Electrical Equipment and Machines: Finite Element Analysis
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Lecture 28
Calculation of Eddy Current Losses

Welcome to 28th lecture. We will continue our discussion on time-harmonic problems and solution of the diffusion equation.

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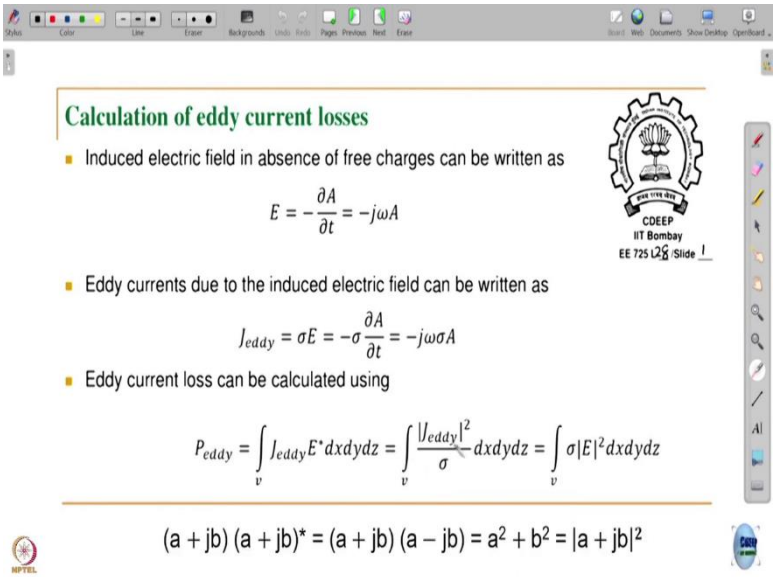

We are going to see 3 to 4 applications of these time harmonic problems. First, we will see the calculation of eddy current losses which is very common in many applications. In fact in electrical machines and equipments, we want to minimize eddy currents. But in some other applications like heating applications, we need eddy currents. So, calculation of eddy currents is important in electromagnetic devices.

Now, induced electric field intensity in absence of free charges is calculated by using

$$E = -\frac{\partial A}{\partial t} = -j\omega A$$

The complete expression of electric field intensity is

$$E = -\nabla V - \frac{\partial A}{\partial t}$$

But if there is no charge accumulation the $-\nabla V = 0$. So, charges are moving and only currents are present and they can be either free currents or induced currents.

An example for the case where there is accumulation of charges is a capacitor. For example, in a capacitor charges get accumulated and $-\nabla V$ term is non zero. If we are considering a case where only currents (either source currents or induced currents) are flowing then there is no charge accumulation.

So, $-\nabla V = 0$ and $E = -\frac{\partial A}{\partial t}, \frac{\partial}{\partial t}$ is converted to $j\omega$ in the frequency domain. Then, eddy currents due to induced electric field will be calculated by using the following expression

$$J_{eddy} = \sigma E = -\sigma \frac{\partial A}{\partial t} = -j\omega\sigma A$$

We have seen the above expression earlier in the diffusion equation. Now, eddy current loss is VI and the corresponding JE will be loss per unit volume because the unit of E is V/m and J is A/m^2. So, 1/m times 1/m^2 will be 1/m^3, so that is why JE will be loss per unit volume. In the following equation of loss we have complex conjugate for E because they are phasors. So always when we take product of electrical quantities we consider one of the quantities as complex conjugate.

$$P_{eddy} = \int_v J_{eddy} E^* dxdydz = \int_v \frac{|J_{eddy}|^2}{\sigma} dxdydz = \int_v \sigma |E|^2 dxdydz$$

If we substitute $E^* = J^*_{eddy}/\sigma$, then we get $\frac{|J_{eddy}|^2}{\sigma} dxdydz$ as given in the bove equation.

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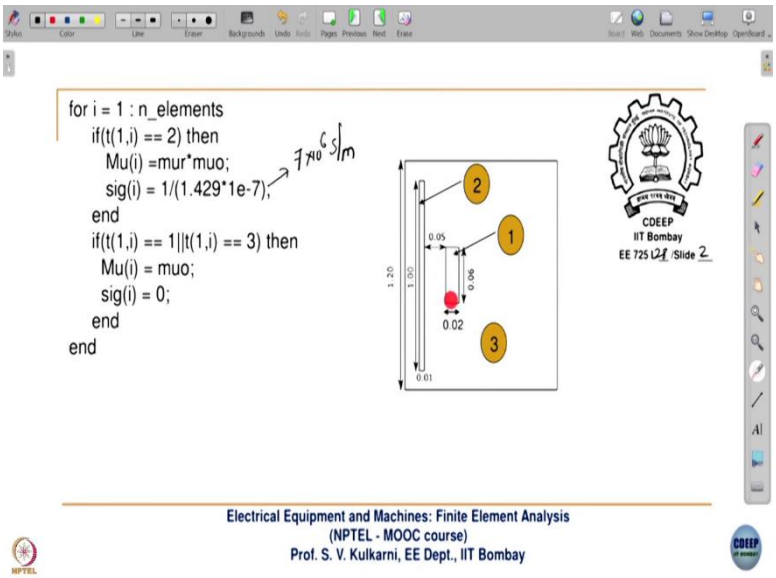

