Electrical Equipment and Machines: Finite Element Analysis
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Lecture 27
Time Harmonic FE Analysis

Welcome to the 27th lecture. In the previous lecture, we saw how to derive some of the entries of the element coefficient for quadratic elements.

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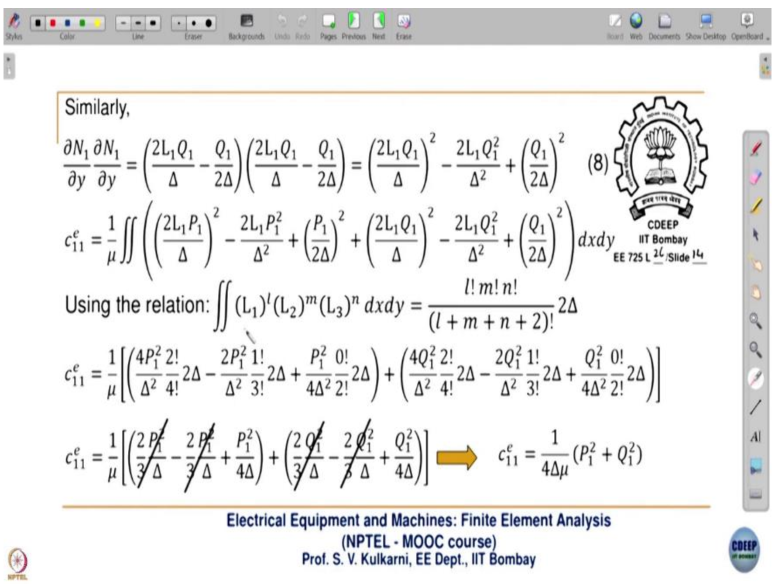

First, we saw the derivation of c_{11} (a diagonal element) whose expression is given below.

$$c_{11}^e = \frac{1}{4\Delta\mu}(P_1^2 + Q_1^2)$$

The above expression was same in case of a linear element. We also saw the following expression.

$$\iint (\mathrm{L}_1)^l(\mathrm{L}_2)^m(\mathrm{L}_3)^n\,dxdy = \frac{l!\,m!\,n!}{(l+m+n+2)!}2\Delta$$

Remember, the left hand side of the expression has N_1, N_2, and N_3 in case of linear elements because for linear elements, $N_1 = L_1$, $N_2 = L_2$, and $N_3 = L_3$. The fundamental formula is in terms of area coordinates as given in the above equation. Incidentally for linear elements, $N_1 = L_1$, $N_2 =$

L_2, and $N_3 = L_3$. That is why for linear elements we have used the expression with N_1, N_2, and N_3.

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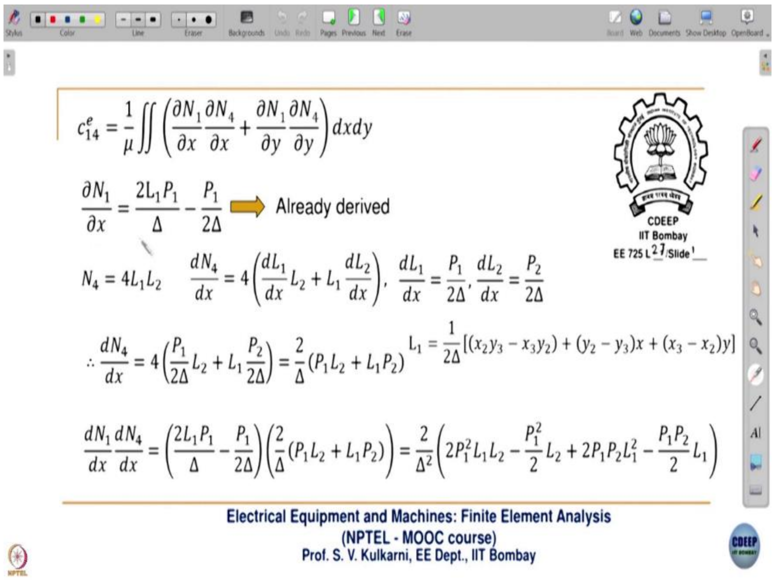

Now let us see the derivation for c_{14}^e entry. The expression for this entry will be different because in case of linear elements, c_{14} doesn't exist as size of its element coefficient matrix is 3×3. c_{14}^e for any element e is given by the following expression.

$$c_{14}^e = \frac{1}{\mu}\iint\left(\frac{\partial N_1}{\partial x}\frac{\partial N_4}{\partial x} + \frac{\partial N_1}{\partial y}\frac{\partial N_4}{\partial y}\right)dxdy$$

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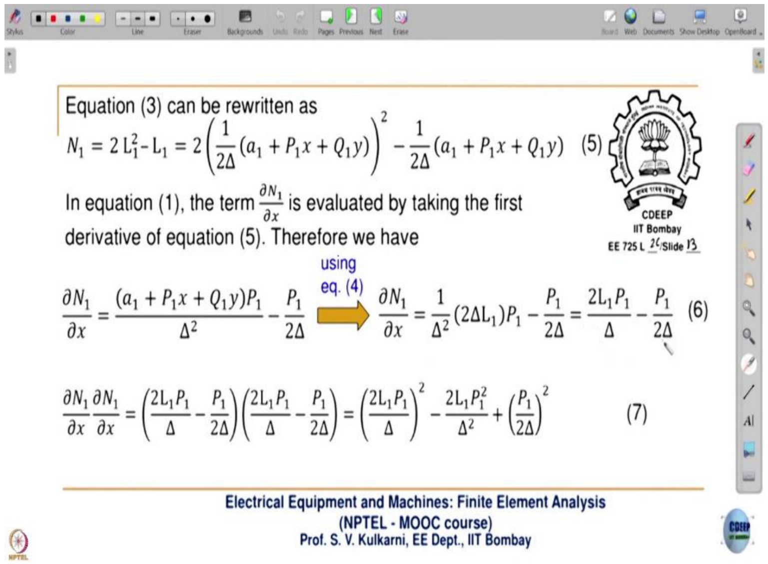

In the previous lecture, we have already derived the following expression for $\frac{\partial N_1}{\partial x}$.

$$\frac{\partial N_1}{\partial x} = \frac{2L_1P_1}{\Delta} - \frac{P_1}{2\Delta}$$

