Electrical Equipment and Machines: Finite Element Analysis
Professor Shrikrishna V. Kulkarni
Department of Electrical Engineering
Indian Institute of Technology, Bombay
Lecture 24
Calculation of Inductance of an Induction Motor and a Gapped-Core Shunt Reactor

Welcome to lecture 24. In this lecture, we will see two case studies. One on calculation of magnetizing inductance of an induction motor and second study would be for calculation of inductance for a gapped-core shunt reactor. For the first case study, let us understand how to calculate no load inductance of a 3-phase induction motor.

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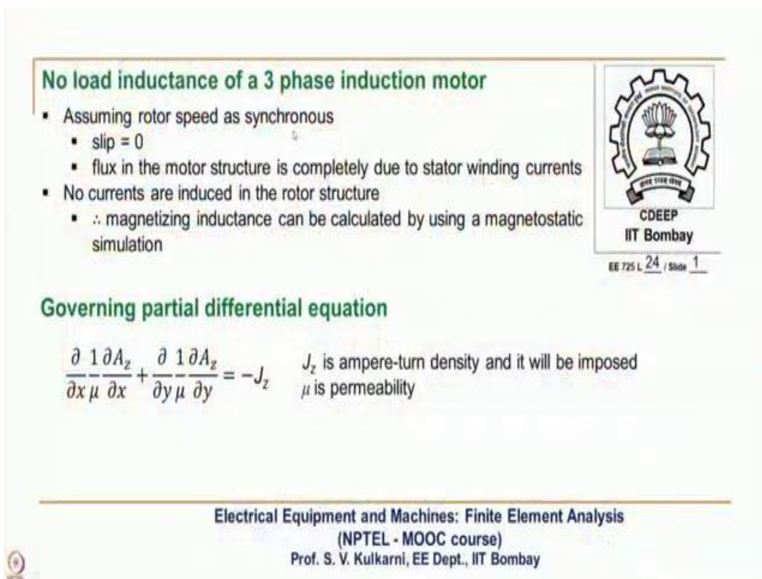

In this case, we will assume that rotor speed is synchronous speed, effectively we are assuming its slip as 0, and hence there are no induced eddy currents in the rotor structure and flux in the motor structure is completely due to the stator winding currents. With these approximations, the magnetizing inductance can be calculated by using a simple magnetostatic simulation. The governing partial differential equation is $\nabla^2 A = -\mu J$

I have already told you in some previous lectures that $\frac{1}{\mu}$ is taken on the left hand side of the above equation to make it more general, so that changes in material properties can be easily accounted in the FEM formulation. The actual form of governing equation which we are going to use in this formulation is given below.

$$\frac{\partial}{\partial x}\frac{1}{\mu}\frac{\partial A_z}{\partial x}+\frac{\partial}{\partial y}\frac{1}{\mu}\frac{\partial A_z}{\partial y}=-J_z$$

Remember, J is in z direction because here we are considering a 2-dimensional approximation and hence A also will be in z direction. That is why it reduces to a simple formulation with unknowns as magnetic vector potential at various nodes in the domain and the direction of A is also fixed in z direction. Now let us go further.

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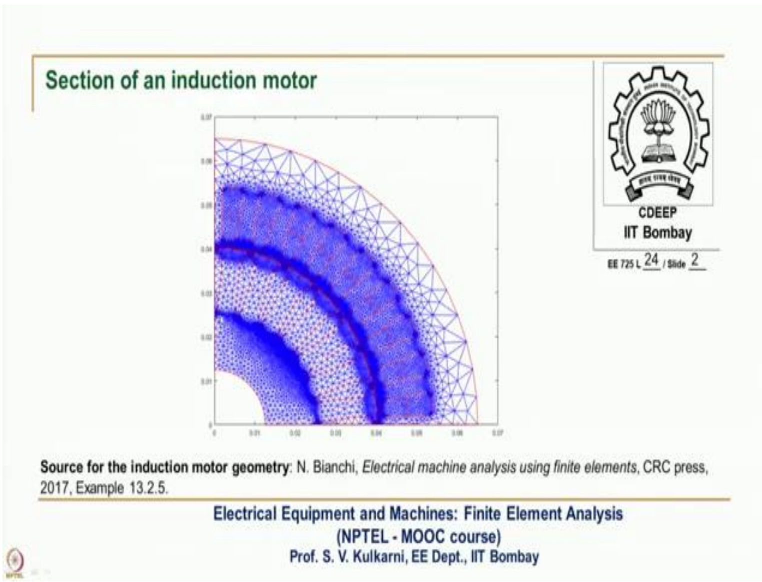

The discretized geometry of the chosen induction motor is shown in the above slide. Geometry details are taken from the reference cited in the slide. Remember that we have modelled only one quarter of the full geometry.