Now, we will take a standard bar plate problem which is a representative case for eddy current loss calculations. In the figure given in the above slide, we have a current carrying bar. Suppose

there is some structural plate in its vicinity. This situation appears in many electrical equipment and machines. So the current carrying conductor will induce eddy currents in the structural plate in its vicinity.

Here, we are interested in finding eddy current loss in the plate due to the current in the bar. This current is going to produce alternating magnetic field around it and that field is going to induce eddy currents in the plate because of the conductivity associated with the plate. If we are interested further in the calculation of temperature rise, then that also can be done. The eddy current losses per unit volume for each finite element can be fed to a thermal FEM code or commercial software to calculate the temperature rise. But in this course, we are restricting ourselves to electromagnetic field calculation and corresponding performance parameters and we will not get into the thermal analysis.

One more thing that we want to remember is we are not interested in eddy current losses induced in the bar. So, we will define $\sigma = 0$ for the current carrying bar. While doing FE analysis one has to be clear about the focus of the problem to reduce the problem complexity. Here since we are interested only in eddy currents in the plate, we need not define σ for the bar in the code. If we define it, we will get losses in the bar also.

If we are interested in calculating losses in the bar as well, then we have to define its conductivity. Suppose the bar is made of copper then we have to define the conductivity in the corresponding subdomain and we have to also worry about the skin effect and the corresponding size of finite elements in the bar. So, that increases the complexity, if we want to avoid that complexity then we do not have to define conductivity for this bar and we are treating that bar as a source which is inducing eddy currents in the plate.

If we are interested in skin effect and effective AC resistance of the bar, then we have to define conductivity and then take care of skin effect and the mesh size in the bar. So, that is the reason we are not defining conductivity in the bar. Here, we are going to the following part of the code and rest of the code will be similar.

```
for i = 1 : n_elements
    if(t(1,i) == 2) then
      Mu(i) =mur*muo;
      sig(i) = 1/(1.429*1e-7);
    end
    if(t(1,i) == 1||t(1,i) == 3) then
     Mu(i) = muo;
     sig(i) = 0;
    end
end
```

In the above code, we will set up a loop which will go through each element. For each element, if the subdomain number of the element is 2 (mild steel plate) then we will define $\mu = \mu_r \mu_0$. μ_r is the relative permeability of mild steel and $\sigma = 1/(1.429 \times 10^{-7})$ because for mild steel $\sigma = 7 \times 10^6$S/m. If the subdomain number of the element is 1 or 3, that means either the domain is air or bar then μ is μ_0. Because we have seen earlier the permeability of copper is also μ_0, as μ_r is close to 1 and $\sigma = 0$ and for air, $\mu = \mu_0$ and $\sigma = 0$. As explained earlier, we are defining $\sigma = 0$ for copper bar because we are not interested in eddy currents in the bar.

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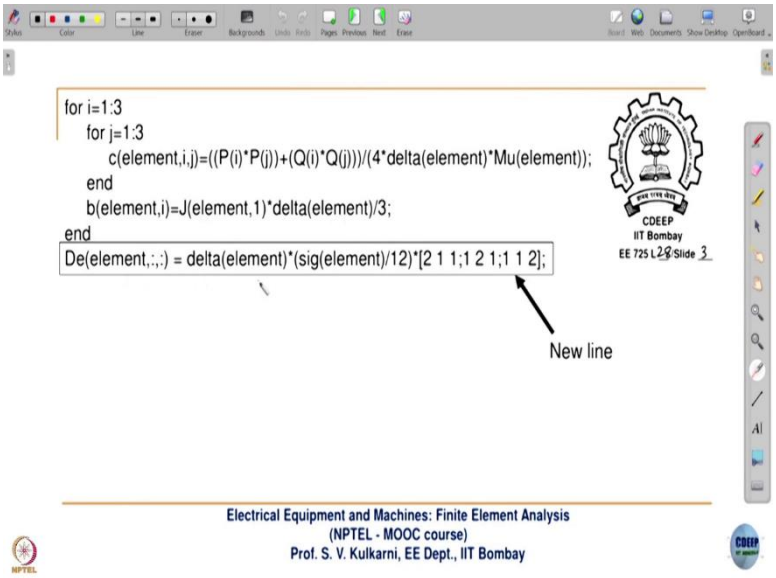

