Electrical Equipment and Machines: Finite Element Analysis
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Lecture 02 - Analytical and Numerical Methods

So, in the previous lecture, we just had a quick introduction to the course and what are we expected to learn in this course.

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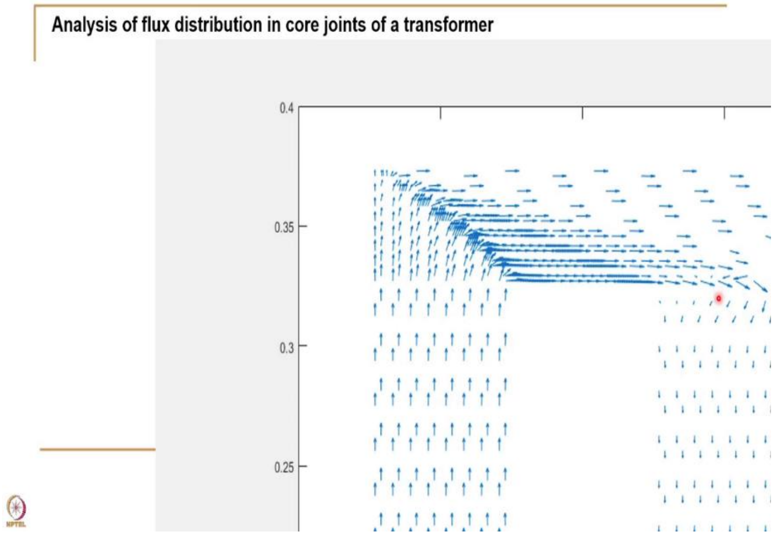

Now, let us see some interesting applications of finite element analysis, first let us see a case study of a transformer. Herein what we are doing is we are analyzing the magnetic circuit of a transformer and how the flux distribution and B vectors vary. They basically vary as a function of time. Here these are two legs of the core and third leg of course is not visible but it is there, this is the horizontal part of the core which is called as yoke.

Now if you see here in the middle joint that means at the middle leg and the yoke joint the flux or the B vector, basically it is rotating as a function of time. See here if you see in the main middle portion, the flux is always in vertical direction, it is either directed upwards and after half cycle it will get directed vertically downwards. So, the direction is either vertically up or vertically down, but here in this joint portion the B vectors rotate with time and that leads to additional losses what are called as rotational hysteresis losses. So, such kind of detailed analysis is possible only with finite element method.

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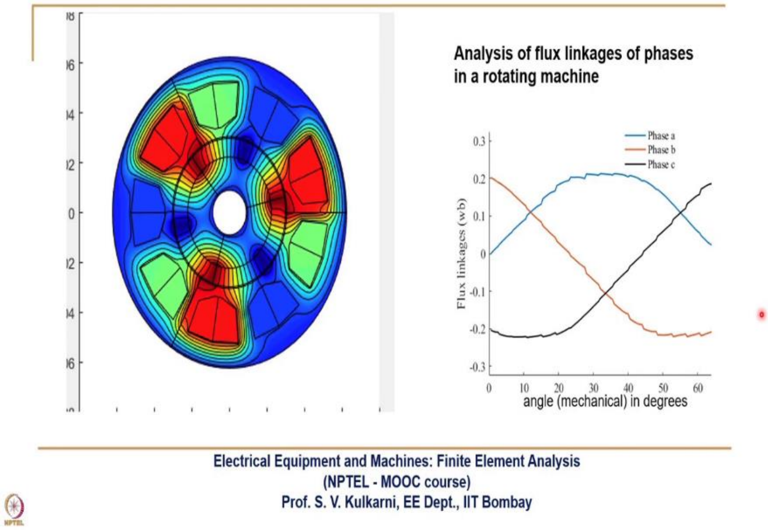

Next, we will see a case study involving a rotating machine. Here you can see as in case of the transformer in the previous case we can study variation of field as a function of time as shown in this plot. Furthermore, we can also calculate performance parameters, for example, we can calculate flux linkages of phases a, b, c with angle in degrees (mechanical degrees).

So, we can calculate such parameters quite easily. And then we can furthermore calculate back EMF and other performance figures related to this rotating machine. Details of this, we will see subsequently in the course.

