Electrical Equipment and Machines Finite Element Analysis Professor Shrikrishna V Kulkarni Indian Institute of Technology Bombay Lecture No 25 Insulation Design Using FE Analysis

Welcome to lecture 25. In this lecture, we will understand how to apply finite element method to analyse and design insulation of high voltage equipment in an effective way. We will be mostly see a power transformer with high voltages imposed on its primary and secondary sides. Before going into finite element method, let us see some basics of high voltage insulation design.

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Types of breakdown mechanisms are jump or bulk oil breakdown, creepage breakdown, corona and partial discharge. We will briefly describe each of these types this in further slides. Then we will see two withstand theories. Finite element method will give you the stress distributions as well as withstand strength inside the problem domain.

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Now, let us understand the jump or bulk-oil breakdown and creepage breakdown. As you can see, the figure on the left hand side of the above slide which is a standard high voltage configuration. It contains an HV lead in the vicinity of a ground plane. If we go on increasing the voltage applied to the high voltage lead then at some point the electric field intensity exceeds a certain value and there could be a direct flash over from the high voltage lead to ground. So that is called as bulk-oil breakdown or jump breakdown.

In creepage breakdown, for some reason, if the high voltage lead has to be supported from this ground plane using an insulating piece as shown in the figure on the right hand side then what we observe in the figure is the equipotential contours are cutting the surface of the insulating piece and there is a tangential electric field intensity all along the insulating surface and the corresponding stress is known as creepage stress.

The thing to note here is that the creepage strength is much lower than the bulk oil strength. That is why it is important to find out the creepage stress and compare it with corresponding creepage strength. In the further slides, we will see how do we calculate that stress and strength. (Refer Slide Time 03:33)



The third type is a partial breakdown mechanism which is called as corona. Now, corona and partial discharges are partial breakdown mechanisms because some breakdown happens only locally. This corona discharge is a well-known phenomenon in high voltage transmission lines, particularly during monsoon season when moisture is there, the air molecules around the high voltage conductor get ionized and there will be a corresponding energy loss. The energy loss is associated with glow discharge as well as sound that you can hear. So, this corona discharge needs to be mitigated and there are established ways to mitigate this phenomenon.

Coming to partial discharge, suppose you have a bulk solid insulation with a void made of either air or oil as shown in the following figure. Then, since the dielectric constant of the void would be less as compared to the dielectric constant of the solid insulation, the electric field intensity is inversely proportional to the dielectric constant (the permittivity). Then, higher stress is going to come across the void and there could be a local breakdown and discharge.



Now, one may wonder that what is great about this? Let, there be a local discharge and complete breakdown is not happening. But if you allow this kind of local discharges for a longer period of time then slowly the insulation is going to get deteriorated and eventually there

could be a complete flashover on one day. That is why again partial discharge activities inside an insulation should be minimized. These corona and partial discharge phenomena can be also analysed by using finite element analysis.

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Now, let us understand how do we design a bulk oil duct. That means instead of having a complete single oil duct between low voltage and high voltage windings of a transformer we generally split the entire oil duct into small oil ducts. The individual small oil ducts and solid insulating barriers which divide the whole oil duct into smaller ducts are shown in the figure on the left hand side in the above slide. This is done because subdivision of oil ducts increases the kV/mm withstand stress. What does it mean?

Suppose if you have a 10 mm oil duct and its withstand stress is 10 kV/mm. If the gap is 20 mm then the withstand stress will not be 20 kV and it maybe 19 kV which is a lesser value. That is the meaning of an increase in kV/mm withstand stress for lower duct gaps. The barriers would arrest the propagation of discharge streamers between two electrodes. One has to remember that barriers should be as thin as mechanically possible, otherwise there will be increased stress in the oil. But one may think that even though the insulation strength or withstand stress for solid insulation is much greater than that of oil then why increasing the thickness of these barriers does not increase the overall withstand of the insulation.