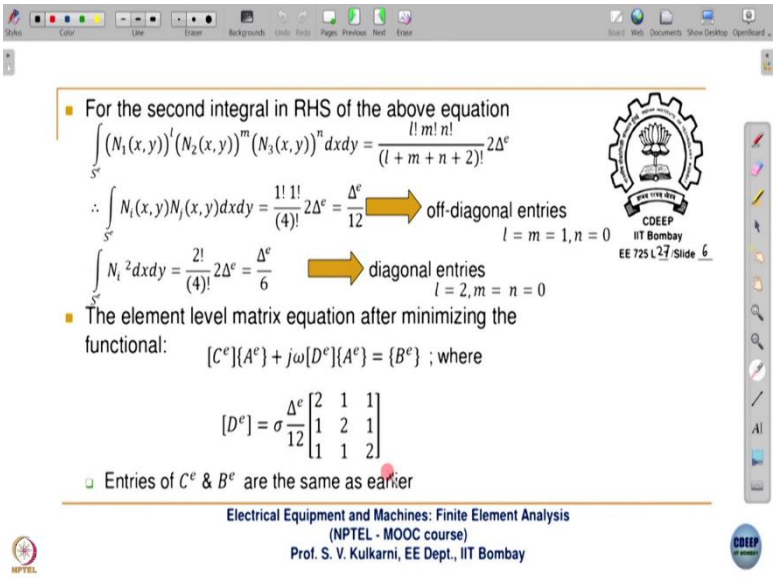

The following command will give the element level coefficient matrix and it is identical to the one that we saw.

```
c(element,i,j)=((P(i)*P(j))+(Q(i)*Q(j)))/(4*delta(element)*Mu(element));
```

Then the element level source matrix is given by the following command.

```
b(element,i)=J(element,1)*delta(element)/3;
```

In this code, matrix D is the only new thing and it is defined by using the following command.

```
De(element,:,:) = delta(element)*(sig(element)/12)*[2 1 1;1 2 1;1 1 2];
```

Mr. Sairam had explained in one of his tutorials on FEM that a small matrix can be written in Scilab as given in the above command. First row, second row and third row are separated by a semi colon. So, the above command is the only new line to be added in the code that we saw in the previous lecture.

Now, we can see how FEM coding is straightforward and simple. If we understand a simple Poisson's equation code which was was discussed and explained, then we can use the same code for diffusion equation if we just add the new parts of the code that we discussed in this slide. Here, we have formed all the element level matrices.

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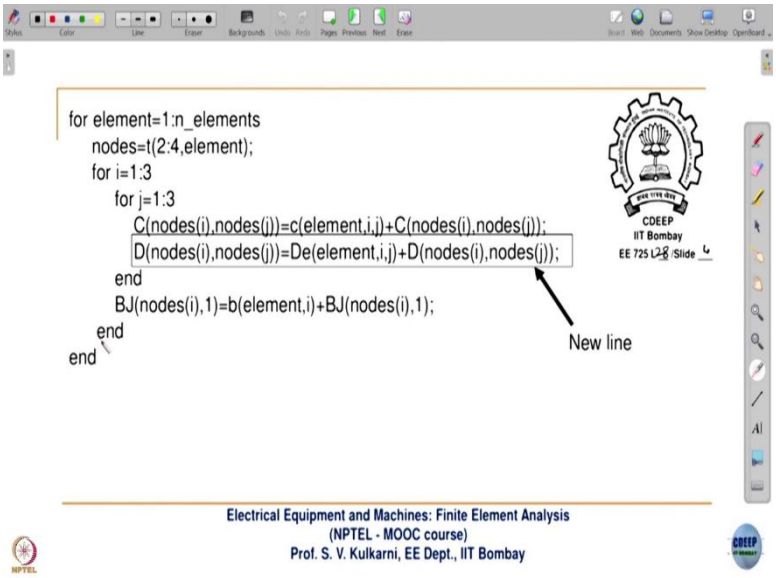

