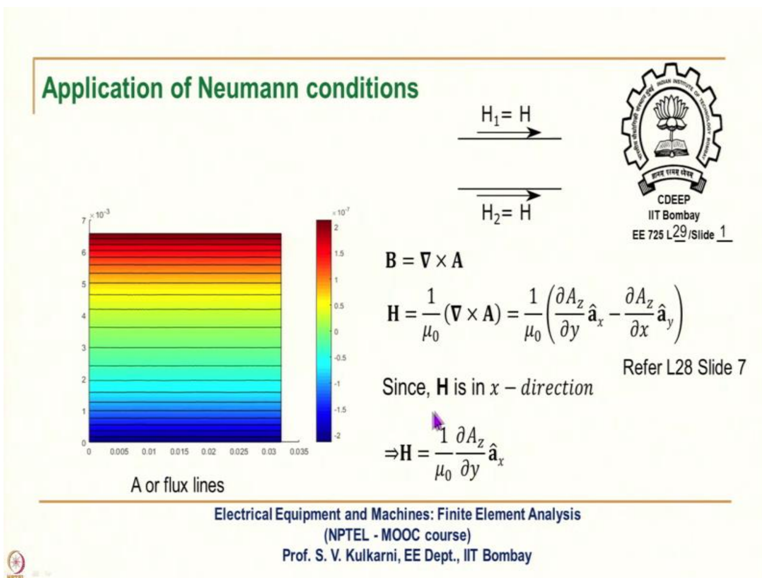
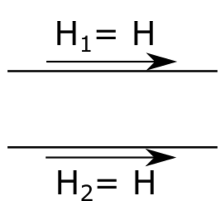


First let us see the application of non-homogeneous Neumann conditions. Let us consider case of a conducting plate, which is excited on its both surfaces by same magnetic field intensity (H) as shown in the following figure.



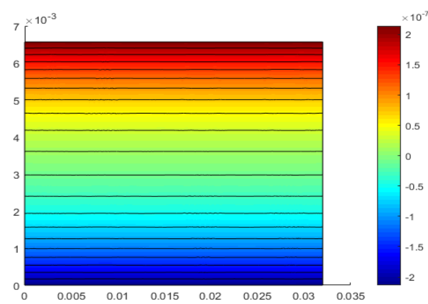
This configuration is quite common, for example, it can be a conducting plate in a rotating machine or a transformer, a core lamination, a winding conductor, etc., excited on its both sides by the same value of H. We know that $\mathbf{B} = \nabla \times \mathbf{A}$, and

$$\mathbf{H} = \frac{1}{\mu_0}(\nabla \times \mathbf{A}) = \frac{1}{\mu_0}\left(\frac{\partial A_z}{\partial y}\hat{\mathbf{a}}_x - \frac{\partial A_z}{\partial x}\hat{\mathbf{a}}_y\right)$$

Remember that A will be only in z direction because we are doing two-dimensional approximation.

This above equation was derived in slide 7 of L 28. Now, in this case, H is only in *x* direction. So, only $\frac{1}{\mu_0}\left(\frac{\partial A_z}{\partial y}\hat{\mathbf{a}}_x\right)$ exists and $\frac{1}{\mu_0}\left(\frac{\partial A_z}{\partial x}\hat{\mathbf{a}}_y\right)$ is not defined on the surface. We are imposing only $\frac{1}{\mu_0}\left(\frac{\partial A_z}{\partial y}\hat{\mathbf{a}}_x\right)$ on the boundary. Inside the conductor domain also H is in *x* direction because there are no air gaps in the domain.

H is going to reduce as we go from the surface of the conductor to the inner region of the conductor. That is the reason why $\frac{\partial A_z}{\partial y}$ is going to change as shown in the following figure.



Remember, the lines shown in the above figure are equi A or equi flux countours. As mentioned in one of the previous lectures, lines and contours can be interchangeably used in electromagnetics and particularly FE analysis.

In the above figure, we can see that A is varying with y. This can be understood from the figure because these colours are changing with y. Here, H_x is imposed and we have to solve the diffusion equation for this geometry and the magnitude of H_x is going to reduce as we go inside the conductor. Correspondingly, $\frac{\partial A_z}{\partial y}$ and A_z are also going to change as we go inside the conductor from both these surfaces as seen in the above figure.