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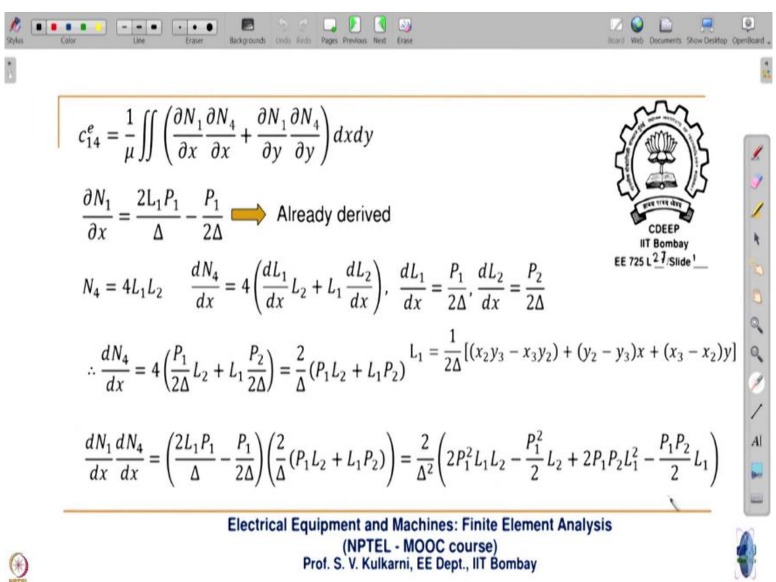

Now, the expression of N_4 is $4L_1L_2$ which is already derived in the previous lecture.

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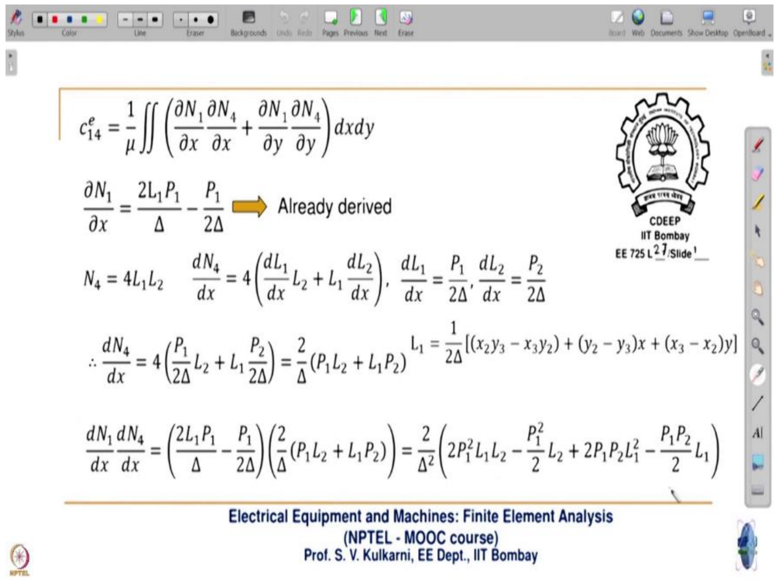

The expression of $\frac{\partial N_4}{\partial x}$ can be derived as given below.

$$N_4 = 4L_1L_2 \qquad \frac{dN_4}{dx} = 4\left(\frac{dL_1}{dx}L_2 + L_1\frac{dL_2}{dx}\right), \ \ \frac{dL_1}{dx} = \frac{P_1}{2\Delta}, \frac{dL_2}{dx} = \frac{P_2}{2\Delta}$$

$$\therefore \frac{dN_4}{dx} = 4\left(\frac{P_1}{2\Delta}L_2 + L_1\frac{P_2}{2\Delta}\right) = \frac{2}{\Delta}(P_1L_2 + L_1P_2)$$

We have already seen the derivative of L_1 with *x*. The expression of $\frac{\partial N_1}{\partial x}\frac{\partial N_4}{\partial x}$ which is required to derive c_{14}^e can be calculated as given below.

$$\frac{dN_1}{dx}\frac{dN_4}{dx} = \left(\frac{2L_1P_1}{\Delta} - \frac{P_1}{2\Delta}\right)\left(\frac{2}{\Delta}(P_1L_2 + L_1P_2)\right) = \frac{2}{\Delta^2}\left(2P_1^2L_1L_2 - \frac{P_1^2}{2}L_2 + 2P_1P_2L_1^2 - \frac{P_1P_2}{2}L_1\right)$$

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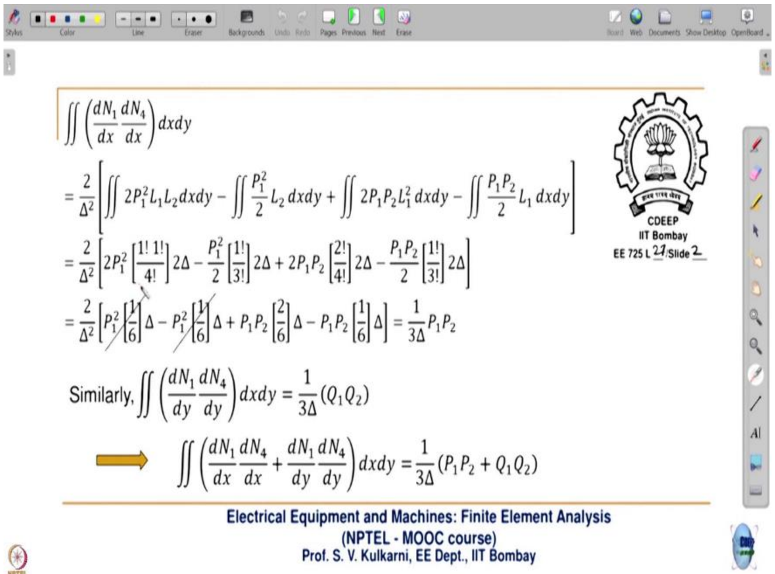

Now, we have to integrate the product over the elemental area.

$$\iint \left(\frac{dN_1}{dx}\frac{dN_4}{dx}\right)dxdy = \frac{2}{\Delta^2}\left[\iint 2P_1^2 L_1 L_2 dxdy - \iint \frac{P_1^2}{2} L_2\, dxdy + \iint 2P_1P_2L_1^2\, dxdy - \iint \frac{P_1P_2}{2} L_1\, dxdy\right]$$