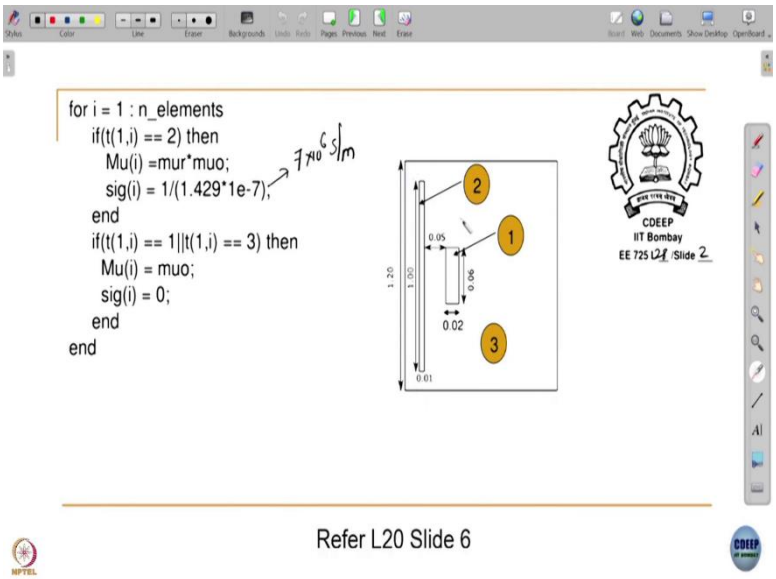

The global matrix is formed as discussed in the previous lecture. Again, we first take the global node numbers which will be second to fourth entries of the corresponding column of the t matrix because the dimensions of t matix is 4 × number of elements. In each column of t matrix, the first entry is the sub-domain number and second, third, and fourth entries are the global node numbers of the corresponding element. By using the following command we can take the global nodes for one particular element under consideration.

```
nodes=t(2:4,element);
```

We are saving the three global node numbers of the element in the nodes matrix. We basically set up the following for loop and using the commands in the for loop we are forming global coefficient matrix C, global D matrix and global source matrix.

```
for i=1:3
    for j=1:3
        C(nodes(i),nodes(j))=c(element,i,j)+C(nodes(i),nodes(j));
        D(nodes(i),nodes(j))=De(element,i,j)+D(nodes(i),nodes(j));
    end
    BJ(nodes(i),1)=b(element,i)+BJ(nodes(i),1);
end
```

New line

After forming the final set of linear equations we have to impose the boundary conditions. Here, for the outermost box, A will be defined as 0.

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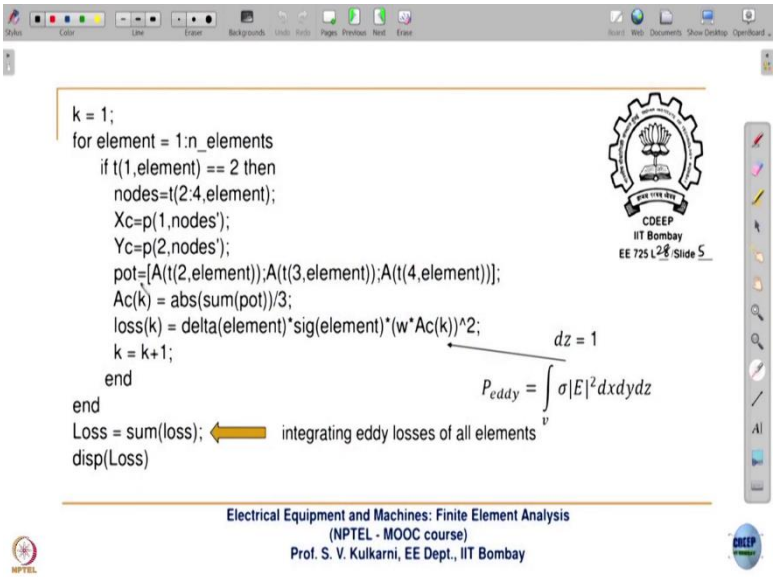

By forming all global matrices, imposing boundary conditions and then inverting the final matrix equation we would have got the solution in terms of nodal potentials. Assuming that we have written a code and we have got the solution, we will see the post processing part of this code.

Again going from element to element, if the sub-domain number of that element is 2 that means the element is in the region of plate where we are interested to calculate eddy currents.

So, that means if that element is inside the plate then we take its global node numbers, the corresponding nodes x and y coordinates of the three global nodes. In the pot variable, we store the magnetic vector potential at the centroid of the element by taking the average of potentials at the three nodes of the element. Then the eddy current loss in the element can be calculated by using the following expression

$$P_{eddy} = \int_v \sigma|E|^2 dxdydz$$

Here, we have dz = 1 because it is a 2-dimensional approximation. So, we will be calculating losses per meter depth. These calculations can be done using the code given in the above slide.