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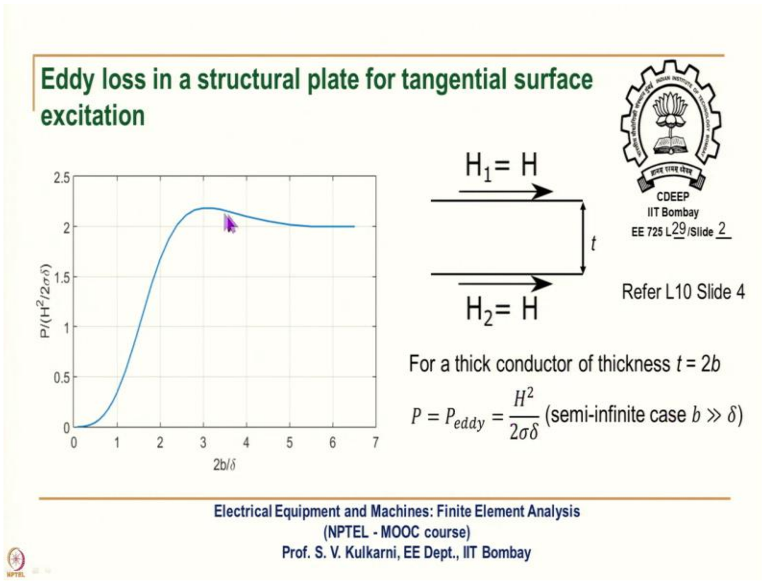

Now, what is the application of the theory that we saw in the previous slide. Let us take a conductor shown in the above slide which is excited by H on both sides. Consider that the conductor is electrically thick. As explained in L 10, the thickness of an electrically thick conductor is much greater than its skin depth.

If the conductor is electrically thick then H value reduces to 0 before the midpoint. Similarly, if the electrically thick conductor is excited on the two surfaces then the H value will go to 0 even before we go to the centerline of this conductor from the two surfaces. So, in other words, the conductor in the above slide will represent two semi infinite cases. We also know from the literature that for semi infinite case, the eddy loss per unit area is given by the following equation

$$P = P_{eddy} = \frac{H^2}{2\sigma\delta} \text{ (Semi infinite case } b \gg \delta)$$

So the plate given in the above slide can be analyzed in terms of two semi-infinite plates. That is why if the thickness (2b) of the conductor exceeds the value of skin depth then the normalized eddy current loss will approach the value of 2.

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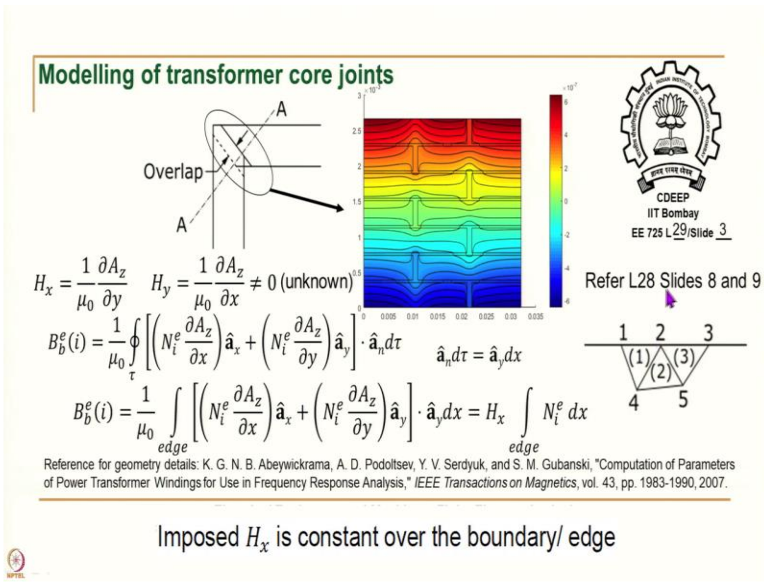
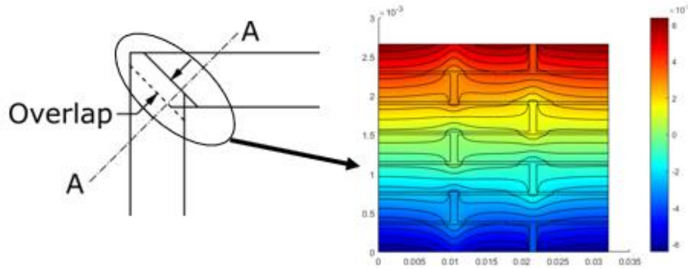


Let us go further and see some more complication in the problem domain. In this slide, we will see how to model transformer core joints particularly for frequency response analysis which is used to find the transformer impedance as a function of frequency. When we have to do time harmonic analysis we consider the core and corresponding losses. Hence, the permeability will be a complex number.