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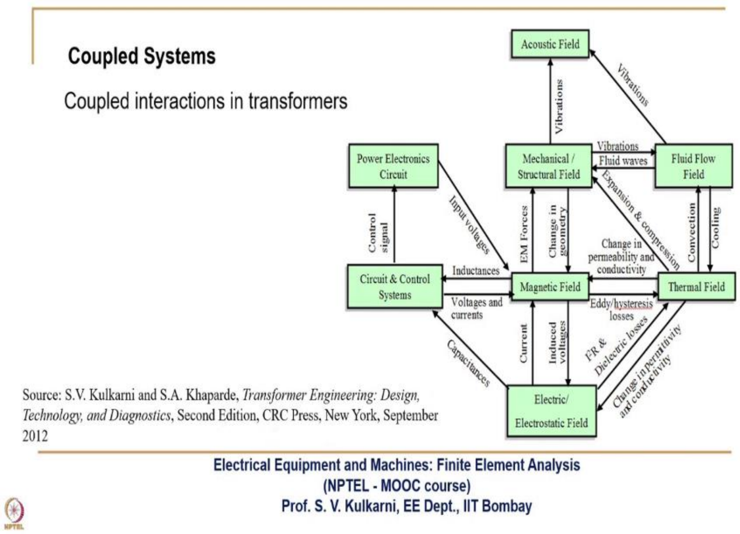

Before showing further case studies, I would like to mention that actual problems in equipment and machines are really complex, they involve many engineering fields. It does not involve just magnetic fields or electric fields, but there are thermal fields, structural fields, acoustic fields, connected networks or circuits and so on. Now, if we want to design, analyze, and optimize a given electromagnetic device, then all these engineering aspects need to be accounted in our analysis. And then the interactions between them need to be accounted accurately.

For example, let us see the interactions between magnetic field and thermal field. An alternating magnetic field leads to eddy and hysteresis losses in core, that leads to temperature rise in the core and that is called as thermal field. This temperature rise may change permeability or conductivity if the temperature rise is significant and that in turn will change the magnetic field. So, this is how the interactions go both ways.

Now, similarly, we can analyze interactions between other coupled fields. In this course, we are basically going to study magnetic fields interacting with the connected circuit. So, basically for example, inrush current in a transformer will be explained in a further slide or finding the skin effect, proximity effect, those kinds of problems can be effectively solved by considering the given electromagnetic device being excited by an external circuit.

So, as I said earlier, we will be restricting in this course, coupled circuit field analysis, this part. But for more details, on other engineering aspects you can refer this book which is dealing with transformers and coupled fields in transformers and there are many other books and published literature which talk about coupled fields in rotating machines and other equipment.

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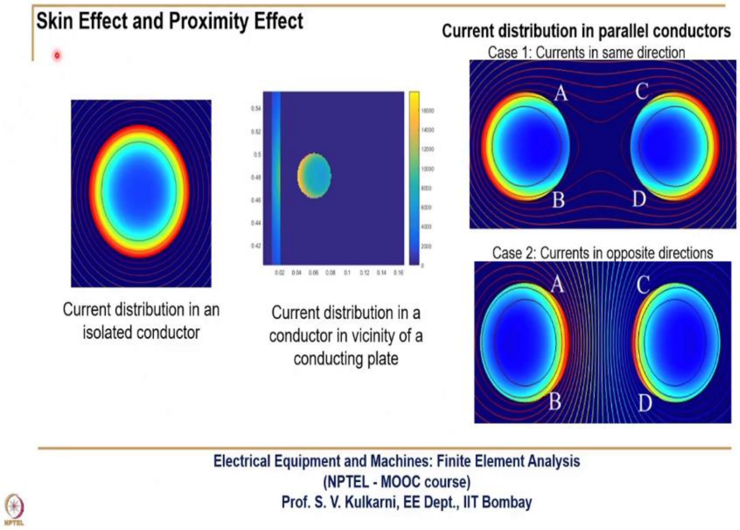

So, in this slide, we are going to analyze skin and proximity effects in conductors. In this first figure, we have an isolated conductor carrying current at some frequency. As frequency increases, the skin effect increases and current tries to be more and more on the surface reducing the effective area and increasing the effective AC resistance and hence the losses. This can be analyzed by using coupled circuit field FE analysis, wherein this conductor is fed by an external voltage source or current source. How do we do that? We will see later in the course.