This integral can be simplified by using the following integral that we have seen in the previous lecture.

$$\iint (\mathrm{L}_1)^l(\mathrm{L}_2)^m(\mathrm{L}_3)^n\, dxdy = \frac{l!\, m!\, n!}{(l+m+n+2)!}2\Delta$$

For example, for $\iint 2P_1^2 L_1 L_2 dxdy$ l = 1, m = 1, n = 0. So the result of this integral is given below.

$$\iint 2P_1^2 L_1 L_2 dxdy = 2P_1^2\left[\frac{1!\,1!}{(1+1+0+2)!}\right]2\Delta = 2P_1^2\left[\frac{1!\,1!}{4!}\right]2\Delta$$

Likewise, we can do it for the other three terms and we get the corresponding expressions. Then, after simplification, we will find the following expression.

$$\iint \left(\frac{dN_1}{dx}\frac{dN_4}{dx}\right) dxdy$$
$$= \frac{2}{\Delta^2}\left[\iint 2P_1^2 L_1 L_2 dxdy - \iint \frac{P_1^2}{2} L_2\, dxdy + \iint 2P_1P_2L_1^2\, dxdy - \iint \frac{P_1P_2}{2} L_1\, dxdy\right]$$
$$= \frac{2}{\Delta^2}\left[2P_1^2\left[\frac{1!\,1!}{4!}\right]2\Delta - \frac{P_1^2}{2}\left[\frac{1!}{3!}\right]2\Delta + 2P_1P_2\left[\frac{2!}{4!}\right]2\Delta - \frac{P_1P_2}{2}\left[\frac{1!}{3!}\right]2\Delta\right]$$
$$= \frac{2}{\Delta^2}\left[P_1^2\left[\frac{1}{6}\right]\Delta - P_1^2\left[\frac{1}{6}\right]\Delta + P_1P_2\left[\frac{2}{6}\right]\Delta - P_1P_2\left[\frac{1}{6}\right]\Delta\right] = \frac{1}{3\Delta}P_1P_2$$

Similarly, the product of derivatives with respect to y, then the expression for the entry c_{14}^e is given below. The factor $1/\mu$ will come in the expression if we are solving Poisson's equation.

$$\iint \left(\frac{dN_1}{dx}\frac{dN_4}{dx} + \frac{dN_1}{dy}\frac{dN_4}{dy}\right) dxdy = \frac{1}{3\Delta}(P_1P_2 + Q_1Q_2)$$

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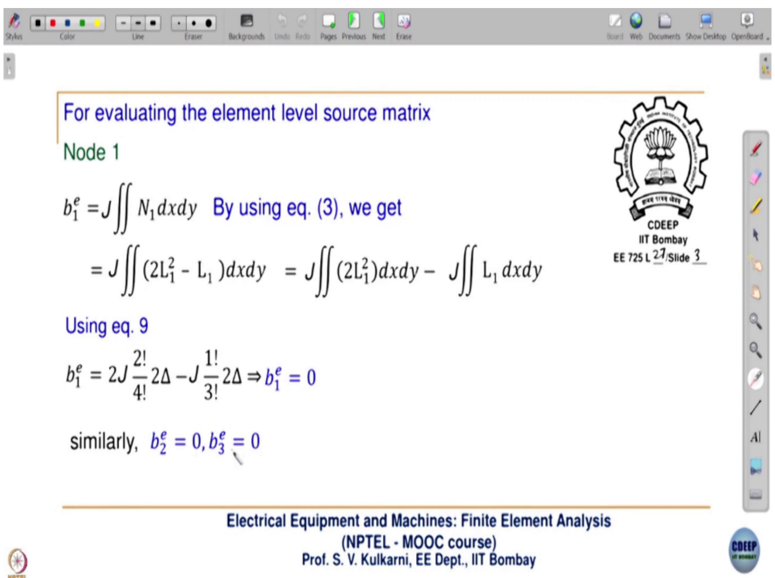

Now we will see the entries of the right hand side matrix which is contributed by the source J. If you remember in case of linear elements, we had $\frac{J\Delta}{3}$ as the contribution from the source (J). Also remember that the J is distributed throughout the element. Now we will see how do we apportion J to the various nodes in the discretized domain.

In FE analysis, we are going from a continuous domain to a discretized domain. The entry for node 1 in an element level source matrix can be determined by using the following expression.

$$b_1^e = J\iint N_1 dxdy$$

Now, we substitute $N_1 = 2L_1^2 - L_1$ and then after simplification we get

$$b_1^e = J\iint (2L_1^2 - L_1)dxdy = J\iint (2L_1^2)dxdy - J\iint L_1\, dxdy$$

Again, use the following expression to solve the above integrals.

$$\iint (L_1)^l (L_2)^m (L_3)^n\, dxdy = \frac{l!\,m!\,n!}{(l+m+n+2)!}2\Delta$$

Here, for $\iint (2L_1^2)dxdy$, $l = 2$, $m = 0$, and $n = 0$. So the solution of this integral is $\frac{2!}{4!}2\Delta$. Using the same procedure, the value of the other integral can be obtained and the expression of b_1^e reduces as given below.

$$b_1^e = 2J\frac{2!}{4!}2\Delta - J\frac{1!}{3!}2\Delta \Rightarrow b_1^e = 0$$

Similarly, b_2^e and b_3^e are 0.