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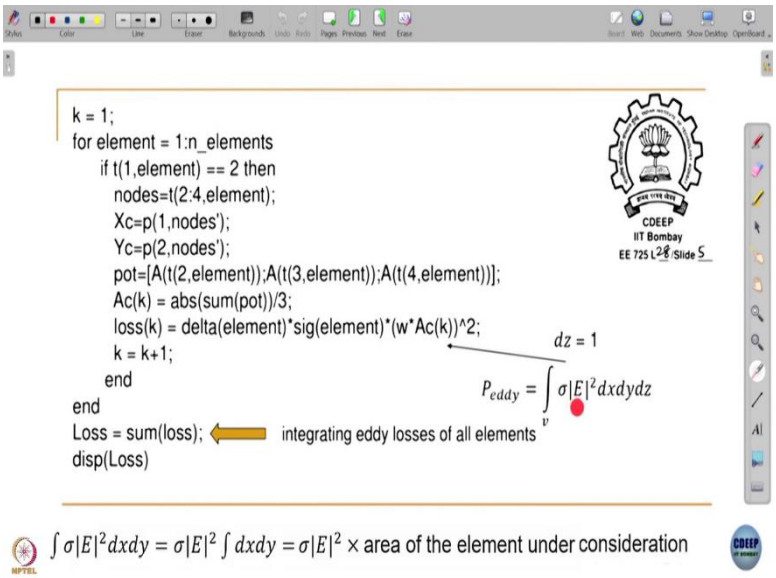

The magnitude of E is calculated by using $-j\omega A$. So, $E^2 (= \omega^2 A^2)$ is calculated for each element. Here, we are taking the average value of A for the corresponding element. Losses in each element can be calculated by using the code given in the above slide. The following commands will add all the losses corresponding to all the elements in the plate and then display the value of loss.

```
Loss = sum(loss);
disp(Loss)
```

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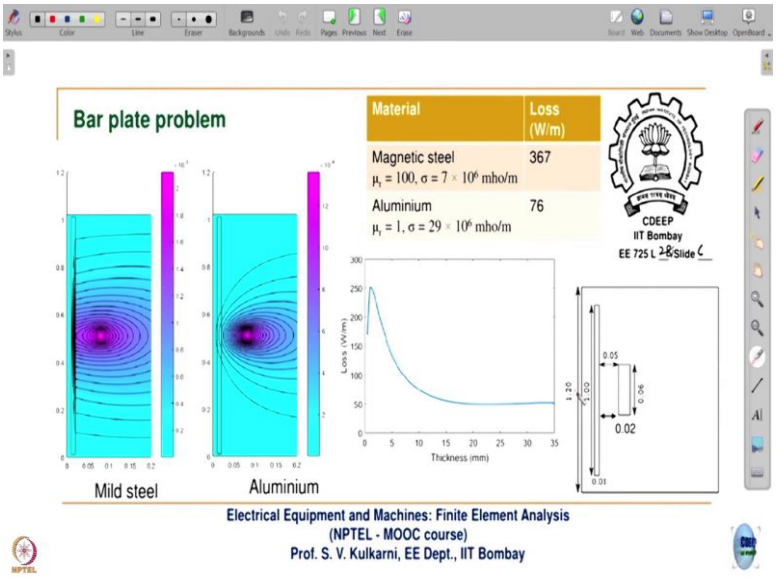
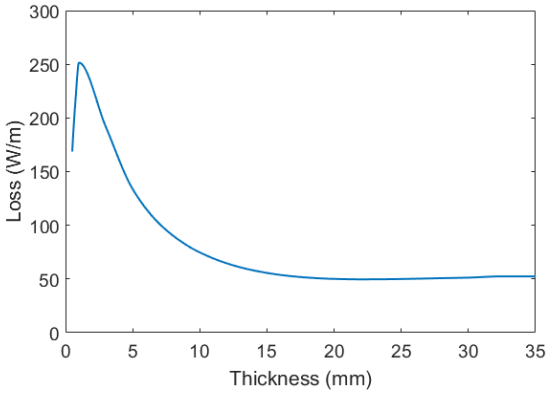


The solutions (potential contours) for mild steel and aluminium plates are shown in the figure on the above slide. For the mild steel case, we can clearly see the skin effect and in case of aluminium, the skin depth is more. For 50 Hz excitation the skin depth for mild steel is around 2.69 mm.

The skin depth for aluminium is more than 10 mm so that is why in the contour plot we can see that one contour is diffusing out of the plate. The diffusion of potential or field is more because skin depth is more. The corresponding losses for the two plates can be calculated after determining the fields and by following the procedure that we just saw. The loss obtained for mild steel is 367 W/m and for aluminium it is 76 W/m.

The value of loss is low for aluminium because in this problem we have used sufficiently thick aluminium. If we use a thin aluminium plate, then the loss will be quite high. So, whenever we use aluminium or copper, the thickness should be sufficiently high, it should be at least equal to skin depth to reduce the losses to a reasonably low value. Otherwise, there will be excessive losses in aluminium plate.

Before going further we will see what happens if the thickness of aluminium plate is very small. As mentioned in the previous discussion, the aluminium sheet thickness should be sufficiently high and it should be at least comparable to its skin depth. So, here in the following figure we can see that when the thickenss of aluminium plate is less than 5 mm then the loss is quite high.