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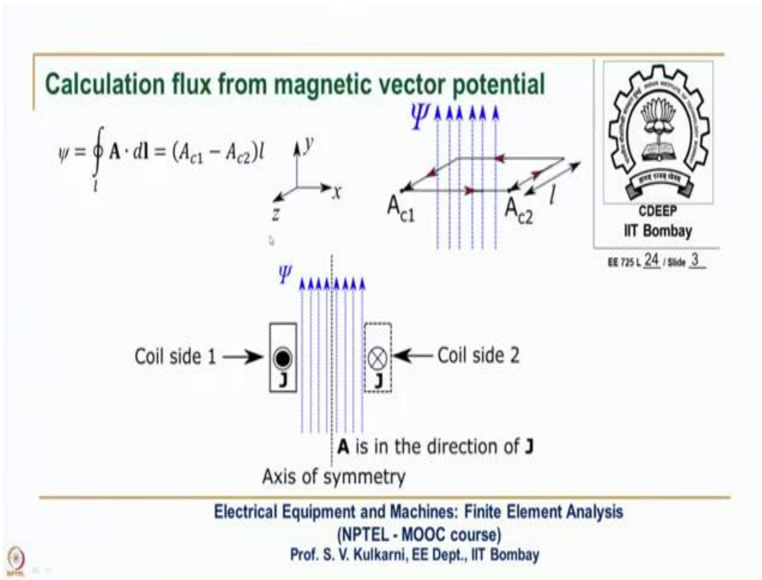
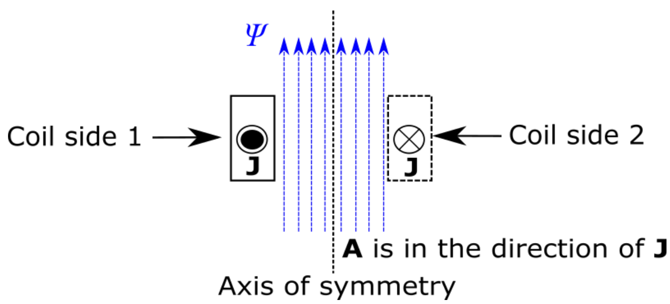
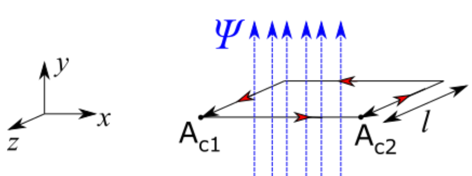


Let us revisit the procedure to calculate flux using magnetic vector potential, which we had seen in the basics of electromagnetics. Now, we will study the procedure with reference to a coil in a rotating machine. Now, consider a coil in a rotating machine as shown in the following figure. Its coil sides are also indicated in the figure.



Consider that the coil is under two opposite poles. We know that the flux is calculated by using $\psi = \oint_l \mathbf{A} \cdot d\mathbf{l}$ integrated over a contour shown in the following figure.



For the contour shown in the above figure, the expression of flux reduces to $\psi = (A_{c1} - A_{c2})l$ Because the two horizontal segments of the contour will not contribute to this integral because length vectors of these segments and A vectors are orthogonal to each other, so the dot product $\mathbf{A} \cdot d\mathbf{l} = 0$. We have seen this in earlier lectures. As mentioned earlier, in a 2-pole machine the two coil sides of a coil are under two opposite poles and the flux is like as shown in the above figure with the current direction as indicated in the figure.

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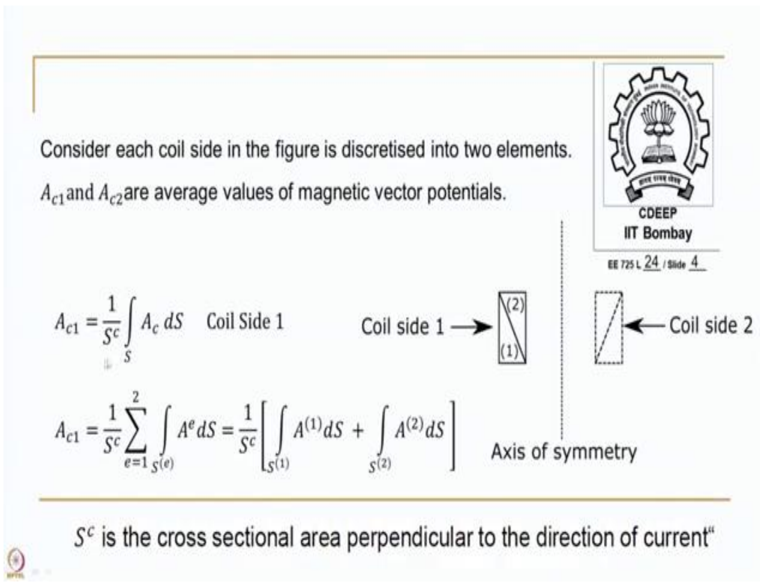
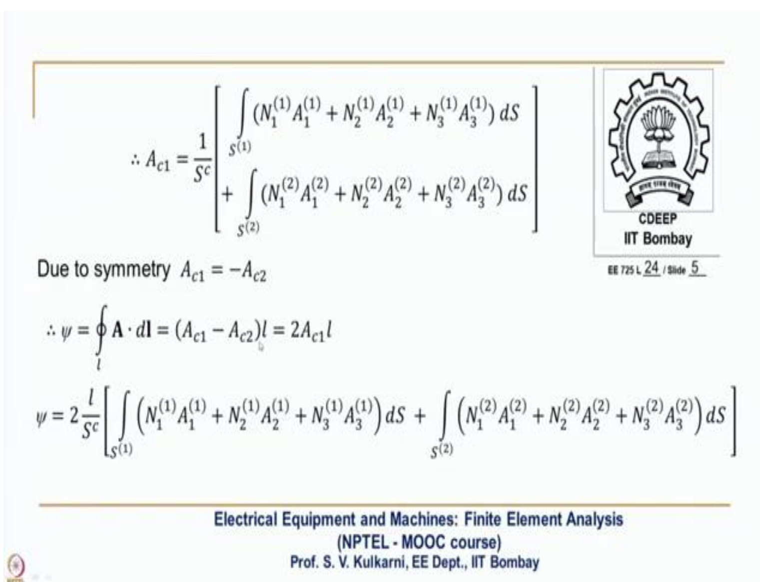
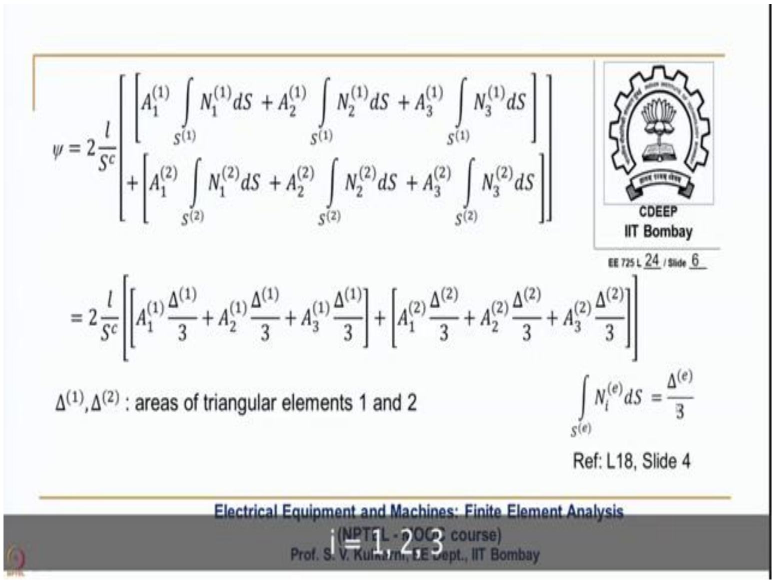
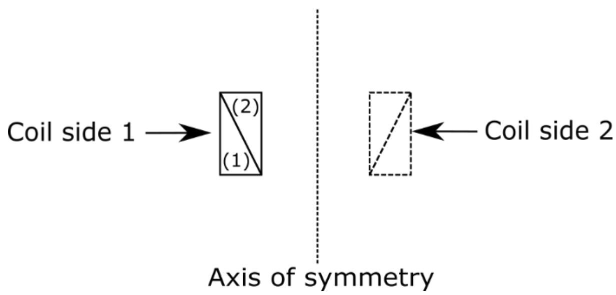