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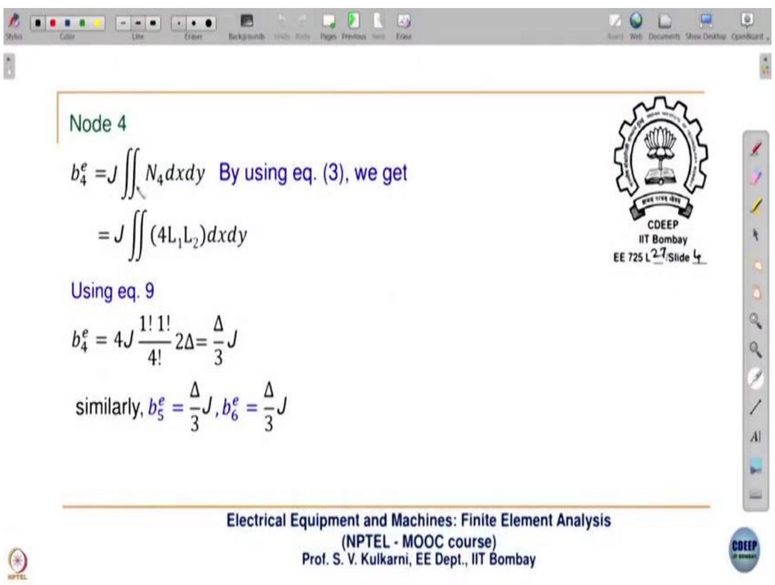

That means, in case of a quadratic element, J does not get apportioned to the main nodes 1, 2, and 3. But they get apportioned to nodes 4, 5, and 6 which are the midpoints of the edges. Now let us see the derivation of b_4^e. The expression for this entry after substituting $N_4 = 4L_1L_2$ is given below.

$$b_4^e = J \iint N_4 dxdy = J \iint (4\mathrm{L}_1\mathrm{L}_2) dxdy$$

The value of the $\iint (4\text{L1L2}) dxdy$ is $\frac{1!1!}{(1+1+0+2)!} 2\Delta = \frac{1!1!}{4!} 2\Delta$ because $l = 1$, $m = 1$, and $n = 0$. With this, the expression of b_4^e entry is as given below.

$$b_4^e = 4J \frac{1!\,1!}{4!} 2\Delta = \frac{\Delta}{3} J$$

Similarly, b_5^e and b_6^e can be calculated as $\mathrm{J}\Delta/3$.

That means for a quadratic element with nodes 1 to 6, J reduces to $\mathrm{J}\Delta/3$ after discretization and it gets apportioned to 4, 5, 6 nodes and not to nodes 1, 2, 3.

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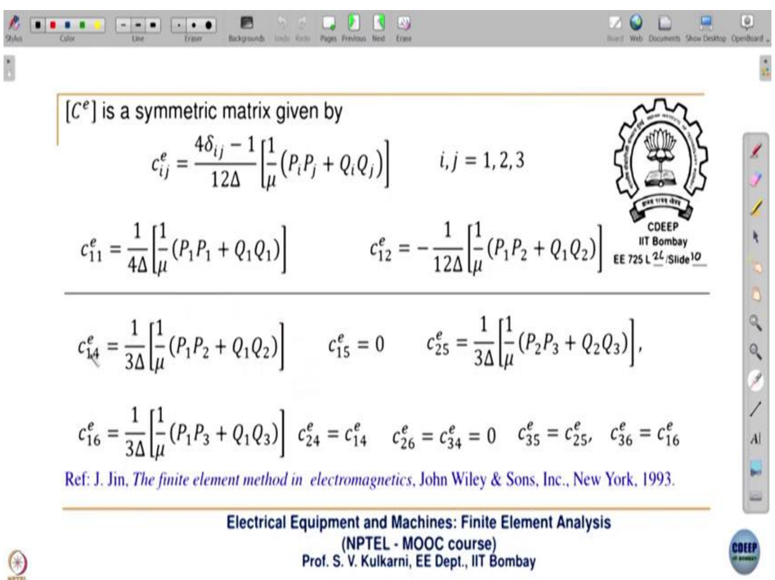

Thus we have seen the entire FE procedure for quadratic elements and we have derived the entire element level b matrix.

Similarly, we derived one off-diagonal entry (c_{14}^e) and one diagonal entry (c_{11}^e) of the element coefficient matrix. By doing this, we would have derived all the entire element coefficient matrices.

Similarly, the other entries of the element coefficient matrix can be derived. Then we also derived b_e matrix which represents the source contribution. After forming element level matrices we have to form a global coefficient matrix $[C]$ and $[B]$. Then the rest of the procedure is same as for linear elements. Then this completes the FE formulation for quadratic elements.

Now we will see the next topic that is the solution of diffusion equation for time harmonic problems which are very common. In fact, most of the electromagnetic devices are AC devices. So the excitation to these devices is time varying. Earlier also, for the electrostatic analysis voltage at every point in a transformer or any other high voltage equipment is time varying. But in that analysis we considered the voltage at its peak value to calculate the maximum value of electrostatic field between two electrodes.

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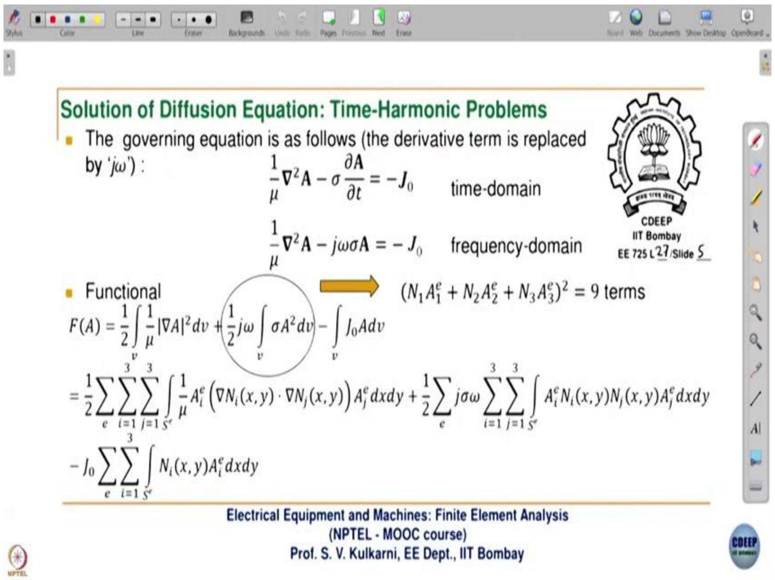

When we are interested to calculate eddy currents we have to consider induction effects. For this, we have to consider time in transient analysis or frequency in time harmonic case. The later one is applicable when all the quantities are sinusoidal. So, we are going to see time harmonic analysis in which all quantities are sinusoidal.

Effectively, we are assuming all the materials in the given problem domain are linear. For example, if we are applying voltage, then B will be sinusoidal and if we are assuming the magnetic material is linear, then H and I will be also sinusoidal and there are no harmonics. Even if there are harmonics in excitation and if the material is linear then for each harmonic we could use this formulation and the total loss is calculated by combining all the effects.