But what is going to happen is, the smaller oil ducts in that case will get stressed to higher stress levels and that could be a problem. Now, to understand this let us analyse one simple case of an oil solid composite system as shown in the following figure.



Here, the thicknesses of these 2 insulations are 10 mm and dielectric constants (or relative permittivity) of oil and solid are 2.2 and 4.4 respectively. We will get $E_1d_1 + E_2d_2 = 100$ kV because the total voltage difference is 100 kV. Since E is inversely proportional to permittivity we can write $E_1 = 2E_2$. Then if you substitute $E_1 = 2E_2$ in the above equation, we will get $E_2 = 3.3$ kV/mm and $E_1 = 2E_2 = 6.6$ kV/mm.

Now, the equivalent oil gap for this configuration can be determined as given below.

Equivalent oil gap:

$$\frac{100 \text{ kV}}{10 + 10 \left(\frac{2.2}{4.4}\right)} = \frac{100 \text{ kV}}{15} = 6.66 \text{ kV/mm}$$

So, you get the equivalent stress as $\frac{100 \, kV}{15 \, mm} = 6.6 \, kV/mm$ as obtained earlier. So, the thing to notice here is that the equivalent oil gap is 15 mm. That means although you have solid barriers and if we have to analyse this in terms of one equivalent oil gap then the thickness of gap will be 15 mm.

If you increase the number of barriers or their thickness, then the equivalent oil gap is going to reduce and it will increase the stress on oil ducts. Since oil has lower dielectric withstand strength that could lead to some problems and that is the reason why the barriers should be as thin as possible from the mechanical strength point of view. Otherwise, there will be an increased stress in oil.

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Now, let us understand how to design an individual duct. For that, we will use cumulative stress and withstand theory. It is well known that smaller the gap or length of any insulation then its strength is higher. That is clear from the following withstand curves which are in blue color.



In the figure, you can see that if the gap is small then the withstand is higher, and as the gap length increases, the withstand becomes lower. As mentioned on the previous slide, it is always better to have smaller ducts for higher strength. Remember when we say stress and strength, their unit is kV/mm.

So, having understood how the withstand stress behaves as a function of gap length, let us understand how to calculate cumulative stress. Consider a gap as shown in the following figure.

$$\underbrace{\begin{smallmatrix} 0 & 1 & 2 & 3 \\ E_{max}, V_0 & E_1, V_1 & E_2, V_2 & E_3, V_3 & E_{min}, V_n \end{smallmatrix}}_{E_{min}, V_n}$$

Let us assume that maximum electric field intensity is at the left end and on the right end the electric field intensity is E_{min} for the duct. Let us divide the gap into equally spaced segments

with nodes at 1, 2, 3 ... n. Let the voltages at these nodes be V_0 , V_1 , V_2 , V_3 , ..., V_n and the distance between adjacent nodes is 1 mm. We already know that E_0 and E_n is equal to E_{max} and E_{min} respectively.

Now $E_1 = \frac{V_0 - V_1}{1 mm}$ and its units is kV/mm. Here, we have to note voltages at all the nodes are in kV. Similarly to calculate E_2 instead of using 1 mm, we will take 2 mm because we are calculating cumulative stress. Cumulative stress means we have to increase the gap length and $E_2 = \frac{V_0 - V_2}{2 mm}$. Similarly $E_3 = \frac{V_0 - V_3}{3 mm}$. So, all these cumulative stress values can be plotted as given in the following graph, and cumulative withstand stress is also shown in the figure.



At every point we can calculate the margin between withstand and the stress, and this margin should be good enough. In case of insulation design, the margin has to be much higher as compared to other design aspects because insulation breakdown is statistical phenomena and repeatability is not as good as compared to other engineering field phenomena. So, that is why you need to have a good amount of margin between strength and stress. The value of margin at various places in the insulation should be more or less equal that means the insulation should be optimally utilized everywhere.