Now consider that each coil side in the figure shown in the previous slide is discretised into two finite elements as shown the following figure.



The coil sides 1 and 2 are discretized into two finite elements as shown in the figure. So the return path for current in an element in coil side 1 is the corresponding symmetrical element in coil side 2. Let A_{c1} and A_{c2} be the average values of magnetic vector potentials in coil side 1 and coil side 2 respectively.

Now, A_{c1} can be determined by evaluating $\int_S A_c\, dS$ over the coil side area and then divide it by S^c because we are calculating the average value of magnetic vector potential in coil side 1. So the overall expression of A_{c1} is given in the following equation.

$$A_{c1} = \frac{1}{S^c}\int_S A_c\, dS$$

Further simplifying using finite element procedure for the two elements, the above integral can be written as a summation over two elements as given below.

$$A_{c1} = \frac{1}{S^c} \sum_{e=1}^{2} \int_{S^{(e)}} A^e dS = \frac{1}{S^c} \left[\int_{S^{(1)}} A^{(1)} dS + \int_{S^{(2)}} A^{(2)} dS \right]$$

With this we are converting the integration over the whole coil area into integrations over element area and e goes from 1 to 2. Now, we can split this summation and then we can write sum of integrals over individual elements as given in the above equation. Remember $A^{(1)}$ and $A^{(2)}$are the approximated magnetic vector potentials over elements 1 and 2 respectively. Whereas A_{c1} is the average magnetic vector potential for coil side 1. Further we are expanding $A^{(1)}$ and $A^{(2)}$ as given below

$$A^{(1)} = N_1^{(1)} A_1^{(1)} + N_2^{(1)} A_2^{(1)} + N_3^{(1)} A_3^{(1)}$$
$$A^{(2)} = N_1^{(2)} A_1^{(2)} + N_2^{(2)} A_2^{(2)} + N_3^{(2)} A_3^{(2)}$$

The expression of A_{c1} is modified as given below

$$A_{c1} = \frac{1}{S^c} \left[\begin{array}{l} \int_{S^{(1)}} (N_1^{(1)} A_1^{(1)} + N_2^{(1)} A_2^{(1)} + N_3^{(1)} A_3^{(1)}) \; dS \\ + \int_{S^{(2)}} (N_1^{(2)} A_1^{(2)} + N_2^{(2)} A_2^{(2)} + N_3^{(2)} A_3^{(2)}) \, dS \end{array} \right]$$

Due to symmetry, $A_{c1} = A_{c2}$. Because by symmetry the magnetic vector potential at mirror points on the right side and left side of the axis will have exactly opposite signs. So the expression of flux ($\psi = (A_{c1} - A_{c2})l$) reduces to $\psi = 2A_{c1}l$. Now, substituting the expression of A_{c1}in $\psi = 2A_{c1}l$ will give

$$\psi = 2\frac{l}{S^c} \left[\int_{S^{(1)}} \left(N_1^{(1)} A_1^{(1)} + N_2^{(1)} A_2^{(1)} + N_3^{(1)} A_3^{(1)} \right) dS + \int_{S^{(2)}} \left(N_1^{(2)} A_1^{(2)} + N_2^{(2)} A_2^{(2)} + N_3^{(2)} A_3^{(2)} \right) dS \right]$$