Now, let us get into the following diffusion equation that we have already seen in the previous lecture.

$$\frac{1}{\mu}\nabla^2\mathbf{A} - \sigma\frac{\partial\mathbf{A}}{\partial t} = -\boldsymbol{J}_0 \qquad \text{time-domain}$$

$$\frac{1}{\mu}\nabla^2\mathbf{A} - j\omega\sigma\mathbf{A} = -\boldsymbol{J}_0 \qquad \text{frequency-domain}$$

In the above equations, $\frac{\partial A}{\partial t}$ and $j\omega A$ terms represent the induced effect and their units are same as current density.

Because $\frac{\partial A}{\partial t}$ is induced electric field intensity (E) and σE is eddy current density. So unit wise or variable wise it is matching in the above two equations. This we have seen earlier in the basics of electromagnetics. In frequency domain, $\frac{\partial A}{\partial t}$ gets replaced by $j\omega A$. The functional for diffusion equation is given below.

$$F(A) = \frac{1}{2}\int_v \frac{1}{\mu}|\nabla A|^2 dv + \frac{1}{2}j\omega\int_v \sigma A^2 dv - \int_v J_0 A dv$$

While deriving functional first we started with Laplace's equation and then we derived for Poisson's equation. Then we also wrote the functional for the diffusion equation. Also the logic to write this functional is simple. Earlier also it was mentioned that we can take the terms (other than Laplace's term) to the right hand side and the sign of the term comes in the functional expression. For example, if we take the diffusion term to the right hand side then its sign changes to plus and in the functional we get A^2.

J_0 in the governing equation is alone. So it gets multiplied with A in the functional expression. Like this, by intuition we can write the functional for a given PDE. But we can derive the functional for the diffusion equation using the procedure that we have seen earlier. A half will appear with the two terms because there is a square term. Again that is due to the rule that was mentioned earlier.

Whenever there is a square term in the functional, half will get multiplied and if there is only A term then there is no half there. So, the above equation is the expression for the functional of the diffusion equation. Now, this equation is for the whole domain. So, if we discretize the problem domain using the FE procedure then the expression of functional reduces to the following equation.

$$F(A) = \frac{1}{2}\int_v \frac{1}{\mu}|\nabla A|^2 dv + \frac{1}{2} j\omega \int_v \sigma A^2 dv - \int_v J_0 A dv$$

$$= \frac{1}{2}\sum_e \sum_{i=1}^{3}\sum_{j=1}^{3}\int_{S^e} \frac{1}{\mu} A_i^e \left(\nabla N_i(x,y)\cdot\nabla N_j(x,y)\right) A_j^e dxdy + \frac{1}{2}\sum_e j\sigma\omega \sum_{i=1}^{3}\sum_{j=1}^{3}\int_{S^e} A_i^e N_i(x,y) N_j(x,y) A_j^e dxdy$$

$$- J_0 \sum_e \sum_{i=1}^{3}\int_{S^e} N_i(x,y) A_i^e dxdy$$

The first summation in all the three terms is representing elements, because we are discretizing the whole domain. So they are summations over all elements and the ∇^2 term in the governing equation results into nine terms for each element.

The corresponding nine entries of element coefficient matrix are represented with $\int_{Se} \frac{1}{\mu} A_i^e \left(\nabla N_i(x,y)\cdot\nabla N_j(x,y)\right) A_j^e dxdy$. The two summations in the first term of the above equation will give us entries of the element coefficient matrix multiplied by A_i and A_j. When we minimize the functional, one of these A_is or A_js will be cancelled.

Then after minimization, one entry of element coefficient matrix and the corresponding potential will remain. So the final set of matrix equation is $CA = B$. The first term in the above equation is going to give the element coefficient matrix. In the summation of the first term, if $i = 1$ and $j = 1$ then that will give us C_{11} and it will be multiplied by A_1^2. When we minimize by differentiating with respect to A_1 then one of the $'A_1's$ will be cancelled. So this will result in $C_{11}A_1$ in the final linear system of equations.

We have seen this number of times. Then the third term in the above equation is also identical and there is no change. So, this term will result into $\frac{J\Delta}{3}$ and it will get apportioned to nodes 1, 2, and 3 for linear elements. Remember that in this formulation we are using linear elements not quadratic elements.

The new or extra term here corresponds to $\frac{1}{2} j\omega \int_v \sigma A^2 dv$ term because we have seen the first and last terms in Poisson's equation also.

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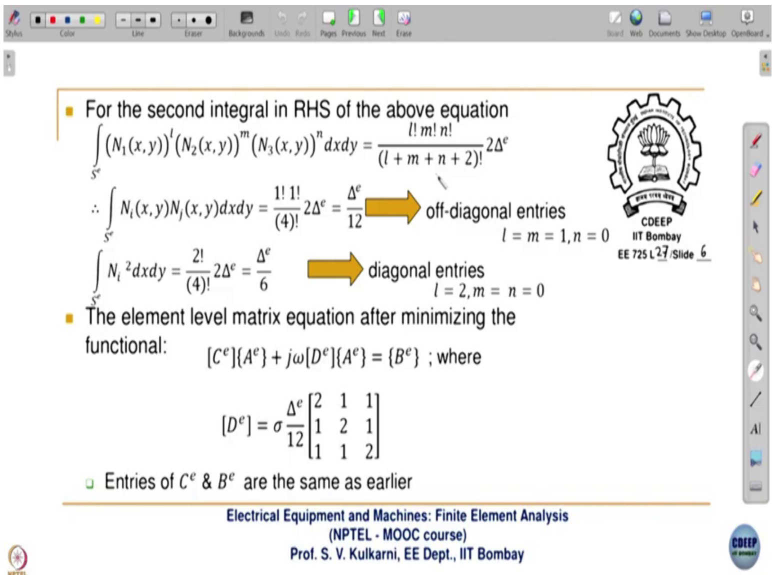

In the diffusion term, if we substitute A in terms of $\sum_{i=1}^{3} N_i A_i$ then we will get the following term.

$$\frac{1}{2} j\omega \int_v \sigma A^2 dv = \frac{1}{2} \sum_e j\sigma\omega \sum_{i=1}^{3} \sum_{j=1}^{3} \int_{Se} A_i^e N_i(x,y) N_j(x,y) A_j^e dxdy$$

As explained here, $(A^e)^2 = (N_1 A_1^e + N_2 A_2^e + N_3 A_3^e)^2$ will give nine terms. Those nine terms are represented with two summations as given in the above equation.