You should not have a case wherein at some point of the insulation the margin is very less and at some other places you have ample margin. So, that is not a good insulation design. Of course, this is a challenging design aspect and it may not be possible to equalize stresses and margins everywhere in the insulation, but our efforts should be in that direction. So, this is how you can design a bulk oil insulation by dividing into sub ducts and then designing individual duct by using cumulative stress and withstand theory.

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Now, let us understand how we can use the finite element method to analyse creepage phenomena and increase the withstand. Consider the HV system as shown in the following figure.



The HV electrode and the ground plane are also indicated in the figure. Then the equipotential contours for this problem are also shown in the figure. The overall withstand strength of the insulation gap can be increased by placing some insulating piece which is in inverted L shape.

With this, although we have increased the overall breakdown strength of the gap, but along the surface of the insulating piece, there will be a significant amount of tangential electric field intensity, and this leads to corresponding creepage stress. Now in the figure, we can see that the equipotential contours are crossing the insulation structure and there will be tangential electric field intensity and the corresponding creepage stress. As mentioned earlier, since creepage strength is smaller than the bulk insulation strength that is why you need to watch creepage stress closely.

The creepage withstand can be increased by changing the shape of the insulating piece as shown in the following figure.



In the above figure, the insulating piece is almost along the equipotential contours. Since the equipotential contours are along the surface, the tangential electric field is reduced considerably, and the corresponding stress is reduced. So this way we can increase the quality and reliability of the insulation.

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How do we assess the creepage stress whether it is below certain allowed value? For this again we will use the same theory of cumulative stress and withstand stress. Now if you consider the vertical segment of the insulating piece, the tangential E_{max} will be somewhere at the top of the segment and E_{min} of the tangential component will be somewhere at the bottom. So, then

starting from the position of E_{max} to the position of E_{min} , you can calculate the cumulative stress and the corresponding cumulative withstand which is given in the following plot.



The equation for cumulative withstand is available in the literature. I have given some references at the bottom of this slide, so you can refer them. Similarly on this horizontal segment of the insulating piece, E_{max} will be somewhere at the right most point and E_{min} will be somewhere at left end point. So, again starting the position of E_{max} to E_{min} , you can calculate and plot cumulative stress and you already have the cumulative withstand so again you can compare margin between withstand and stress at various points to ensure that you have a certain amount of minimum margin available at any portion.

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Now, let us go to another case study wherein we have to design lead to ground insulation. The system shown in the following figure contains high voltage lead and ground and the lead is covered by some insulation.



Consider the insulation is paper. Now we will calculate the stress by using finite element method and maximum stress will be at a point on the lead which is closest to the ground plane in oil. The maximum stress will not be there at the point in the solid because electric field intensity is inversely proportional to dielectric constant. So, the electric field will be higher in the oil at the interface between oil and paper and in the oil, the electric field intensity will be higher as compared to the corresponding point in the paper insulation.

So, the finite element method will give you the maximum stress at the point at the interface between oil and paper which will be typically at the minimum distance between the insulated lead and the ground plane. Withstand stress can be calculated by using stressed oil volume which is defined by the following expression which is generally calculated by first computing the area between E_{max} and 90% of E_{max} contour which is indicated in the following figure.



On the contour shown in the figure the stress values will be 90% of E_{max} , and this shaded area will be stressed oil area that contains the points where the stress is in between 100% stress and 90% stress values.

We have to multiply this area by the lead length which is perpendicular to the plane shown in the above figures. So, that lead length multiplied by the stressed area will give stressed oil volume and strength will be a function of stressed oil volume, and you can find that equation in the references given in the slide. The value of E_{max} can be calculated by FEM and it should

be considerably less than the strength calculated by the function of stressed oil volume. So, this is how you can design a high voltage lead to ground gap.