We have explained earlier that complex permeability will be used to represent a lossy magnetic material. The imaginary part of the complex permeability represents losses. In this problem, we will find the effective complex permeability of the core joint shown in the following figure.



A typical joint of a leg or limb and yoke is shown in the figure on the left hand side. So, this joint is the intersection of the vertical portion and the horizontal portion of the core of a transformer. If we take a cross section (AA), then we will get the problem domain shown in the figure on the right hand side. There are air gaps in the problem domain. The air gaps will be staggered as shown in the figure. This configuration will make the whole structure of the transformer as mechanically stable. In the joint region, air gap is never allowed to be along the same vertical line.

We again impose H_x on both horizontal boundaries top and bottom of this problem domain. But we do not have any information about H_y. Because, of the air gaps the flux contours hit the top and bottom surfaces or lines at some angle as shown in the above figure.

The value of H_y vary along the two horizontal boundaries or lines. So, H_y is unknown and we impose only H_x component along the two boundaries. Again we have seen the expressions of H_x and H_y in L 28. So, H_x is given by the following expression and we are imposing this condition and H_y is not zero and it is unknown on the boundary.

$$H_x = \frac{1}{\mu_0}\frac{\partial A_z}{\partial y} \qquad H_y \neq 0 \text{ (unknown)}$$

As we have seen earlier, the entries of boundary condition on the right-hand side matrix B_b of the final linear system of equation is given below.

$$B_b^e(i) = \frac{1}{\mu_0}\oint_{\tau}\left[\left(N_i^e\frac{\partial A_z}{\partial x}\right)\hat{\mathbf{a}}_x + \left(N_i^e\frac{\partial A_z}{\partial y}\right)\hat{\mathbf{a}}_y\right]\cdot\hat{\mathbf{a}}_n d\tau$$

In the above expression, i stands for node number, B is the right-hand side matrix, b is the contribution of boundary condition to this right-hand side matrix, and e represents the element number under consideration. We have seen the above integral for an i^{th} node in slides 8 and 9 of L 28.

Now $\hat{\mathbf{a}}_n$ will be simply $\hat{\mathbf{a}}_y$ because for the horizontal line and the corresponding surface in 3D, the unit normal will be in y direction if we are considering the plane of the slide as xy plane. So, if we substitute $\hat{\mathbf{a}}_y$ in the place of $\hat{\mathbf{a}}_n$, we will get the dot product of the second term in above integral as 0.

The dot product with the first term will be nonzero and now in place of $\frac{1}{\mu_0}\frac{\partial A_z}{\partial y}$ we substitute H_x which is the imposed boundary condition and the above integral reduces to the following equation.

$$B_b^e(i) = \frac{1}{\mu_0}\int_{edge}\left[\left(N_i^e\frac{\partial A_z}{\partial x}\right)\hat{\mathbf{a}}_x + \left(N_i^e\frac{\partial A_z}{\partial y}\right)\hat{\mathbf{a}}_y\right]\cdot\hat{\mathbf{a}}_y dx = H_x\int_{edge} N_i^e\, dx$$

Again we saw the solution for the above integral in L 28. The contribution of boundary condtion for any node will come from the corresponding contour integral over the considered element. Remember, for each node of the element under consideration, we will get the above integral that corresponds to the boundary condition. The contributions of the inner segments will be cancelled as explained in L 28.

Also, we have seen the evaluation of the above integral in L 28. We calculate the contributions of all such boundary edges by using the procedure discussed in L28, for the imposed boundary condition.

Each of these individual elements on the corresponding edges will contribute to the B_b matrix. All the contributions of the nodes can be calculated by the procedure discussed in L28 and then the overall global matrix can be calculated accordingly. Then, further calculations can be made to get the value of magnetic flux density (B) and using the calculated values of B, the average value of B over the whole domain under consideration is calculated. The average value of B is divided by the imposed H_x to calculate the value of complex permeability. For more details, see the reference given below.