So, that is why we need to have sufficient thickness for the electromagnetic shield. The aluminium and copper are called as electromagnetic shields, because they work as a shield by virtue of the eddy currents induced in them and the repelling action. There are other shields which are called as magnetic shields, for example, if we want to shield a structural component then we have to place a high permeable magnetic material to shield the aluminium plate.

Then, the magnetic material will take all the flux through it and will not allow the flux to go into the structural plate which has to be shielded, so these materials are called as magnetic shields. Aluminium and copper materials are used as electromagnetic shields as they work on the principle of eddy currents and the corresponding repelling effect of the field.

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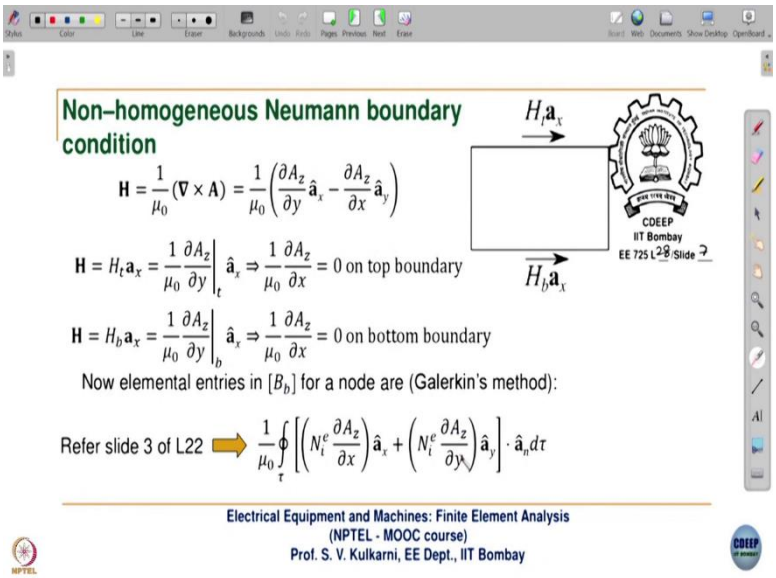


Let us go further and we will talk about non-homogeneous Neumann boundary conditions. This is a little advanced topic but still it is important to understand because we have been talking about homogeneous Neumann condition. Dirichlet conditions are two types, one is homogenous and the other is non-homogeneous. If voltage is 0 then it is homogenous Dirichlet and when voltage is non-zero then it is non-homogeneous Dirichlet condition which is obvious.

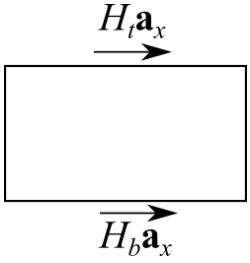
We have used homogeneous Neumann condition in the case of a parallel plate capacitor to neglect fringing. These conditions are effectively imposed on the vertical sides of that parallel plate configuration. Then one may be wondering when would this non-homogeneous Neumann conditions would be helpful.

Llet us see the application of these boundary conditions. We start with $\mathbf{B} = \nabla \times \mathbf{A}$ and then

$$\mathbf{H} = \frac{1}{\mu}(\nabla \times \mathbf{A}) = \frac{1}{\mu_0}\left(\frac{\partial A_z}{\partial y}\hat{\mathbf{a}}x - \frac{\partial A_z}{\partial x}\hat{\mathbf{a}}y\right)$$

Now, if we expand $\nabla \times \mathbf{A}$ with $\mathbf{A}$ having z component then we will have only two components of $\mathbf{H}$ as given in the above equation.

For the geometry shown in the following figure non-homogeneous Neumann boundary conditions are imposed on the top and bottom boundaries and we will see the application of this later.



Imposing non-homogeneous condition is equivalent to imposing tangential magnetic field intensity on the top (H_t) and bottom (H_b) plates or surfaces in 3D.

Always remember the geometry shown in the above figure is a 2D approximation but actually the configuration is 3D. So, now if we say $\mathbf{H} = H_t\mathbf{a}_x$ is imposed on the top plate then effectively we impose $\frac{\partial A_z}{\partial x} = 0$.

Then only H will become $H_t\hat{\mathbf{a}}_x$, so $\frac{\partial A_z}{\partial x}$ has to be 0 then only the $\hat{\mathbf{a}}_y$ component of H will be 0. So, that effectively means that we are imposing $\frac{\partial A_z}{\partial x} = 0$ and $\frac{\partial A_z}{\partial y} = \mu_0 H_t$ on the top surface or boundary. Effectively, we are imposing the non-homogeneous boundary condition $\frac{\partial A_z}{\partial y} = \mu_0 H_t$.

The $\frac{\partial A_z}{\partial y}$ is a normal derivative of A_z in this problem. Similarly, $\mathbf{H} = H_b\mathbf{a}_x$ is imposed on the bottom surface or boundary. Now we will derive the entries of element level boundary condition matrix (b matrix) for all the nodes.