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Let us understand how do we extend an FEM code for calculating stressed oil volume and corresponding strength. We know the electric field intensity is calculated by using $\mathbf{E} = -\nabla V$. V in electrostatic FEM formulation is given by the following expression.

$$V^e = N_1 V_1^e + N_2 V_2^e + N_3 V_3^e$$

 N_1 , N_2 , and N_3 are the shape functions of 3 nodes of the element under consideration. Now substituting the expression of V in $\mathbf{E} = -\nabla V$, you will get the following expression.

$$\mathbf{E}^e = -V_1^e \mathbf{\nabla} N_1 - V_2^e \mathbf{\nabla} N_2 - V_3^e \mathbf{\nabla} N_3$$

Remember that V_1 , V_2 , and V_3 are not functions of x and y, that is why they are taken out of the ∇ operator. ∇ operator acts only on N_1 , N_2 , and N_3 which are functions of x and y. The expression for N_1 is given below.

$$N_1 = \frac{1}{2\Delta} \{ (x_2 y_3 - x_3 y_2) + (y_2 - y_3) x + (x_3 - x_2) y \}$$

Now, ∇N_i is given by the following expression

$$\boldsymbol{\nabla}N_i = \frac{P_i \hat{\mathbf{a}}_x + Q_i \hat{\mathbf{a}}_y}{2\Delta}$$

Here, $P_1 = y_2 - y_3$ and $Q_1 = x_3 - x_2$. We have seen these expressions earlier. By substituting all these expressions in $\mathbf{E} = -\nabla V$, we get the expression for E as

$$\mathbf{E}^{e} = -\frac{1}{2\Delta} \{ (V_{1}^{e}P_{1} + V_{2}^{e}P_{2} + V_{3}^{e}P_{3})\hat{\mathbf{a}}_{x} + (V_{1}^{e}Q_{1} + V_{2}^{e}Q_{2} + V_{3}^{e}Q_{3})\hat{\mathbf{a}}_{y} \}$$

Then the magnitude of E will be simply $\sqrt{E_x^2 + E_y^2}$ for the element under consideration.

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Now let us see how to calculate electric field and stressed oil volume using FEM solution (nodal V values). First we will set up a for loop that goes from 1 to the total number of elements using the command "for element=1:n_elements" which we have seen in the earlier codes. By using the command "nodes=t(2:4,element);" we will get the global node numbers of the element under consideration into the nodes matrix. This matrix is a column matrix of having 3 entries. The three global node numbers get assigned to 3 entries of the nodes matrix.

Then the two commands "Xc=p(1,nodes'); Yc=p(2,nodes');" are having 3 entries and the 3 entries will be *x* and y coordinates of the corresponding 3 global nodes. Here, we are initializing P and Q matrices with zeros and then P_1 , P_2 , P_3 , Q_1 , Q_2 , Q_3 entries are calculated using the formulae mentioned in the previous lectures. The area of the element will be calculated by using the command "delta(element)= 0.5*abs((P(2)*Q(3))-(P(3)*Q(2)));".

The expressions of E_x and E_y that we have seen in the previous slide can be coded as given below.