Again using the following formula in terms of N_1, N_2, and N_3 the above integral is solved.

$$\int_{S^e} (N_1(x,y))^l (N_2(x,y))^m (N_3(x,y))^n dxdy = \frac{l!\,m!\,n!}{(l+m+n+2)!} 2\Delta^e$$

Here we are going back to the same old formula which is in terms of shape function, because in this formulation we are using linear elements. Since the first and third terms of the functional expression are same as earlier, we are concentrating on the second term which is given by the following integral.

$$\int_{S^e} N_i(x,y)N_j(x,y)dxdy$$

That means in the above formula, $l = 1$, $m = 1$, and $n = 0$ and the solution of the above integral is as given below.

$$\int_{S^e} N_i(x,y)N_j(x,y)dxdy = \frac{1!\,1!}{(4)!}2\Delta^e = \frac{\Delta^e}{12}$$

So, the off-diagonal entries for the diffusion term will be $\Delta^e/12$. Diagonal entries will come when $i = j$ and the integral that corresponds to the diffusion term is given below.

$$\int_{S^e} N_i{}^2 dxdy = \frac{2!}{(4)!}2\Delta^e = \frac{\Delta^e}{6}$$

Because, N_i^2 means $l = 2$, $m = n = 0$ and the integral that corresponds to the diffusion term will result in $\Delta^e/6$ as given in the above equation. So, $\Delta^e/6$ will be the diagonal entries. The element level matrix equation after minimizing the functional will be given by the following equation.

$$[C^e]\{A^e\} + j\omega[D^e]\{A^e\} = \{B^e\}$$

In the above equation, $[C^e]\{A^e\}$ and $\{B^e\}$ are same as in Poisson's equation. Here we have to note that A^e and B^e are column vectors and they are enclosed in curly brackets.

The $[D^e]$ is given by the following matrix

$$[D^e] = \sigma\frac{\Delta^e}{12}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

Using $\int_{Se} N_i(x,y)N_j(x,y)dxdy$, we have got the diagonal and off-diagonal entries. Only one A appears in the final matrix equation because after minimization one of the A_is will go. So only one A_i will remain after minimization.

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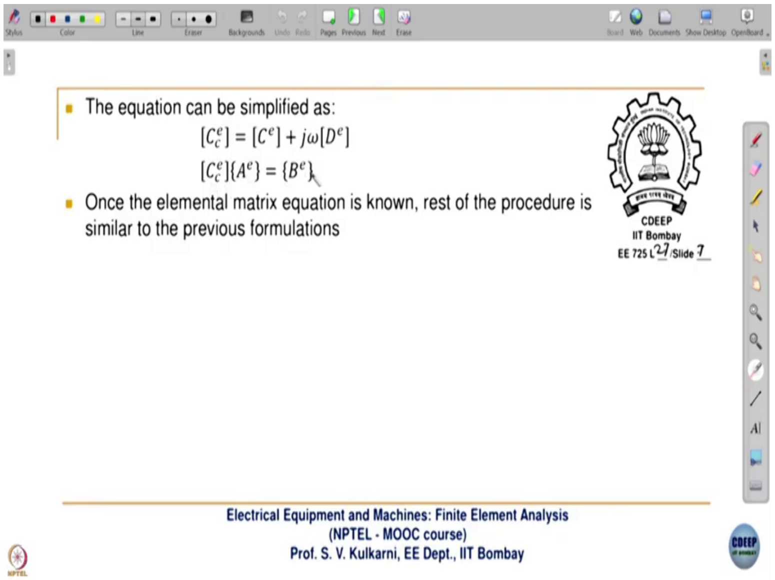

The final matrix equation is $[C_c^e]\{A^e\} = \{B^e\}$. As compared to Poisson's equation here, the coefficient matrix is complex. We have to remember that here, A is a phasor because we are formulating in frequency domain. In $[B^e]$, each of the entries is some function of J (current density) which is also a phasor quantity. So the whole matrix equation is in phasor form. The entries of matrices $[C^e]$ $[B^e]$ are same as earlier formulations.

Now, to bring the final matrix equation in a form like $CA = B$, we can consider that $[C_c^e] = [C^e] + j\omega[D^e]$. c in the subscript of $[C_c^e]$ stands for complex matrix. Then we finally get the following matrix equation.

$$[C_c^e]\{A^e\} = \{B^e\}$$

In the above equation, the unknown variables are magnetic vector potentials. This matrix equation is at the element level. Now we combine all the element level matrices by following the usual procedure of formation of global matrices and then we would get the solution (nodal magnetic vector potentials $\{A\}$) of the whole domain.

So, $[C_c]\{A\} = \{B\}$ is the global matrix equation. From this equation, we can calculate $\{A\}$ as given below

$$\{A\} = [C_c]^{-1}\{B\}$$

Before that we have to apply the boundary conditions. Remember that the B matrix was only coming from current density (J). Till now the source is being represented in B matrix. But for a problem domain in FEM there could be a boundary. So, we have to impose appropriate boundary conditions as discussed in the previous lectures. Finally, we can solve this global equation in terms of $[C_c]\{A\} = \{B\}$.