Reference for geometry details: K. G. N. B. Abeywickrama, A. D. Podoltsev, Y. V. Serdyuk, and S. M. Gubanski, "Computation of Parameters of Power Transformer Windings for Use in Frequency Response Analysis," *IEEE Transactions on Magnetics*, vol. 43, pp. 1983-1990, 2007.

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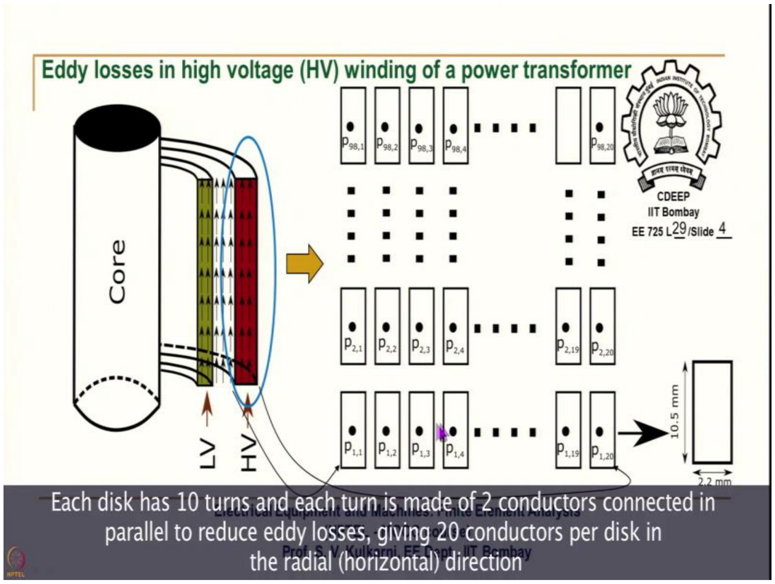

We will see how to calculate eddy current losses in a coil. This is very important. Here as an example, we are considering the high voltage winding of a power transformer. In the above slide, a three-dimensional view is shown in the left hand side figure. In the figure, only a core leg and windings are shown. The leakage field at an instant of time is directed vertically upwards and we are not showing the fringing field at the ends of the windings. In this figure, we are just showing the axial field, but there will be radial flux.

The objective is to calculate the eddy current loss of high voltage winding which is subjected to the alternating leakage magnetic field. We will use the eddy loss formula that we have already seen in L 10. Before calculating eddy current loss we will see the geometry of HV winding a little bit more in detail. The high voltage winding is shown in the figure on the right hand side and it is not to the scale. The starting and ending points and the radial depth of the HV winding are also shown in the figure.

The high voltage winding will have a number of conductors placed as shown in the figure. Each rectangle in the figure represents a disk and the dimensions of each rectangle are also shown in the above slide.

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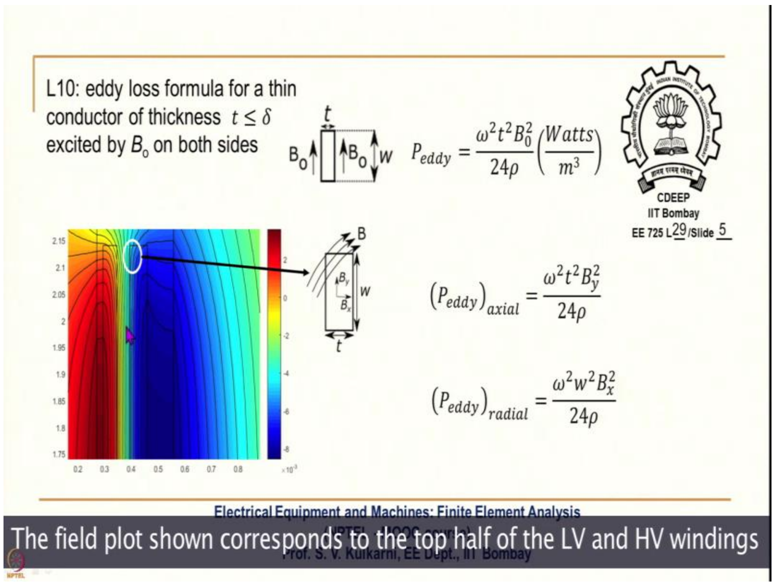
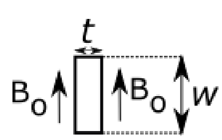