$$\begin{split} & \mathsf{Ex}(\mathsf{element}) = -(((\mathsf{V}(\mathsf{nodes}(1),1))^*\mathsf{P}(1)) + ((\mathsf{V}(\mathsf{nodes}(2),1))^*\mathsf{P}(2)) + ((\mathsf{V}(\mathsf{nodes}(3),1))^*\mathsf{P}(3))) / (2^*\mathsf{delta}(\mathsf{element})); \\ & \mathsf{Ey}(\mathsf{element}) = -(((\mathsf{V}(\mathsf{nodes}(1),1))^*\mathsf{Q}(1)) + ((\mathsf{V}(\mathsf{nodes}(2),1))^*\mathsf{Q}(2)) + ((\mathsf{V}(\mathsf{nodes}(3),1))^*\mathsf{Q}(3))) / (2^*\mathsf{delta}(\mathsf{element})); \\ & \mathsf{Ey}(\mathsf{element}) = -((\mathsf{V}(\mathsf{nodes}(1),1))^*\mathsf{Q}(1)) + ((\mathsf{V}(\mathsf{nodes}(2),1))^*\mathsf{Q}(2)) + ((\mathsf{V}(\mathsf{nodes}(3),1))^*\mathsf{Q}(3))) / (2^*\mathsf{delta}(\mathsf{element})); \\ & \mathsf{Ey}(\mathsf{element}) = -((\mathsf{V}(\mathsf{nodes}(1),1))^*\mathsf{Q}(1)) + ((\mathsf{V}(\mathsf{nodes}(2),1))^*\mathsf{Q}(2)) + ((\mathsf{V}(\mathsf{nodes}(3),1))^*\mathsf{Q}(3)) / (2^*\mathsf{delta}(\mathsf{element})); \\ & \mathsf{Ey}(\mathsf{element}) = -((\mathsf{V}(\mathsf{nodes}(1),1))^*\mathsf{Q}(1)) + ((\mathsf{V}(\mathsf{nodes}(2),1))^*\mathsf{Q}(2)) + ((\mathsf{V}(\mathsf{nodes}(3),1))^*\mathsf{Q}(3)) / (2^*\mathsf{delta}(\mathsf{element})); \\ & \mathsf{Ey}(\mathsf{element}) = -(\mathsf{E}(\mathsf{V}(\mathsf{nodes}(1),1))^*\mathsf{Q}(1)) + (\mathsf{E}(\mathsf{V}(\mathsf{nodes}(2),1))^*\mathsf{Q}(2)) + (\mathsf{E}(\mathsf{V}(\mathsf{nodes}(3),1))^*\mathsf{Q}(3)) / (2^*\mathsf{delta}(\mathsf{element})); \\ & \mathsf{Ey}(\mathsf{element}) = -(\mathsf{E}(\mathsf{E}(\mathsf{element})))^*\mathsf{Q}(\mathsf{element})) + (\mathsf{E}(\mathsf{E}(\mathsf{nodes}(3),1))^*\mathsf{Q}(\mathsf{element})) + (\mathsf{E}(\mathsf{E}(\mathsf{nodes}(3),1))^*\mathsf{Q}(\mathsf{element})) + (\mathsf{E}(\mathsf{element})) + (\mathsf{E}(\mathsf{E}(\mathsf{element}))) + (\mathsf{E}(\mathsf{E}(\mathsf{element})) + (\mathsf{E}(\mathsf{element})) + (\mathsf{E}(\mathsf{element})) + (\mathsf{E}(\mathsf{element})) + (\mathsf{E}(\mathsf{E}(\mathsf{element})) + (\mathsf{E}(\mathsf{element}))$$

The magnitude of electric field intensity of the element under consideration can be calculated by using the following statement.

```
Enet(element)=sqrt((Ex(element)^2)+(Ey(element)^2));
```

 E_x , E_y , and E_{net} for all elements will be stored in the corresponding matrices when we run the for loop shown in the above slide.

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h = 1; for i = 1:n, elements				555
if $t(1 i) = 1$ then				5/00512
Eoil(h) = Enet(i):				$ \left[\begin{array}{c} \varphi \\ \varphi \end{array} \right] $
ele oil(h) = i:				
h=h+1;				इतर रायर केस
end				CDEEP
End				IIT Bombay
E100 = max(Eoil);				05 44
E90 = 0.9*E100;				EE 725 L 20 / Slide 11
k=1:	Conductor	Electric	SOV	Strength
for i = 1:h-1	diameter	Stress		
if ((Eoil(i)>=E90) && (Eoil(i)<=E100)) then stressele(k) = ele_oil(i);	10 mm	6.47×10 ⁶ V/m	37.6 cm ³	10.35 kV _{rms} /mm
k=k+1;				
end				
end				
SOV = sum(delta(stressele))*5; // 5m is the dep	th in the z dime	nsion		
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Now, we go further and understand how to calculate stressed oil volume. We run the following for loop and check whether the sub domain number is 1 for the element under consideration.

```
h = 1;
for i = 1:n_elements
if t(1,i) == 1 then
Eoil(h) = Enet(i);
ele_oil(h) = i;
h=h+1;
end
end
```

Subdomain number should be 1 because in the Gmsh interface number 1 is assigned to oil and number 2 is assigned to paper insulation on the lead. So, if the subdomain number of the element is 1, then in the matrix Eoil you store the corresponding net electric field intensity of that element.