Same conductor when it is brought close to a metallic conducting plate, then this skin effect gets skewed as shown here. This is because the alternating magnetic field produced by this current carrying conductor, induces eddy currents in this plate and because of this proximity effect and corresponding interaction, the skin effect gets skewed and that further reduces the effective area of this conductor and it increases its effective AC resistance and losses.

If we now see the case of two parallel conductors, the first case is currents are carried in the same direction, then the skin effect in both conductors get skewed as shown and currents are concentrating in regions which are not facing each other. It can be proved that the impedances offered by these regions are lesser as compared to the other regions of this conductor. This is because of the fact that these regions link lower flux as compared to other regions and that is why they have lower inductance and lower impedance and that is why they carry higher currents.

Case 2: currents are in opposite direction. Now here in this case the currents try to concentrate in regions which are facing each other. And this is because of the fact that now here the flux is mostly concentrating in the region between the conductors as against the first case where flux was mostly around the conductors and flux was not there in the region between the two conductors. This basically reverses the phenomenon. Now the impedance offered by these regions is much lesser as compared to other regions and that is why the current concentrates more in these regions and hence, again the effective AC resistance goes up and the losses go up. So, such analysis can be easily done by coupled circuit field analysis and in fact, we will see in one of later lectures how this can be done.

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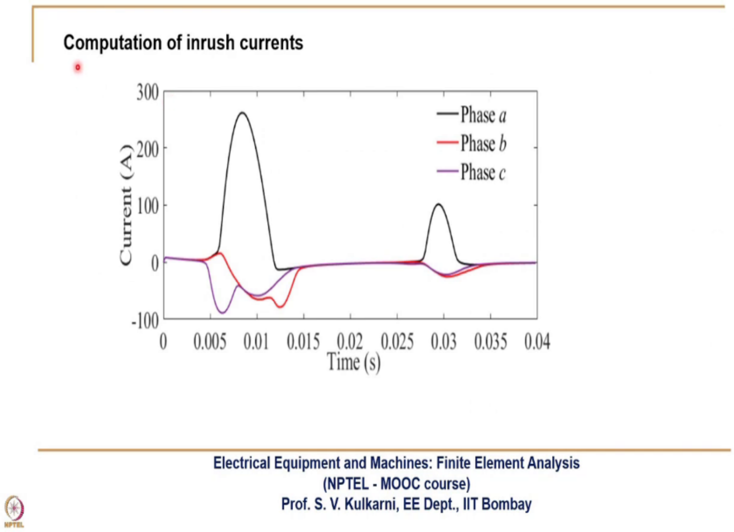

Finally, we will see how do we compute inrush currents in a transformer or an induction motor. So, here a case study involving a transformer is shown wherein it is switched-on to a voltage source and depending upon the instant of switching and the magnetic circuit condition, phases a, b, c will draw currents and these currents will be different because instant of switching will be favorable to one phase but may not be for others. For example, here for phase a, the instant of switching is unfavorable as compared to the other phases, that is why the inrush current is quite high as compared to b and c phases.

Now, such kind of analysis can again be done by using transient voltage fed coupled circuit field analysis involving magnetic non-linearity. So, there are three complications here. First of all, it is a transient analysis because with respect to time we want to plot current. Second is, it is a nonlinear analysis because the magnetic circuit is nonlinear, it saturates and hence the currents drawn can be quite high and thirdly, this is coupled circuit field analysis because some voltage source is feeding this transformer and this transformer is switched on to that voltage source. So, it is a coupled circuit field analysis. It is a transient nonlinear coupled circuit field analysis, how do we do such kind of analysis we will see later in the course.

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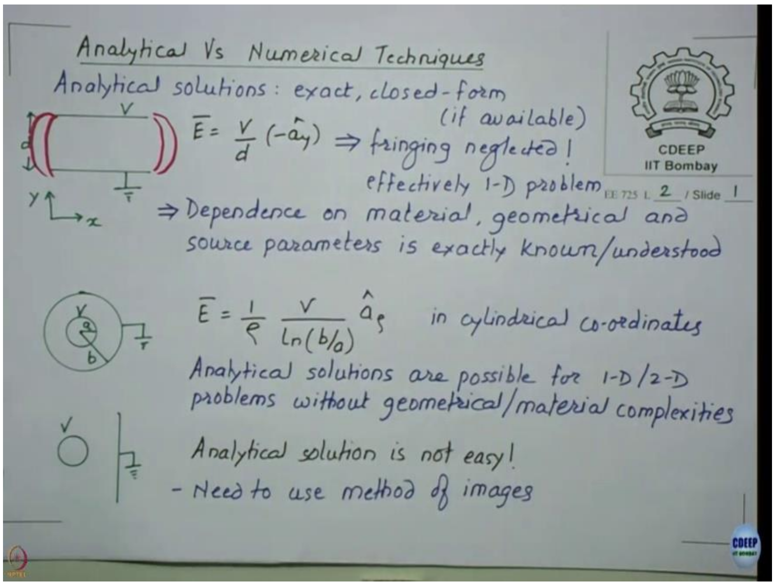