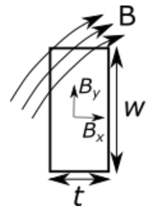
The actual leakage field plot for a transformer is shown in the above slide. There is axial flux only in the middle portion of the winding and at the ends fringing and radial flux will be predominant. In lecture 10, we have seen the following expression of eddy loss formula for a thin conductor of thickness t which is less than or equal to skin depth and the conductor is excited by B_0 on both sides as shown in the following figure.

$$P_{eddy} = \frac{\omega^2 t^2 B_0^2}{24\rho}\left(\frac{\text{Watts}}{\text{m}^3}\right)$$



In the above expression, ρ is the resistivity, B_0 is the peak value of flux density, t is the thickness of the conductor in the direction perpendicular to the flux. In the above figure, the flux is in vertical direction. So, t will be perpendicular direction to the flux as shown in the figure. Remember the expression is loss per unit volume.

At the ends of the winding, we have the radial flux component as shown in the following figure.



For the radial flux component, the perpendicular dimension will be W (the width of the conductor). Then the eddy current loss formula will be modified as given below.

$$\left(P_{eddy}\right)_{radial} = \frac{\omega^2 w^2 B_x^2}{24\rho}$$

Using these formulae, we can individually calculate losses due to axial and radial fields and then sum them up for the winding under consideration.

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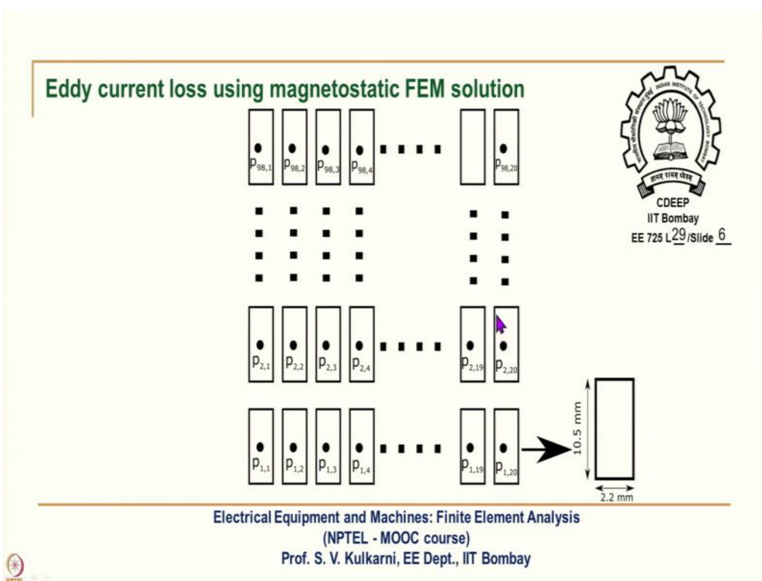

Now, let us go to the problem domain (the whole HV winding) which is shown in the above slide. The title of this slide is “Eddy current loss using Magnetostatic FEM simulation”. Now, this looks odd because we are finding eddy current losses and it is a time varying case but we are using magnetostatic FEM solution for that.

This approach is allowed because the thickness of the conductor and the overall dimension of the conductors are such that they are comparable or less than the skin depth. We can consider them as electrically thin conductors. The eddy currents induced in the individual conductors are not significant and can be assumed that these eddy currents do not influence the leakage field. So, if eddy current reaction is neglected then we can effectively use magnetostatic FEM simulation to calculate the values of B_x and B_y at the conductor centers. We can use the two formulae that we have seen in the previous slide for calculating eddy current losses in each conductor due to axial and radial fields.