We also store the element number in the matrix ele_oil, because this matrix will be useful when we calculate the total stressed area and volume. Now, the maximum of all the entries in Eoil matrix will be determined using the command "E100 = max(Eoil);". Suppose there are 100 finite elements in the oil region then using this command we are finding out which element has the maximum electric field intensity, and that value will be stored in E100. E90 which is 90% of E100 will be determined by using the command "E90 = 0.9*E100;".

Now, we will find the elements in which the electric field intensity is between 90% and 100% of the E_{max} and that can be simply obtained by using the following set of commands.

```
k=1;
for i = 1:h-1
    if ((Eoil(i)>=E90) && (Eoil(i)<=E100)) then
        stressele(k) = ele_oil(i);
        k=k+1;
    end
end
```

The if condition in the above for loop checks if the electric field intensity of an element is in between 90% and 100% of E_{max} . If the value is within this limit then, we note down the corresponding element number in the matrix stressele. So, in stressele matrix, we note down all those elements in oil which are having stress value between 90% and 100% of E_{max} .

Then we calculate the total stress oil volume as the sum of areas of all the elements with the stress between 90% and 100%. The total stressed area is multiplied by length in z direction perpendicular to the plane of the paper. Here the length is 5 m. Using the following command, we will get the stressed oil volume.

```
SOV = sum(delta(stressele))*5; // 5m is the depth in the z dimension
```

The strength is a function of stressed oil volume that we have obtained from finite element analysis. Then using this stressed oil volume, we calculate the strength and we compare the strength with the stress value. If the electric stress is much lower than strength, then it is a good insulation design.

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The last topic of this lecture is how to use the design of experiments technique to design a good insulation system. Again, we take the same configuration with insulated lead to ground which is shown in the above slide. Now, if we want to optimize this insulation system then the variables are the bare diameter, the insulation thickness, and the gap.

Now, the question that arises here is what should be the optimum combination of these a, b and c values? A designer of this system would generally know what are the values of a, b, c, he/she can vary in a certain range which will depend on established design and manufacturing practices. It also depends on the availability of raw material in standard dimensions. So, first we note down the total range of values of the 3 factors a, b, c. Here, a is allowed to vary from 8 to 12 mm, b is allowed to vary from 2 to 4 mm and gap (c) is allowed to vary from 22 to 25 mm as given in the above table.

Now, we will find the optimum combination of these parameters. Instead of doing a number of simulations with various combinations, you can do a set of nine experiments as per this design of experiments technique, and then we have nine combinations of the 3 factors. For example, in the first combination all the factors are at level 1. For each combination, we get the corresponding E_{max} values given in the above table.

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Now, we do the analysis of means. The values in the following matrix are means.

	а	b	с
1	6.750	7.176	6.880
2	6.378	6.356	6.407
3	6.151	5.748	5.992

For example, the value 6.750 is mean for variable a at level 1 for 3 levels of the other parameters which are given in the table given in the previous slide. The mean value given in the above table will be average of these 3 E_{max} values. Similarly, you can calculate means for the other cases. Now, for the considered levels of the factors we need to find the most influential factor in deciding the performance figure, in this case it is the maximum electric field intensity.

From the above table, you can see that the means of b are having a maximum range of variation from for 5.7 to 7.1. That means b is more influential in the considered ranges of 3 factors and it is evident from the percentage effect calculation given in the following table.

	% Effect
а	11.295
b	63.398
С	24.365

It is also evident from the following figure that when b is varied the change is much more as compared to the other 2 factors.



All this analysis will tell you that you can vary the value of b to get maximum advantage in terms of achieving a better performance figure. So, these are some basic things about design of experiments technique. You can see the references given in the above slide to understand more about the power and potential of this technique. This can be applied for the design of any other engineering aspect. Now, we will conclude lecture 25. Thank you.

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