So, now having understood the importance of finite element method in general let us understand to some extent the difference between analytical and numerical techniques. The analytical techniques are the ones which typically give you exact or closed form solutions. Like for example, you have a parallel plate capacitor excited by some voltage difference V and its electric field intensity is just $\frac{V}{d}$. So, in this course, to explain many of the concepts, I will take help of very simple devices like capacitors, inductors, and transformers.

So, here this E is equal to $\frac{V}{d}$ is basically the exact or closed form solution. But here this closed form solution was possible because we neglected the fringing at the ends. If I actually ask you to consider this fringing also, then the problem becomes suddenly very difficult and analytical formulation becomes complex.

But the advantage of these closed form solutions (if available) is that dependence of the field variable on the governing influencing parameters is exactly known. For example, here we know that electric field intensity is inversely proportional to the distance. So, looking at the equation, you can directly understand the dependence and that is the advantage. Whereas when you use a numerical technique like FEM, you do not get that dependence readily. There what you have to do is, you have to do then number of parametric studies. You have to vary the parameters in

FEM analysis and then generate curves like the one I showed in the last lecture that rotor bar breakage resistivity versus torque. Those kinds of graphs have to be generated and then you will get the dependence after a series of parametric studies.

In case of exact solution through analytical formulation, you readily have a governing equation and influence of parameters can be readily seen. So, now, let us increase the complexity of the geometry little bit. Now, we are going to see a system of two cylindrical conductors which is analyzed in cylindrical coordinate system. The inner cylindrical conductor is at a voltage level V and the outer conductor is at ground potential.

Now, again starting from Laplace's equation, you can easily derive this analytical formula. So, now the analytical formula becomes little bit complex, but still it is very much manageable. But such analytical solutions are possible only for one dimensional and two dimensional problems and that too without geometrical or material complexities.

Now, the same geometry if I modify a little bit and make this outer cylindrical conductor as a straight one, suddenly the problem becomes almost very difficult or impossible for doing hand calculations. Yes, but you know researchers earlier they found ways out to find solution for this also.

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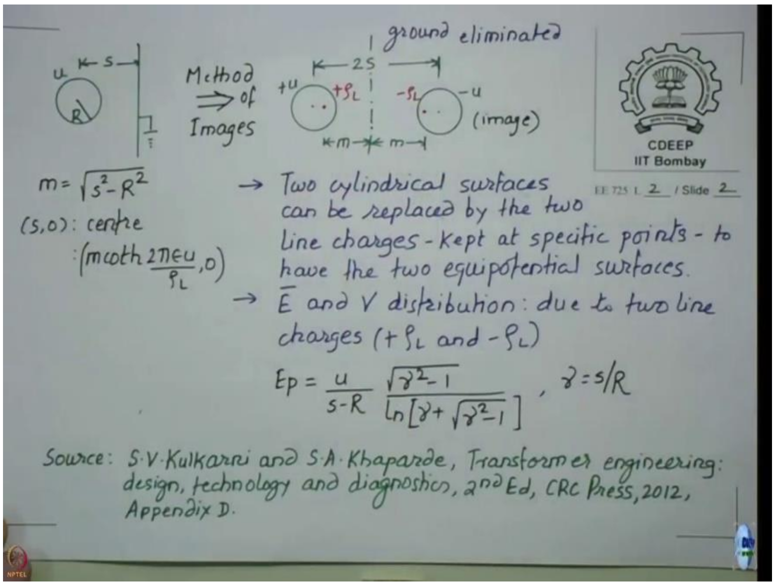

So, what did they do? They basically use what is known as method of images. In method of images basically you are replacing this ground by an image conductor with a negative potential so that you will get a potential here which is of zero. This voltage is –u, and here the voltage is +u, right.