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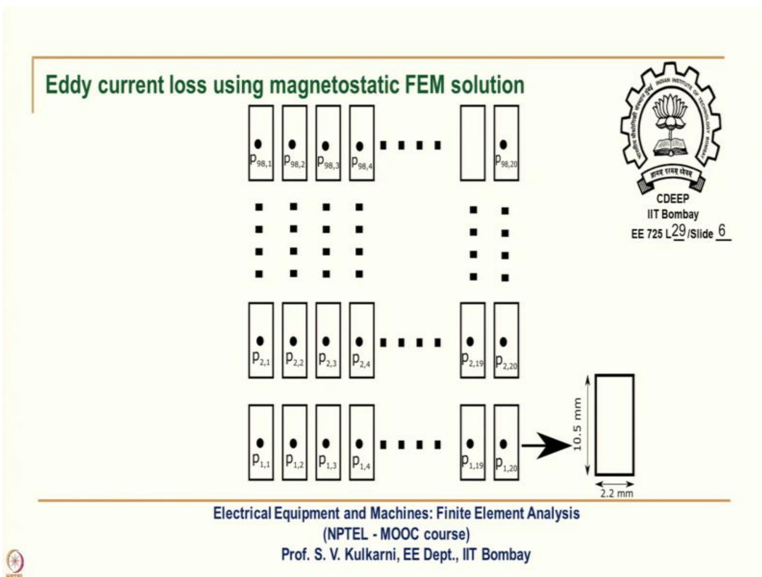

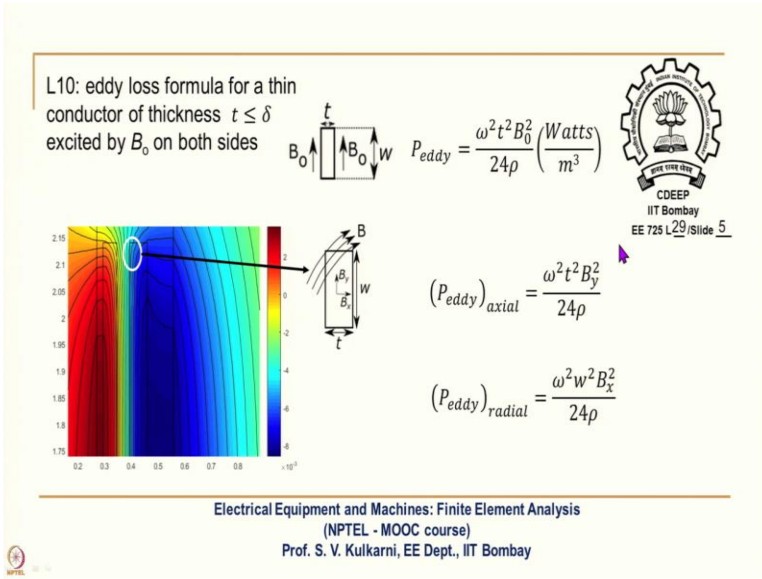

Effectively we are calculating field values using static simulation and then we are using the expressions that are derived starting by solving diffusion equation. So, we are using static FEM simulation to calculate magnetic fields, then using classical eddy current theory we calculate the eddy current losses of each conductor analytically, and then we will sum up all the losses for the entire winding.

For that, we need to find out the positions of each of this conductor and the corresponding representative point. So, for each conductor, we will assume that the flux density of the conductor is represented using the value of B at the centroid. Then we have to first find out the element number in which this center point lies. Then using the information of the element, we get the corresponding B_x and B_y of that element. These values are further used to calculate eddy current losses. In one of the previous lectures, we have seen the procedure to find the element in which the point lies.

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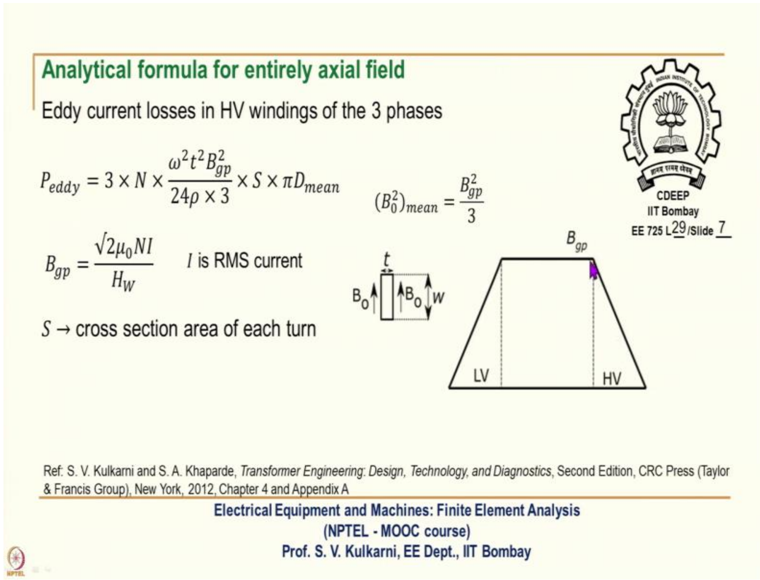
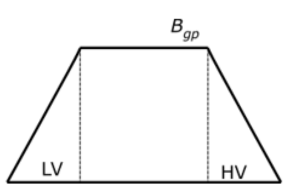