Now, consider the following expression that is derived using Galerkin's method.

Refer slide 3 of L22 ⇨ $$\frac{1}{\mu_0}\oint_{\tau}\left[\left(N_i^e\frac{\partial A_z}{\partial x}\right)\hat{\mathbf{a}}_x + \left(N_i^e\frac{\partial A_z}{\partial y}\right)\hat{\mathbf{a}}_y\right]\cdot\hat{\mathbf{a}}_n d\tau$$

In the above equation, the slide number is also indicated for the reference. So, in the process of deriving FE formuation using Galerkin's method, we get the above equation. The rest of the terms in the FE formulation will remain the same. Because in this problem we are imposing the boundary conditions. In the next slides, we will see how these boundary conditions will get reflected in our final set of matrix equation.

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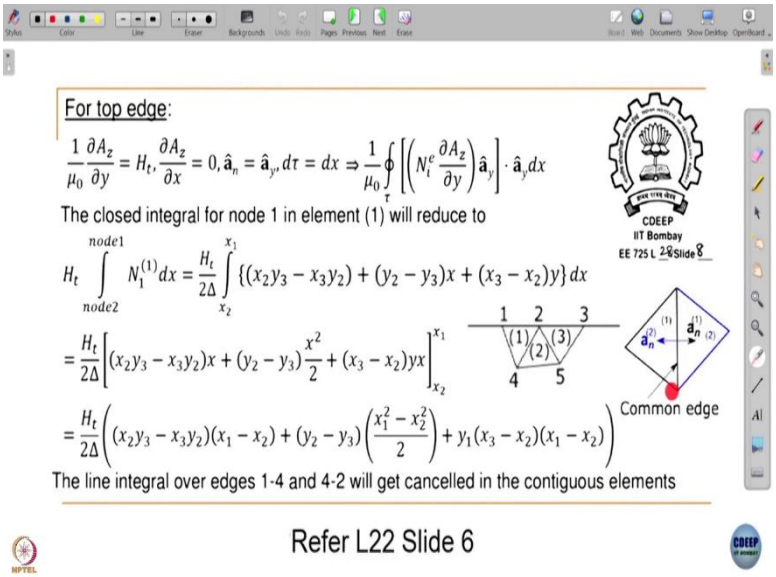
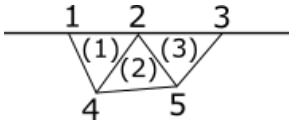


In the following figure, we are considering the top edge of the geometry that we have seen in the previous slide.



For this boundary, $\mathbf{a}_n$(normal vector) will be $\mathbf{a}_y$. The boundary conditions that we are imposing in the present problem is

$$\frac{1}{\mu_0}\frac{\partial A_z}{\partial y} = H_t, \frac{\partial A_z}{\partial x} = 0$$

and $d\tau = dx$ because the contour for this boundary will be dx. By substituting these expressions, the contour integral that we saw in the previous slide reduces to the following equation.

$$\frac{1}{\mu_0}\oint_{\tau}\left[\left(N_i^e\frac{\partial A_z}{\partial y}\right)\hat{\mathbf{a}}_y\right]\cdot\hat{\mathbf{a}}_y dx$$

Only one term will remain in the contour integral. Now, we have to evaluate the above integral. Let us take the above geometry which is a part of the top surface or top boundary. For the top boundary, there maybe a number of elements but here we are just considering three elements. We have to evaluate the above integral for each of these elements and determine the boundary conditions for the corresponding nodes. We had also seen in case of weighted residual technique that we write weighted residual statement for each node. In linear formulation, if an

element has three nodes then there will be three weighted residual statement and one statement for each node.

Let us consider the integral for node 1 in element 1 then N_i^e will be N_1. Now, we are evaluating the above integral for N_1 considering the value of $\frac{1}{\mu_0}\frac{\partial A_z}{\partial y}$ as H_t.

Node 1 is part of element number 1 and in weighted residual technique we had already seen that the common edges do not contribute to the boundary matrix because the normal vectors $(\mathbf{a}_n's)$ on the common edges are oppositely directed and the value of the shape function is also same. So the individual contributions get cancelled.

So, that is why the line integral over edges joining nodes 1 and 4 and nodes 4 and 2 will get cancelled. Only the contributions for the nodes on the boundary will remain. So, we have to evaluate the above integral only on the outermost boundary and in this lecture we are taking the two segments joining nodes 1 and 2 and nodes 2 and 3.

The closed integral for node 1 in element 1 reduces to the following equation.

$$H_t \int_{node2}^{node1} N_1^{(1)} dx$$

Here, we are considering the value of H_t as constant over the boundary. So, H_t comes out of the integral.