So, now since you are having a potential on this line which is zero, you have effectively replaced this ground by an image conductor. So, one problem in terms of geometrical complexity is eliminated by using an image conductor. Secondly, what you have to do is these cylindrical conductors the original conductor and the image conductor can be replaced by equivalent line charges one with ρ_L and one with $-\rho_L$, while ensuring boundary conditions. What are boundary conditions? That on this circle here and this circle here you should get +u and –u. The position has to be adjusted so that you get equipotential surfaces or contours in 2D plane here and then you can eliminate these conductors also.

So, now the geometry reduces to simply two line charges. Now for a line charge, you have the analytical solution. Very simple, straightforward solution is available and then you can use those formulae for positive and negative line charges and superimpose at any point in the whole domain, to calculate the potential or electric field intensity.

So, this is how you do it, but this involves a lot of mathematics. If you see Appendix D in this reference and a lot of maths is involved to derive this formula for EP and what is EP? EP is the maximum stress at point P. So electric field intensity at this point is then given by this formula, where gamma is given by $\frac{S}{R}$, S is this distance between center and this ground plane and R is this radius and here m is given by $\sqrt{S^2 - R^2}$, where m is the position of these two line charges with respect to this ground plane.

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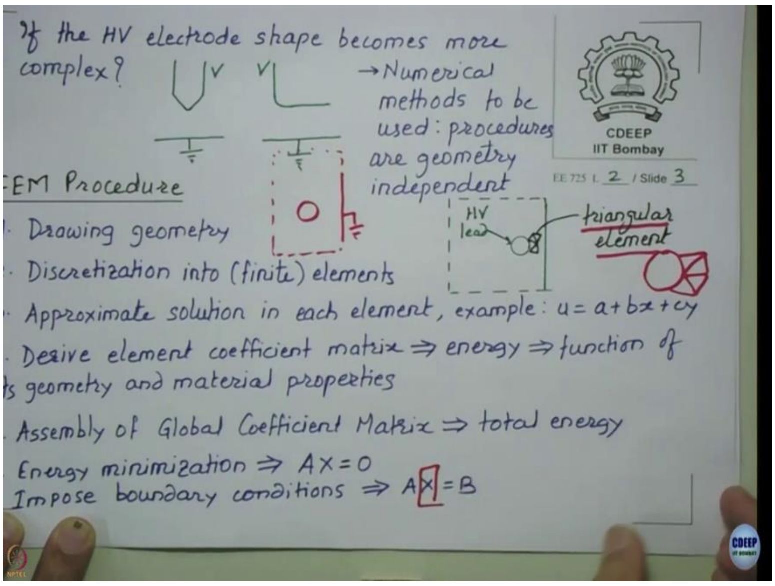

Now, if we go a step further, if we make the electrode's shape even more complicated, like one of the electrodes is having a sharp point or the electrode is something like this, then of course, the analytical methods become very difficult. Even for such things also you will find in the literature, the researchers have done some approximations and they have come up with some approximate formulae, but then they become geometry dependent. Every time you encounter such complicated arrangement of electrodes, you will have to derive a separate formula.

The main advantage of numerical techniques is that formulations in these techniques are geometry independent, that means, let the geometry be of any shape the FE procedure is the same. So, that is the main advantage of finite element method or any such numerical technique.

So, what is the finite element method procedure? So first you have to draw geometry like the one here. The geometry is the previous case wherein we analyzed a high voltage lead near the ground electrode. So first you have to draw the geometry, which is very straightforward in any of the CAD packages. Now, next step is you have to make this problem bounded. As I mentioned during the first lecture, finite element method is a method which requires a bounded structure. So, you have to make this problem which is closed. Now, where should this fictitious boundary be placed? It should be placed far away from this electrode as shown here. Now you can see here for this electrode, what we are interested is the stress distribution in this part, we are not interested in field distribution here, but at the same time, you need to take these three boundaries far away from this electrode so that the boundary conditions on these three do not influence unnecessarily field conditions in this zone. So, more about this we will understand details of FE procedure and its applications.

So once you have this bounded structure, then you have to discretize into finite elements and then approximate the solution in each element. Now, these finite elements as shown here are the triangular elements, the whole region is filled with such triangular elements. So, for example, if this is my lead, the triangular elements would be something like this and so on.