Now, we will see how to verify the eddy current losses calculated by using the obtained FEM solution. Always we should have some verification for any kind of FE analysis. The obtained solution can be verified by using an approximated analytical solution or if we have some verification problem in the literature. If all the geometrical and electrical parameters and solutions are available, then we can first solve that problem to get confidence and then we can use the developed formulation to analyze our problem.

In this case, we are using an analytical formula to verify the developed FE analysis. In analytical formulation, we have to do some approximations. Here, we are analytically integrating the effect over the entire radial depth of the high voltage winding.



In one of the earlier lectures, we have seen the above figure of the ampere turn diagram associated with the leakage field in the LV and HV windings and gap between the windings in a transformer.

From the above figure, one can see that the flux in the gap region is uniform and in LV and HV winding, the leakage field reduces as we go away from the gap towards the other end. So, if we analytically integrate the flux density in the winding region, then the average value of flux density in the winding region is given by the following equation

$$(B_0^2)_{mean} = \frac{B_{gp}^2}{3}$$

The derivations of these terms are given in the following reference.

Ref: S. V. Kulkarni and S. A. Khaparde, *Transformer Engineering: Design, Technology, and Diagnostics*, Second Edition, CRC Press (Taylor & Francis Group), New York, 2012, Chapter 4 and Appendix A

We are analytically integrating the flux density over the entire winding region because in the eddy loss formula we have a B^2 term. So, we have to find out a mean value of B_0^2 and use that mean value in the formula. This mean value will represent the field distribution in the whole winding. So, then the calculations become straightforward. Remember, flux density $\left(B_{gp}\right)$ in the gap is defined by the following equation.

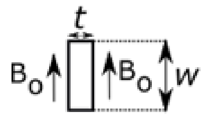
$$B_{gp} = \frac{\sqrt{2}\mu_0 N I}{H_W}$$

Here, H_W is the height of the winding and I is the RMS current because we have $\sqrt{2}$ term in the above equation. So, always remember that the above formula represents the peak values of B_{gp}. The peak value eddy current loss is calculated by using the following equation.

$$P_{eddy} = 3 \times \frac{\omega^2 t^2 B_{gp}^2}{24\rho \times 3} \times S \times N \times \pi D_{mean}$$

In the above equation, S is the cross sectional area of each turn. In this analytical formulation, we are considering axial field only.

So, if we consider the axial field as shown in the following figure then the corresponding thickness is t and it is in the perpendicular dimension to the axial field. That is why t appears in the above expression.



The loss calculated using the above expression is for N turns because there is a factor N. Also, we have a factor 3 in the above expression because this loss corresponds to the three phases. So, $3 \times \frac{\omega^2 t^2 B_{gp}^2}{24\rho \times 3} \times N$ will give eddy loss per unit volume for all turns and three phases. Now, we have to multiply this by the volume which is nothing but cross sectional area of each turn times πD_{mean} as given in the above expression. Here, we are taking the diameter as mean diameter and it again simplifies the calculation.