We bring out magnetic vector potentials out of the integrals as given below because only N_is are the functions of x and y.

$$\psi = 2\frac{l}{S^c}\left[\left[A_1^{(1)}\int_{S^{(1)}} N_1^{(1)}dS + A_2^{(1)}\int_{S^{(1)}} N_2^{(1)}dS + A_3^{(1)}\int_{S^{(1)}} N_3^{(1)}dS\right] + \left[A_1^{(2)}\int_{S^{(2)}} N_1^{(2)}dS + A_2^{(2)}\int_{S^{(2)}} N_2^{(2)}dS + A_3^{(2)}\int_{S^{(2)}} N_3^{(2)}dS\right]\right]$$

In lecture 18, slide number 4 we have seen the following expression for integral of shape function.

$$\int_{S^{(e)}} N_i^{(e)}dS = \frac{\Delta^{(e)}}{3}$$

Then substituting the integral values in the expression of ψ, you can simplify the expression of ψ as given below.

$$\psi = 2\frac{l}{S^c}\left[\left[A_1^{(1)}\frac{\Delta^{(1)}}{3} + A_2^{(1)}\frac{\Delta^{(1)}}{3} + A_3^{(1)}\frac{\Delta^{(1)}}{3}\right] + \left[A_1^{(2)}\frac{\Delta^{(2)}}{3} + A_2^{(2)}\frac{\Delta^{(2)}}{3} + A_3^{(2)}\frac{\Delta^{(2)}}{3}\right]\right]$$

Here, $\Delta^{(1)}$ and $\Delta^{(2)}$ are the areas of the triangular elements 1 and 2.

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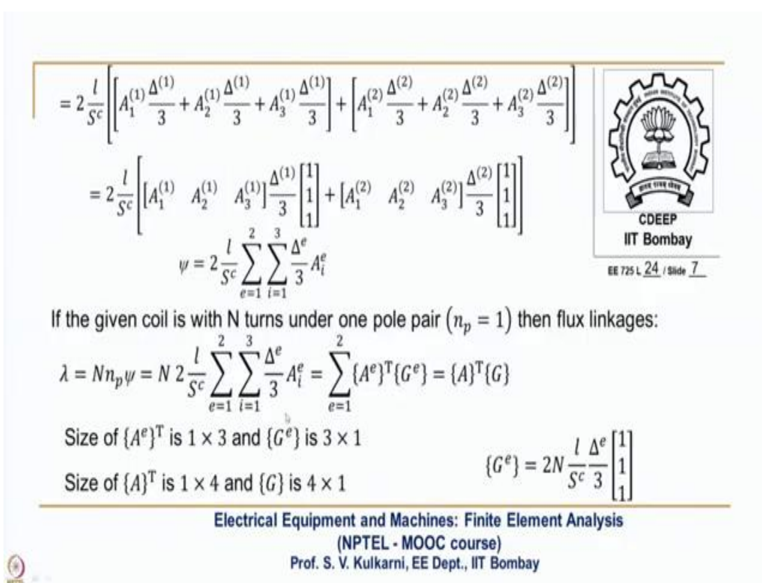

Now the expression of ψ can be written as a product of two matrices as given below.

$$\psi = 2\frac{l}{S^c}\left[\begin{bmatrix} A_1^{(1)} & A_2^{(1)} & A_3^{(1)} \end{bmatrix}\frac{\Delta^{(1)}}{3}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} A_1^{(2)} & A_2^{(2)} & A_3^{(2)} \end{bmatrix}\frac{\Delta^{(2)}}{3}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right]$$

So $\begin{bmatrix} A_1^{(1)} & A_2^{(1)} & A_3^{(1)} \end{bmatrix}$ is a row vector and it is multiplied with column vectors $\frac{\Delta^{(1)}}{3}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\frac{\Delta^{(2)}}{3}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Now the above expression can be written elegantly using two summations as given below.

$$\psi = 2\frac{l}{S^c}\sum_{e=1}^{2}\sum_{i=1}^{3}\frac{\Delta^e}{3}A_i^e$$

Remember that the 2 in the above expression corresponds to two coil sides and the summation is over two elements (here) and the individual element has a summation over its 3 nodes. The above expression is for 1 turn because we considered only 1 turn coil in the formulation. If there are N turns and more than one pole pair, then we have to take as given in the following expression.

$$\lambda = Nn_p\psi = N\,2\frac{l}{S^c}\sum_{e=1}^{2}\sum_{i=1}^{3}\frac{\Delta^e}{3}A_i^e = \sum_{e=1}^{2}\{A^e\}^{\mathrm{T}}\{G^e\} = \{A\}^{\mathrm{T}}\{G\}$$

For simplicity we are took number of pole pairs as 1 in the derivation. The total flux linkages is nothing but $Nn_p\psi$. Here, we are considering $n_p = 1$ so n_p can be replaced by 1 as given in the above expression. Now the expression of ψ can be written elegantly as done in the above derivation. Here, G^e is element level matrix and its expression is given below

$$\{G^e\} = 2N\frac{l}{S^c}\frac{\Delta^e}{3}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

The size of $\{A^e\}^T$ is 1×3 because A^e is a column vector whose dimension is 3×1 and its transpose will be 1×3. So the size of $\{A^e\}^T$ will be 1×3 and G^e will be 3×1 and the product of these two will give a 1×1 matrix. Now, the product $\{A^e\}^{\mathrm{T}}\{G^e\}$ is over one element and summing it over both the elements will give $\psi = \{A\}^{\mathrm{T}}\{G\}$. So we go to the global level. These global matrices are corresponding to both the elements.