Now, we are substituting the standard expression for N_1 in the above integral and we are integrating with respect to x. So, the integral will get simplified as given below.

$$H_t \int_{node2}^{node1} N_1^{(1)} dx = \frac{H_t}{2\Delta} \int_{x_2}^{x_1} \{(x_2y_3 - x_3y_2) + (y_2 - y_3)x + (x_3 - x_2)y\}\, dx$$

$$= \frac{H_t}{2\Delta}\left[(x_2y_3 - x_3y_2)x + (y_2 - y_3)\frac{x^2}{2} + (x_3 - x_2)yx\right]_{x_2}^{x_1}$$

$$= \frac{H_t}{2\Delta}\left((x_2y_3 - x_3y_2)(x_1 - x_2) + (y_2 - y_3)\left(\frac{x_1^2 - x_2^2}{2}\right) + y_1(x_3 - x_2)(x_1 - x_2)\right)$$

In the derivation, in the place of x we substitute x_1 and x_2 and in place of y we can either substitute y_1 or y_2 because y is constant over the considered boundary. Since we are talking of node 1 we will substitute y_1. So, when the whole boundary integral is evaluated it will reduce to the following expression.

$$H_t \int_{node2}^{node1} N_1^{(1)} dx = \frac{H_t}{2\Delta}\left((x_2y_3 - x_3y_2)(x_1 - x_2) + (y_2 - y_3)\left(\frac{x_1^2 - x_2^2}{2}\right) + y_1(x_3 - x_2)(x_1 - x_2)\right)$$

This entry will effectively go into the boundary condition matrix. The above expression corresponds to the node 1 in the B matrix.

Now, node 2 is common for elements 1 and element 3. The boundary integral is evaluated for node 2 two times, one integral as part of element 1 and the other integral for element 3. Node 2 is a part of element 2 and there will not be any contribution to the boundary condition matrix. Because none of the three edges of element 2 are on the boundary.

All the 3 sides of element 2 are common to other elements and by the same logic, the contributions will get cancelled. For node 2, the contributions will be from elements 1 and 3.

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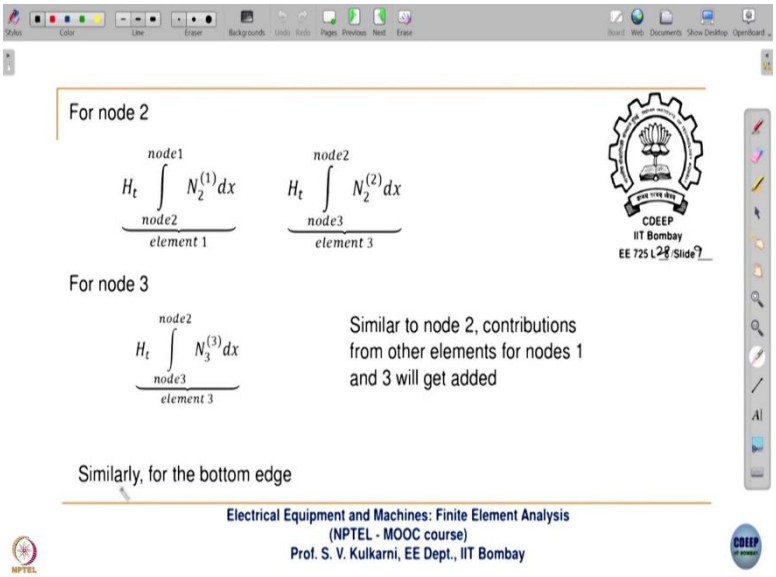

The contributions of the boundary integral for node 2 from elements 1 and 3 are given below.

$$\underbrace{H_t \int_{node2}^{node1} N_2^{(1)} dx}_{element\ 1} \qquad \underbrace{H_t \int_{node3}^{node2} N_2^{(3)} dx}_{element\ 3}$$

H_t remains the same for both the elements. Similarly, for node 3 the boundary integral contribution from element 3 is given below

$$\underbrace{H_t \int_{node3}^{node2} N_3^{(3)} dx}_{element\ 3}$$

Remember that these node 1 and node 3 may be common to some other element. So, there will be additional contributions for node 3 and for node 1. So, that is how we can evaluate the boundary condition matrix and we have evaluated the integral for the top edge and that too only for the three nodes and there may be more number of nodes on this boundary.

For all those nodes, these expressions will have to evaluated and they will go into the corresponding boundary condition matrix which is the right hand side matrix. Remember, this matrix is only for the boundary conditions and there will be contributions from the source matrix if there is J somewhere in the domain.

If we use linear elements in the formulation and if J exist in the domain, then the contributions in the source matrix will be $J\Delta/3$ for all the 3 nodes and they will get appended to the B matrix. We will stop here and then will continue the discussion in the subsequent lectures.

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