Now why triangular elements? Yes, we can use any other shape also, but a triangular element is much easier to code, and that is why we are using it. Later we will discuss more about this. Then approximate the solution in each finite element. For example, u which is the voltage is $a + bx + cy$. One of the approximations could be this. Then you derive the element coefficient matrix which represents the energy of that finite element, which is a function of its geometry and material properties.

Then you assemble all such element coefficient matrices and form what is known as global coefficient matrix. Then after having formed global coefficient matrix which represents the energy of the entire system, minimize the energy in one of the FE approaches. There are some other approaches also, which we will discuss later. But then, typically for this case that will give you a matrix equation which is $AX = 0$. So, this does not have unique solution unless you impose boundary conditions.

So, boundary conditions are: this lead is at potential V and the whole thing is at ground potential. When you do that, you get a form of equation which is $AX = b$. It can be easily solved to determine the unknown, which is X, which gives potentials at various points in the domain.

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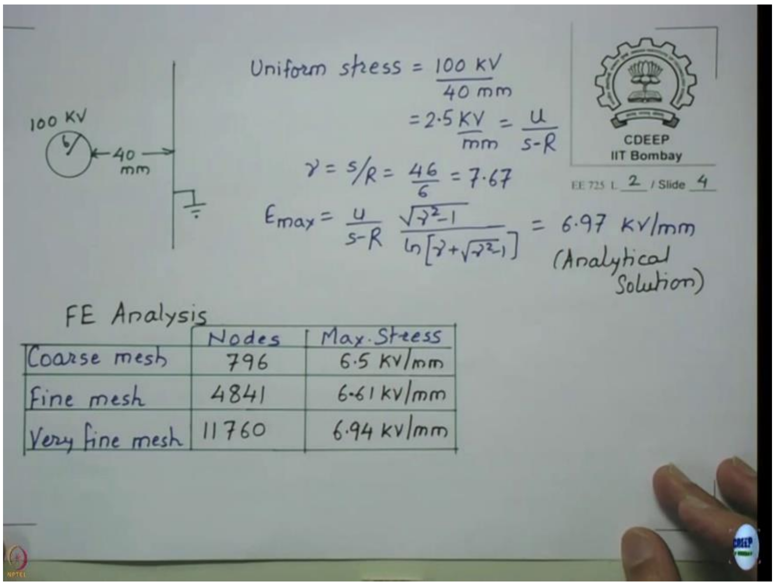

Now, using this finite element method procedure, we can calculate the stresses at various points in the domain.

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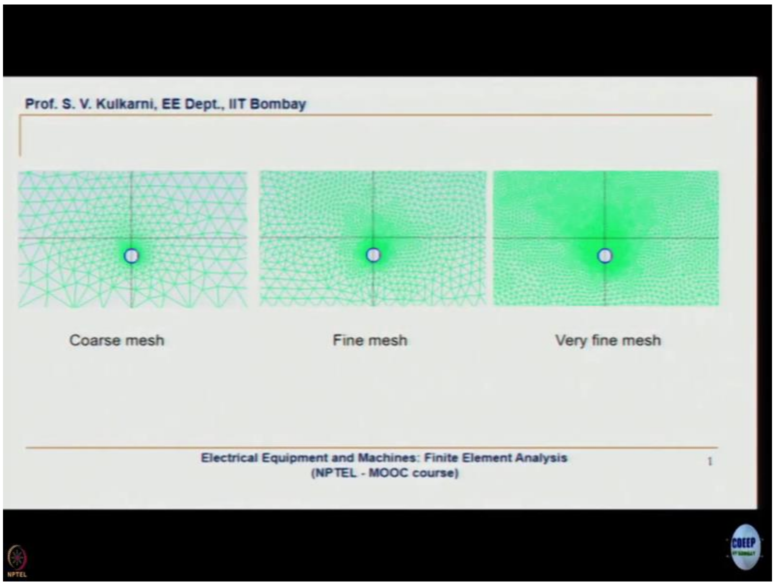