Here, the size of global matrices will be 1×4 for A^T and for G the size will be 4×1. Because even though there are two elements, the total number of nodes are 4 and two nodes are common, so the size of A^T is 1×4 and similarly the size of G matrix would be 4×1. So for this two element example, we have quickly understood how can we go from element level to global level.

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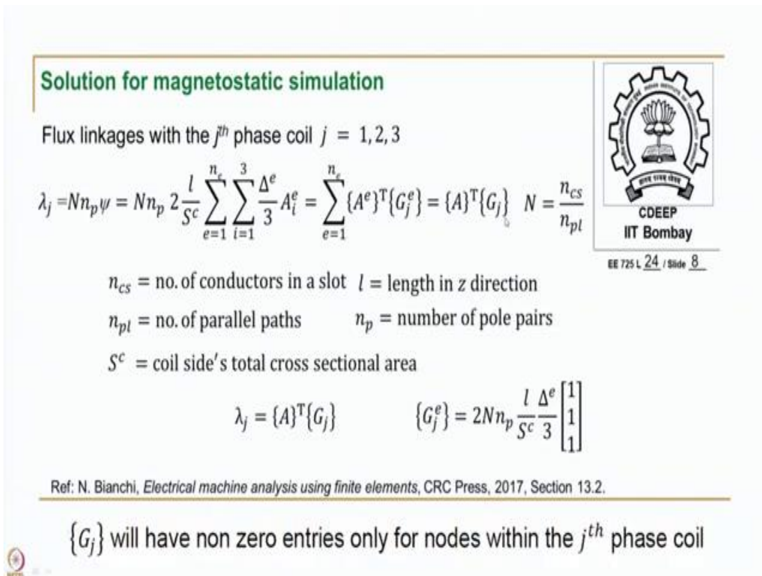
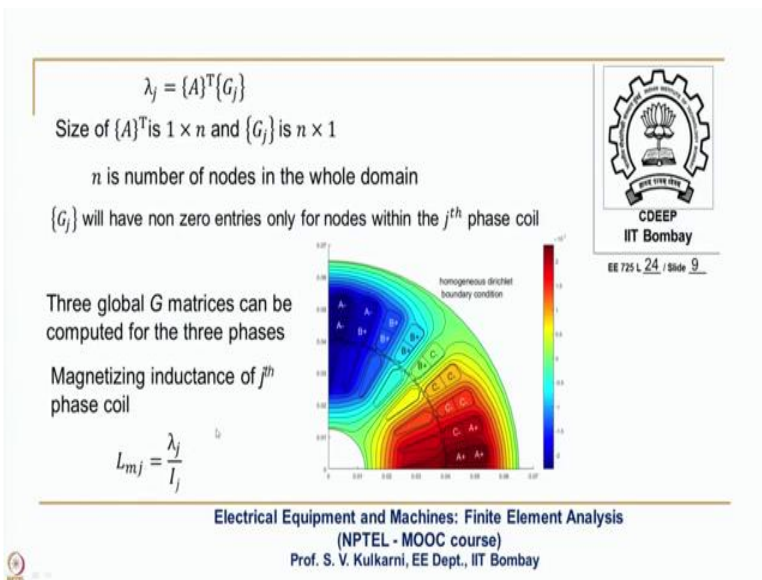




Now, we go to the 3 phase induction motor example that we saw in the first two slides. Here, we are considering its j^{th} phase coil and j can be either 1, 2 or 3. We are considering the following expression of flux linkages with n_p that we have already derived.

$$\lambda_j = N n_p \psi = N n_p\, 2\frac{l}{S^c}\sum_{e=1}^{n_e}\sum_{i=1}^{3}\frac{\Delta^e}{3}A_i^e = \sum_{e=1}^{n_e}\{A^e\}^{\mathrm{T}}\{G_j^e\} = \{A\}^{\mathrm{T}}\{G_j\} \quad N = \frac{n_{cs}}{n_{pl}}$$

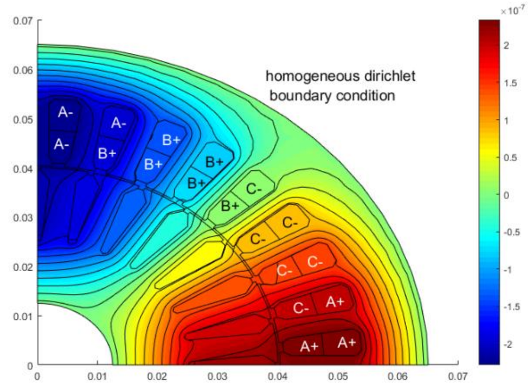
Here, we are not assuming $n_p = 1$ because there can be more than 1 pole pair in a motor. So n_p is the number of pole pairs in the motor. N is the number of turns for an induction motor and it is given by $\frac{n_{cs}}{n_{pl}}$, n_{cs} is total number of conductors in a slot and n_{pl} is the number of parallel paths. So $\frac{n_{cs}}{n_{pl}}$ will give the electrical number of turns and S^c is total cross sectional area of coil sides. In the earlier slide, the limits of summation of elements are going from 1 to 2 but here e goes from 1 to n_e (the total number of elements in the domain). The above expression is for j^{th} phase coil, so G_j^e is written.