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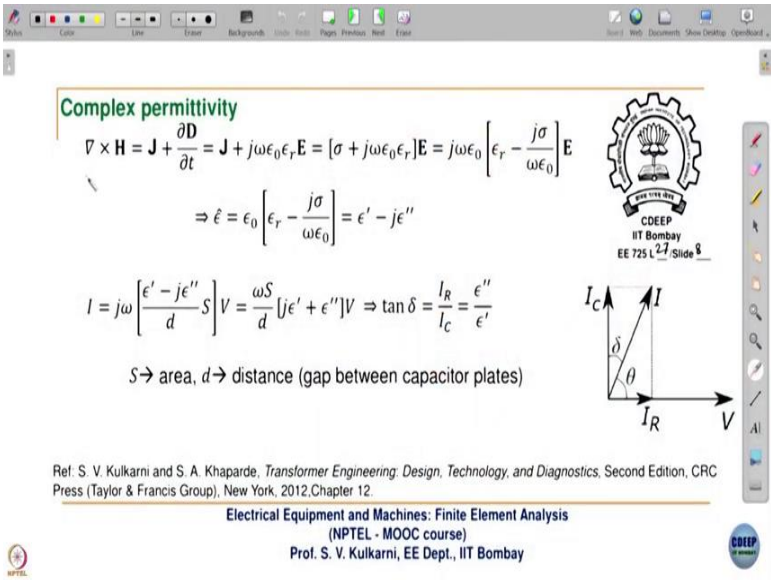


Now, we will quickly discuss two improtent points which we did not cover in basics of electromagnetics and these are very important for time harmonic analysis. They are complex permittivity and complex permeability.

The derivation given in the above slide are very simple and straightforward and it starts from Maxwell's equation. We know that $\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$. In frequency domain, if we replace $\frac{\partial}{\partial t}$ by $j\omega$ J by $\sigma\mathbf{E}$, and $\mathbf{D}$ by $\epsilon_0\epsilon_r\mathbf{E}$ we get $\nabla \times \mathbf{H} = \sigma\mathbf{E} + \epsilon_0\epsilon_r\mathbf{E}$. Then take $j\omega\mathbf{E}$ as common and get the right hand side of the curl equation as $j\omega\epsilon_0\left[\epsilon_r - \frac{j\sigma}{\omega\epsilon_0}\right]\mathbf{E}$. So we get the curl equation as given below.

$$\nabla \times \mathbf{H} = j\omega\epsilon_0\left[\epsilon_r - \frac{j\sigma}{\omega\epsilon_0}\right]\mathbf{E}$$

Now we will call this $\left[\epsilon_r - \frac{j\sigma}{\omega\epsilon_0}\right]$ as complex permittivity as given below.

$$\hat{\epsilon} = \epsilon_0 \left[\epsilon_r - \frac{j\sigma}{\omega \epsilon_0} \right] = \epsilon' - j\epsilon''$$

The complex permittivity has real and imaginary components. The imaginary component is representing losses because it has a σ term which represents finite conductivity of the insulation. For example, a practical capacitor will be represented by a parallel connection of R and C elements and an ideal capacitor will be represented with only pure C. So for an ideal capacitor we have only ϵ'. If it is a practical capacitor or a lossy capacitor, R also will be there and that will be represented by ϵ''.

Now, if we represent a lossy capacitor with the complex permittivity $\epsilon' - j\epsilon''$, then we can formulate current as

$$I = j\omega \left[\frac{\epsilon' - j\epsilon''}{d} S \right] V$$

Because, the current passing through a capacitor is $I = j\omega CV$, where $C = \frac{\epsilon S}{d}$. Here, $\epsilon = \epsilon' - j\epsilon''$ is a complex number, S is the cross-sectional area, and d is the distance between the electrodes. Then the expression of current is simplified as given below.

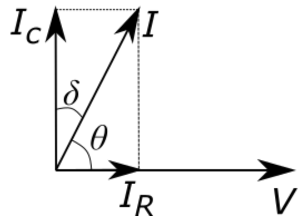
$$I = \frac{\omega S}{d} [j\epsilon' + \epsilon'']V$$

The above equation is the final expression of current. Now, one of the common diagnostic terms for a capacitor is $\tan \delta$ which is defined as the ratio of the resistive component to the capacitive component of the current and it is mathematically represented as

$$\tan \delta = \frac{I_R}{I_C} = \frac{\epsilon''}{\epsilon'}$$

The above expression of $\tan \delta$ is obtained by substituting the resistive component $\left(\frac{\omega S}{d} \epsilon V \right)$ and reactive component $\frac{\omega S}{d} \epsilon'' V$ of the current. The difference between circuit representation and field representation is that the lossy component is represented by ϵ'' (imaginary part) and the non-lossy component is represented by ϵ' (real part).

The current passing through a lossy capacitor is represented by the following phasor diagram.



In the above figure, I is the resultant of resistive (I_R) and reactive (I_c) components of the current and then we have $\delta = 90 - \theta$. The $\tan\delta$ is the ratio of the opposite side (I_R) and the adjacent side (I_c). The complex permittivity will be useful to find out losses in a dielectric material using FE formulation. So, complex permittivity can be used in time harmonic analysis of dielectric matrial with finite conductivity.

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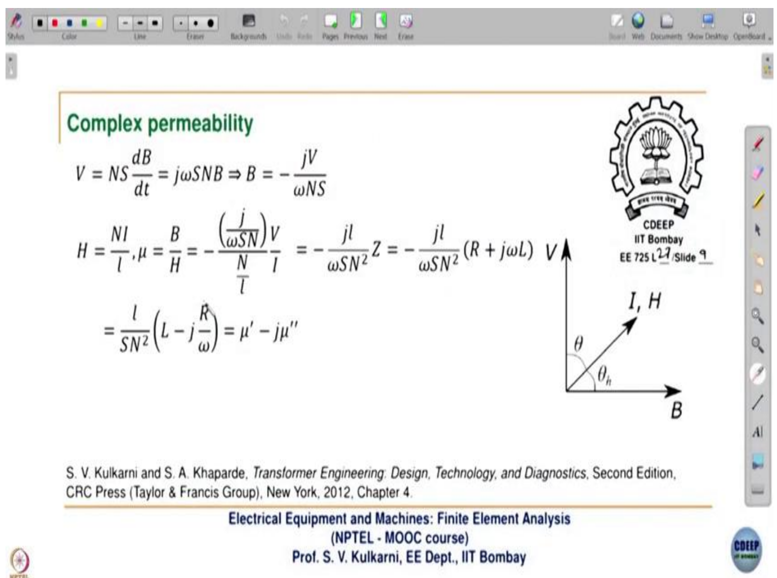


The next concept is complex permeability. This will be derived by starting with $V = N\frac{d\psi}{dt}$. We know that $\psi = BS$ and induced voltage can be written in the time-harmonic analysis as

$$V = NS\frac{dB}{dt} = j\omega SNB$$

We can rearrange the above equation to determine the expression of B as

$$B = -\frac{jV}{\omega NS}$$

Then we know that $H = \frac{NI}{l}$ and $\mu = \frac{B}{H}$. Now, determine the expression of permeability by using the above expressions of B and H as given below.