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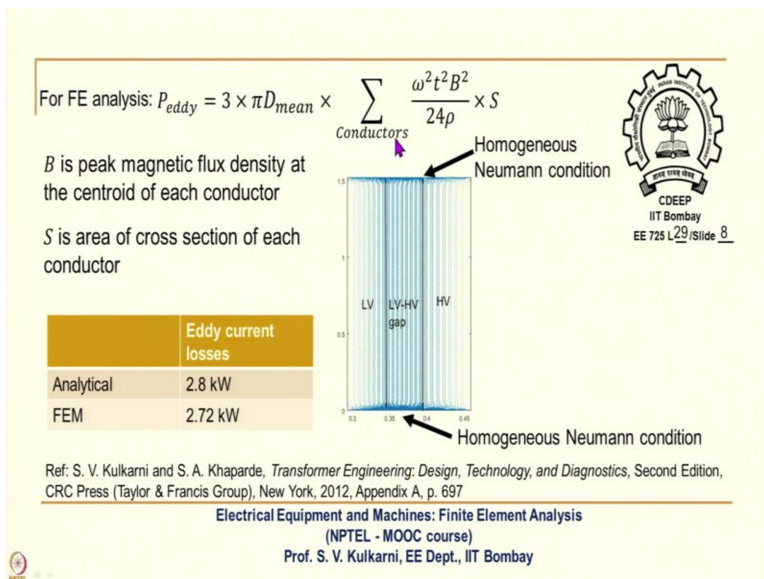
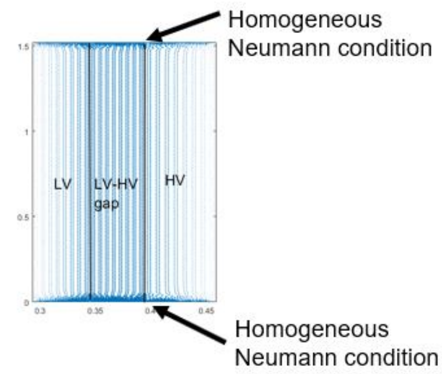


Now, we will verify the developed FE formulation. In the analytical formulation we have considered axial field. So, in the FEM simulation also we should have axial field only to validate the results.

In the FEM simulation, the fields are made completely axial by taking the horizontal boundaries at top and bottom, very close to the low voltage and high voltage windings and imposing homogeneous Neumann condition. So, the derivative of potential $\left(\frac{\partial A_z}{\partial n}\right)$ with respect to the normal of the two boundaries is 0 and this condition will make all the flux lines as straight and they impinging normal to the horizontal boundaries as shown in the following figure.



So, we have constrained the flux to remain axial throughout the high voltage winding. Now, the eddy loss calculated by using the FE analysis is compared with the loss value calculated using the analytical formulation. Before doing that, let us understand the formula to calculate eddy current loss (P_{eddy}) using FE analysis.

Now, we are not analytically integrating the loss as we did for approximate analytical formulation. Here, we are going to calculate the values of B_x and B_y for each conductor and the eddy current loss is calculated in the individual conductor.

We need to worry about only B_y because the flux is completely axial and it is in y direction and the corresponding thickness is t as we saw in the analytical formulation. The eddy current loss in the entire HV winding can be calculated using the following expression.

$$P_{eddy} = 3 \times \pi D_{mean} \times \sum_{Conductors} \frac{\omega^2 t^2 B^2}{24\rho} \times S$$

In the above equation, $\sum_{Conductors} \frac{\omega^2 t^2 B^2}{24\rho} \times S$ is the eddy loss per unit length, because cross sectional area (S) is already considered. Now, this eddy loss per unit length is multiplied by πD_{mean} of the entire winding. So, now $\pi D_{mean} \times \sum_{Conductors} \frac{\omega^2 t^2 B_y^2}{24\rho} \times S$ term becomes eddy loss for the entire one phase of HV winding. Then multiply it by three to calculate the loss for all the three phases of the high voltage winding. So, this is how we calculate the eddy current loss using a magnetostatic FE simulation. For more details about the theory and the corresponding explanation see the following reference.

Ref: S. V. Kulkarni and S. A. Khaparde, *Transformer Engineering: Design, Technology, and Diagnostics*, Second Edition, CRC Press (Taylor & Francis Group), New York, 2012, Appendix A, p. 697

We have already explained that B is the peak magnetic flux density at the center of each conductor. Using the calculated B value, we calculate the eddy loss per unit volume for each conductor and then multiply it by the area of the conductor. After that, sum it over all conductors and multiply by $3 \times \pi D_{mean}$. So, the above expression gives the total eddy loss for the entire three phases of the HV winding.

Eddy losses calculated using analytical and FEM formulations are compared in the following table.

	Eddy current losses
Analytical	2.8 kW
FEM	2.72 kW

The two results are quite close and this gives us confidence that the finite element formulation that we have adopted is correct. We can use this formulation for any complication either in geometry or ampere turn distribution because, in one of the very first lectures I told that one of the major advantages of FEM is that it is more or less independent of geometrical complications.

So, this is how we can validate our finite element formulation by solving a problem with simplified flux distribution and comparing the results to get confidence. Then the formulation is used to solve more complex problems. So, we will stop here and start a new topic in the next lecture. Thank you.

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