The size of A will be $n \times 1$ so the size of A^T will be $1 \times n$ and G_j will be $n \times 1$. So the size of $A^T G_j$ will be 1×1. Because λ (flux linkages) would be a scalar number whose size is 1×1. Here, n is the number of nodes in the whole domain. G_j matrix will have non-zero entries only for the nodes in the j^{th} phase coil.

Similarly, for the matrix that corresponds to some other coil will have non zero entries corresponding to the nodes in that coil. So three global G_j matrices can be computed for 3 phases as just explained briefly and then the magnetizing inductance of the j^{th} phase coil can be calculated by using the following equation .

$$L_{mj} = \frac{\lambda_j}{I_j}$$

Using the formulation that we have derived you got the flux plot shown in the following figure. We are getting the flux contours as expected.



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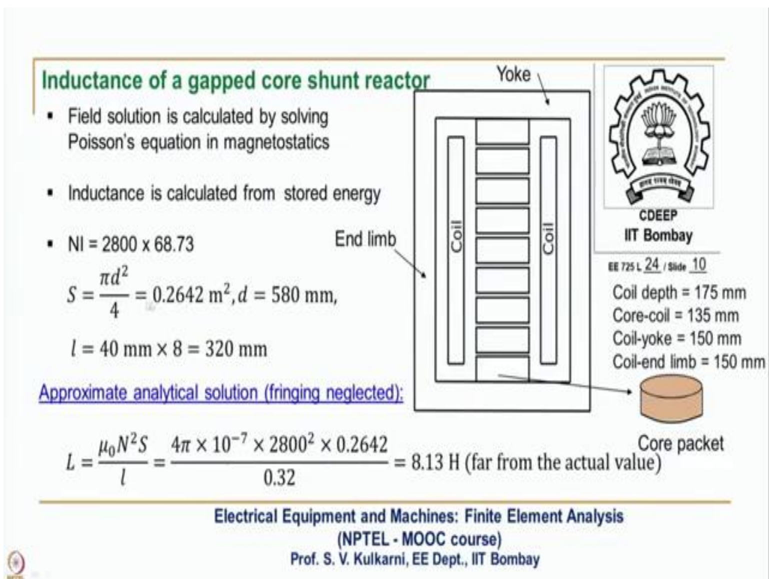
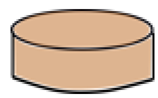


Now, we will see the second case study of this lecture which is about the calculation of inductance for a gapped-core shunt reactor. The shunt reactor shown in the figure on the above slide has a central core with air gaps. Although they are called as air gaps but there is some non-magnetic and non-metallic material in all the gaps between core packets, so that this whole structure is mechanically stable. The height of these gaps are adjusted to get a particular value of inductance. Now, what is the inductance?

Inductance is calculated by using the expression $L = \frac{\mu_0 N^2 S}{l}$. In basics of electromagnetics, we have seen that inductance is given by $\frac{N^2}{\mathcal{R}}$ and here $\frac{l}{\mu_0 S}$ is the reluctance. The reluctance of the whole magnetic circuit is predominantly contributed by the air gaps. Because the reluctance of air gap is very high as compared to the magnetic material because all the rectangular bricks that you can see in cross section are magnetic materials with high permeability. For example, the relative permeability of this magnetic material is 1000, with a very simplified calculation one can say that 1 mm of air gap is equivalent to 1000 mm of this high permeable magnetic material. So most of the reluctance is offered by these air gaps to the flux and hence most of the energy is stored in the air gaps. Remember each of the rectangular packets that you see in the cross section is actually a circular packet as shown below. So the core packets are having circular cross section. When you take a front view, it looks like a rectangle.


Core packet

So, when you look from the top you will get a circular cross section. The field solution can be calculated by using Poisson's equation in magnetostatics. Now let us see the dimensions of the shunt reactor. The diameter of core packet is 580 mm, coil depth is 175 mm, core to coil distance is 135 mm, coil to yoke distance is 150 mm and coil to end limb distance is 150 mm.

The ampere turns defined for this shunt reactor are 2800 × 68.73, because the number of turns is 2800 and current is 68.73 and area of cross section is $\frac{\pi d^2}{4} = 0.2642\ d = 0.58$ m here. The total length of all 8 air gaps together is 320 mm.

If you substitute these values in the formula of inductance then you will get inductance as 8.13 H. Later we will see that this value is far from the actual value because the entire flux does not remain entirely in the air gaps in the central magnetic path. Practically, flux comes out of the central limb and it fringes between the air gaps and there will be some flux in the coil and the flux outside the core also contributes to the inductance which is not accounted in this analytical calculation.

Also, as mentioned in the basics of electromagnetics that wherever there is a flux, the corresponding flux tube can be associated with some inductance. So the inductance calculated using the analytical expression is representing the inductance due to flux contained within the central core and the flux which is outside this magnetic path is not considered. Finite element method will account all the flux and get the accurate value of inductance as we will see now.