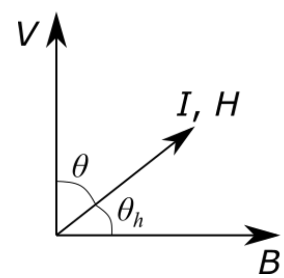
$$\mu = \frac{B}{H} = -\frac{\left(\frac{j}{\omega SN}\right)}{\frac{N}{l}} \frac{V}{I}$$

Then, $\frac{V}{I} = Z$. By doing this, we are trying to find out the equivalent circuit for a lossy magnetic material. Using complex permittivity, we represented an equivalent circuit for a lossy capacitor. Going further, Z is replaced by $R + j\omega L$. Further, we simplify the permeability as given below.

$$\mu = -\frac{jl}{\omega SN^2} Z = -\frac{jl}{\omega SN^2}(R + j\omega L) = \frac{l}{SN^2}\left(L - j\frac{R}{\omega}\right) = \mu' - j\mu''$$

Then we get a real term $(\mu')$and an imaginary term (μ'') for complex permeability. Here, we have to notice that $V = +N\frac{d\psi}{dt}$ and it is from a circuit viewpoint and this we have discussed in basics.

$V = NS\frac{dB}{dt}$ represents that V leads ψ or B by 90^{o} as shown in the following phasor diagram.



In the above figure, phasor V will lead B by 90^{o}. Since we are talking about a lossy case I will lag V by some angle θ. The moment we have μ'' then the material is representing a lossy magnetic material. That means, in the corresponding circuit, I will lag V by some angle θ. Here, we will have $VI\cos\theta$ as the corresponding loss.

If the material is purely inductive and lossless then this angle θ will be 90^o and if it is a perfect resistive material then the angle θ will be zero. Now this loss corresponds to hysteresis loss in the material and in the case of hysteresis phenomena, H leads B by some hysteresis angle θ_h.

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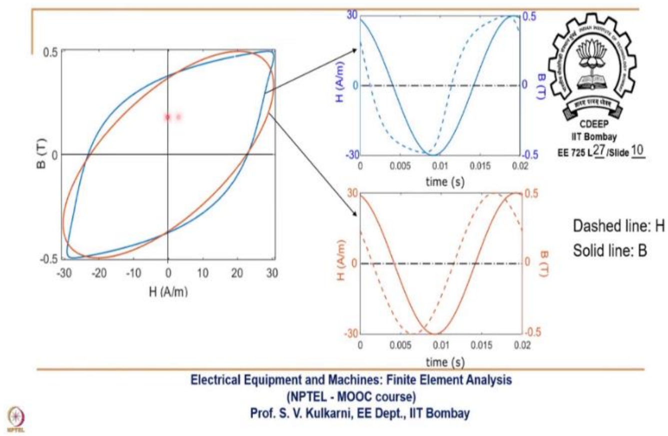
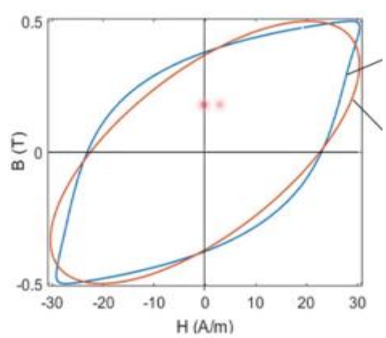
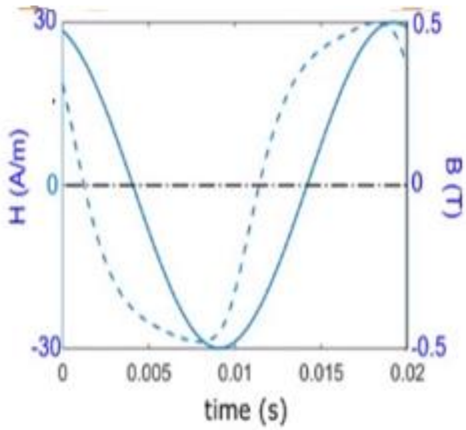


To further understand elliptic or complex permeability, let us now study the original hysteresis curve which is in blue color in the following figure.

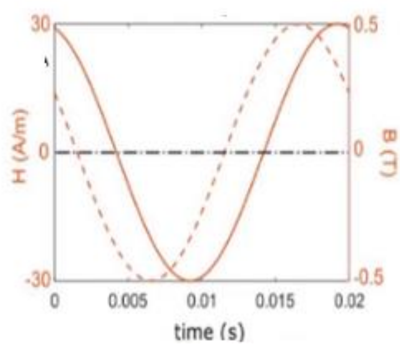


As we know, for a ferromagnetic material there is a hysteresis angle between B and H. So, H goes to zero first and then B goes to zero when the curve is traversed in anticlockwise direction. Now, we plot B and H with time separately as in shown in the following figure.



If we force B to be sinusoidal which can be done during the experimental measurements then H has to be non-sinusoidal as per the blue hysteresis curve in the previous diagram.

Here, H is having a fundamental component as well as some harmonics. In the above figure, H (dotted waveform) also represents the corresponding harmonics. These harmonics are not available for time harmonic formulation. In time harmonic formulation, all the field quantities or field variables should be sinusoidal at one frequency. So in this analysis, we can neglect the harmonics in the H field and we only consider its fundamental component. If we do that, then we get H waveform as sinusoidal as shown in the following figure.



In the above figure, both B and H fields are sinusoidal and now we can use time harmonic formulation and the corresponding BH loop will be elliptical as given by orange loop in the previous figure and that loop is having both B and H as sinusoidal with fundamental components only. Although we have neglected harmonics in H, the hysteresis angle (θ_h) is preserved.

Then, with this simplification and the assumption that harmonics in H being neglected, we get the BH curve as given by orange color and the corresponding permeability is called as elliptic or complex permeability. The time harmonic formulation in terms of complex or elliptical permeability which was discussed on the previous slide can be used to calculate losses in the frequency domain FEM analysis. We will stop here and continue our discussion on diffusion problems in the next lecture.

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