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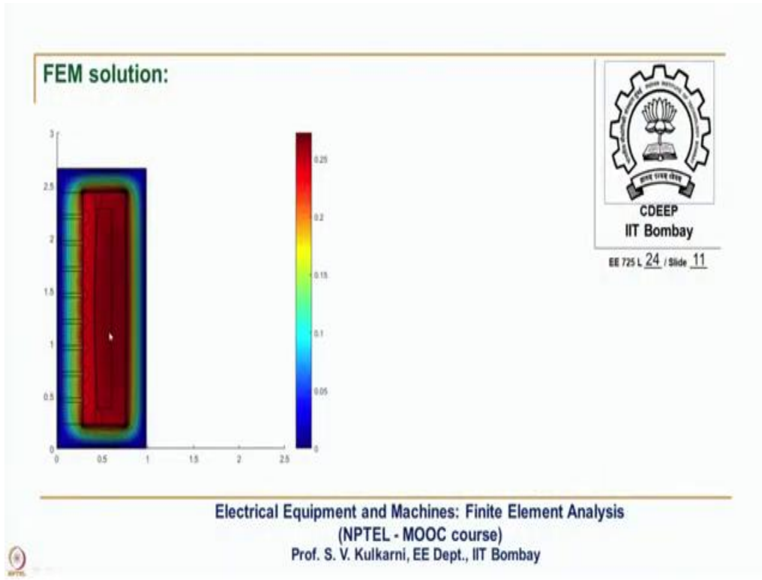

This is the flux plot obtained using finite element code. Here we have modelled only half the geometry, but there is another half which is identical. Since there is a symmetry we have considered only half the geometry. Remember we are solving this problem in xy coordinate system and we will be getting the energy per meter depth. So we have to multiply this by πD_{mean} which is the circumferential length to calculate the entire energy stored in respective parts of this geometry.

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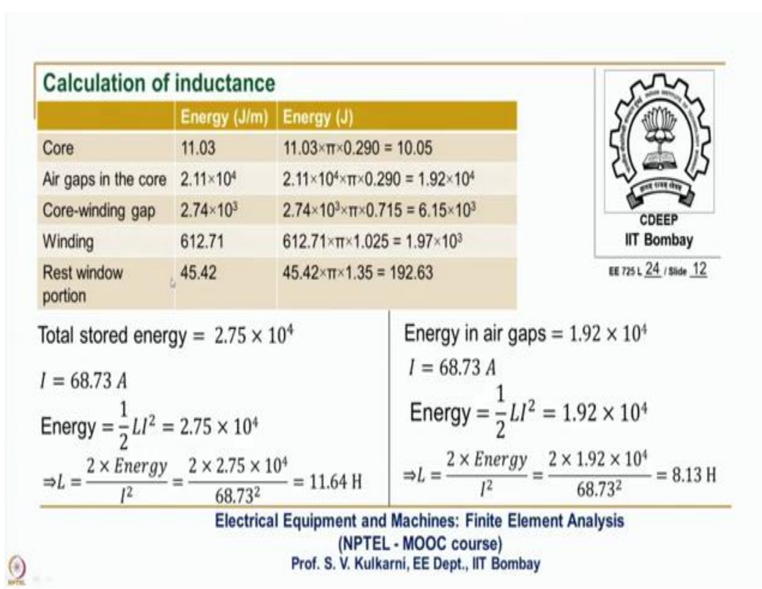

Now, we will see how to calculate the inductance value. Here, we have divided the whole geometry into 5 parts, they are core, air gaps in the core, core winding gap, winding, and rest of the window portion. When we say core, it is only the core which is a high permeable magnetic part. Second is air gaps in the central leg. Third is the region between core and coil. Next is the actual coil and the rest of the portion is between coil and end limb.

Remember we have modelled only one half of the whole geometry and the other half will be considered by multiplying by πD_{mean} which will give us the total circumferential length because we have to remember that the central core with gaps as well as coil are circular and they are symmetrical about the central axis of the shunt reactor. The values of energies in each of the part is given in the following table.

	Energy (J/m)	Energy (J)
Core	11.03	$11.03 \times \pi \times 0.290 = 10.05$
Air gaps in the core	2.11×10^4	$2.11 \times 10^4 \times \pi \times 0.290 = 1.92 \times 10^4$
Core-winding gap	2.74×10^3	$2.74 \times 10^3 \times \pi \times 0.715 = 6.15 \times 10^3$
Winding	612.71	$612.71 \times \pi \times 1.025 = 1.97 \times 10^3$
Rest window portion	45.42	$45.42 \times \pi \times 1.35 = 192.63$

In the above table, you can see that maximum energy is stored in the air gaps and it is equal to 1.92×10^4, then the next highest value is 6.15×10^3 which is the energy between core to winding. This is the second-highest because the flux is fringing between the gaps, so there is considerable energy in this region. But in the winding, it is further less whereas in the core and the rest of the winding portion energy is very less. Energy in the core is very less because its permeability is very high and the reluctance is very low, so the energy stored in it is very low.

For the same flux density the energy density will be calculated by using $\frac{1}{2}\mu H^2$. Since μ is high, for the same B value, H required to magnetize will be very small and if H is small then $\frac{1}{2}\mu H^2$ will be very small. So now total stored energy in the shunt reactor is the addition of all the 5 energy values given in the above table and remember the total energy given below is in Joules.

$$\text{Total stored energy} = 2.75 \times 10^4$$

We have already seen that the energy values given by the FEM code are in J/m. For each part those energy values are multiplied by the corresponding πD_{mean}. The corresponding mean diameters of the 5 sections of the geometry are given in the table. Now the current fed in the simulation is $I = 68.73$ A and now if you equate the total energy to $\frac{1}{2}LI^2$ you will get the value of inductance as 11.64 H.

If we just calculate the inductance by considering the energy in air gaps which is 1.92×10^4 J then we get inductance value as 8.13 H which matches with the value calculated by the simple analytical formula. But as said earlier the other 4 energy values are not accounted in this inductance that is why we get the value of inductance which is much smaller as compared to the actual value (11.64 H).

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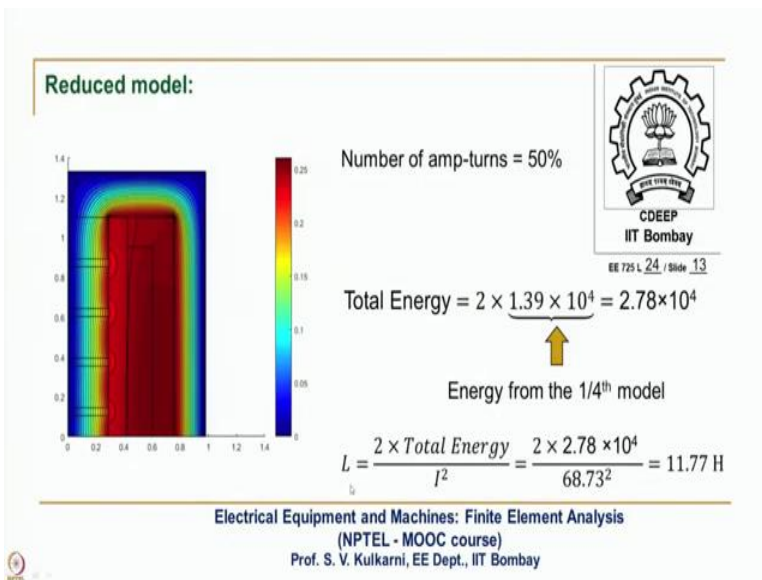

The same solution can be obtained by further reducing the model. Earlier we obtained the solution by considering only half the model. Now in the case shown in the above slide, we will consider only one fourth of the geometry because we can exploit the symmetry around the horizontal axis which is passing through the middle of the coil. Earlier we exploited the symmetry along the vertical axis and now we are exploiting symmetry along the horizontal axis.

The number of ampere turns fed to the simulation will be 50% of the total value used in the previous simulation. You have to multiply the total energy obtained from this one fourth model to get the energy stored in top and bottom parts about the horizontal axis. The total energy that was obtained using the reduced model is 2.78×10^4 J which is quite close to that obtained by considering the half model. The inductance calculated by following the procedure that we have seen in the previous slide is 11.77 H.

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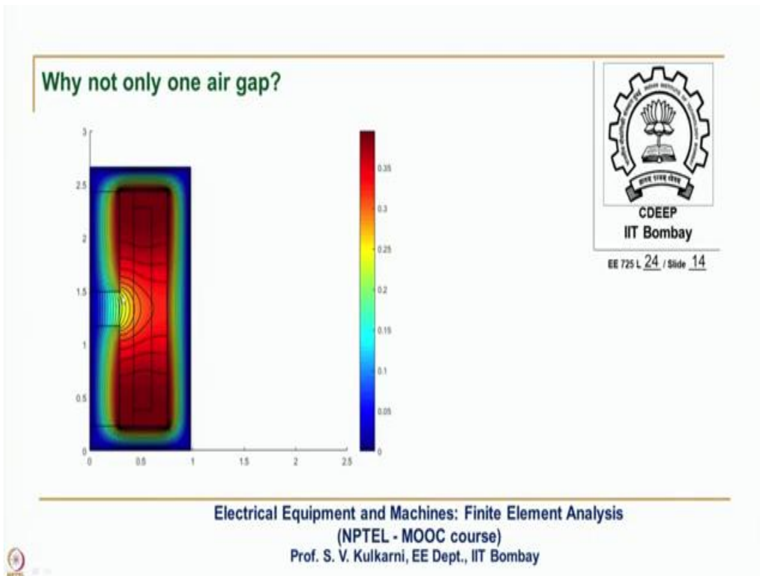

Now, the last point to discuss here is, why to consider so many air gaps in the middle core part and why can’t we use one single air gap? That question may arise in minds of those who are not knowing the design aspects of a typical gapped-core shunt reactor. There are two problems associated with the single air gap even though it is much easier from the point of view of design as well as from manufacturing point of view.

The first problem is you can see that the fringing is very high and so the effective length of these contours is also more. So the inductance that you would get by having one air gap would be quite different than the one obtained by using multiple gaps. Secondly, the heavy fringing flux hits the winding area particularly at the middle portion of the winding as shown in the figure on the above slide. This flux can cause excessive eddy current losses and hotspots in the winding. This may even damage the winding within no time. So that is why we should have multiple air gaps which are of smaller heights to avoid all these problems. You can estimate the inductance also quite accurately and get its value within the expected range. Thank you.

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