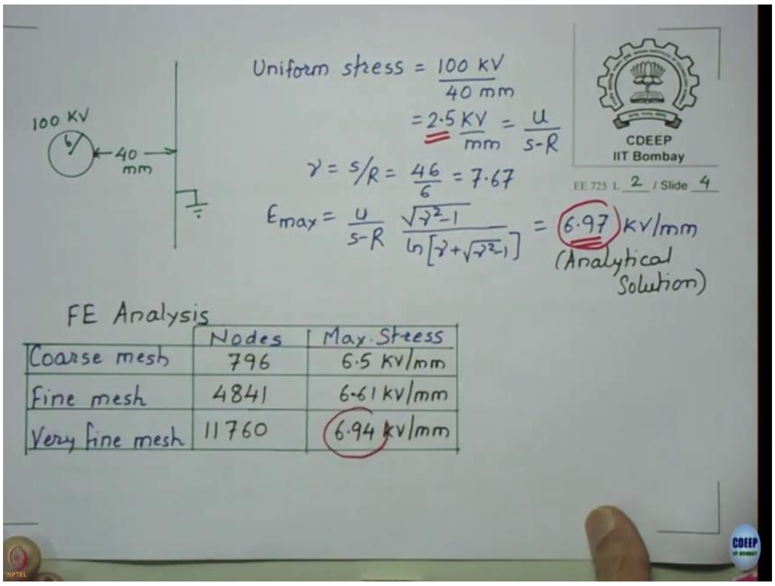

For example, you can see in this slide here there are three cases. One is coarse mesh, fine mesh and very fine mesh. This is a zoomed part very close to the lead and as you increase the mesh density or reduce the finite element mesh size, you get much better result as can be observed from this table.

So, as you increase the mesh density that means you increase the number of nodes and finite elements from 796 to almost 12,000, the maximum stress which occurs at this point, which is the minimum distance between this high voltage lead and ground, the stress comes more or less very close to the analytical value, which is calculated from the formula that we saw in one of the previous slides.

And here you can see the uniform stress in a case where 100 kV is the potential difference, 40 mm is the clearance, 6 mm is the radius of the electrode is 2.5 kV/mm. So, you have 2.5 kV per mm as the uniform stress if this was a parallel plate geometry, but because of this radius of the electrode this field distribution here is non-uniform and basically this E max, 6.97 kV divided by this 2.5 kV which is uniform stress gives the enhancement factor. As compared to uniform field stress, you are getting 6.97 kV. So, this is the enhancement in the field due to the non-uniform field conditions.

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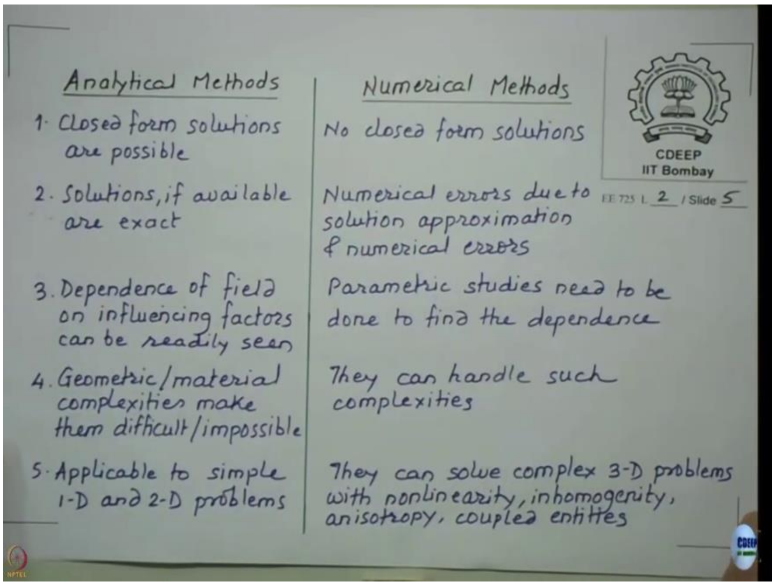

Now, quickly I am summarizing the difference between analytical and numerical techniques, in analytical techniques closed form solutions are available, here they are not available, available analytical solutions are exact, you need to do a lot of improvements like in mesh size and all that to get accurate results while using FEM, that is very much possible. Dependence of field on influence factor can be readily seen in analytical formulae, here in FEM we have to do number of parametric studies.

Complexities cannot be easily handled in analytical methods, complexities in geometry and material can be handled in FEM. Analytical: applicable for 1D or 2D problems. Whereas, there is no such limitation in case of numerical techniques, they can easily solve 3D problems also with available commercial codes, or if you develop your own codes even for 3D problems.

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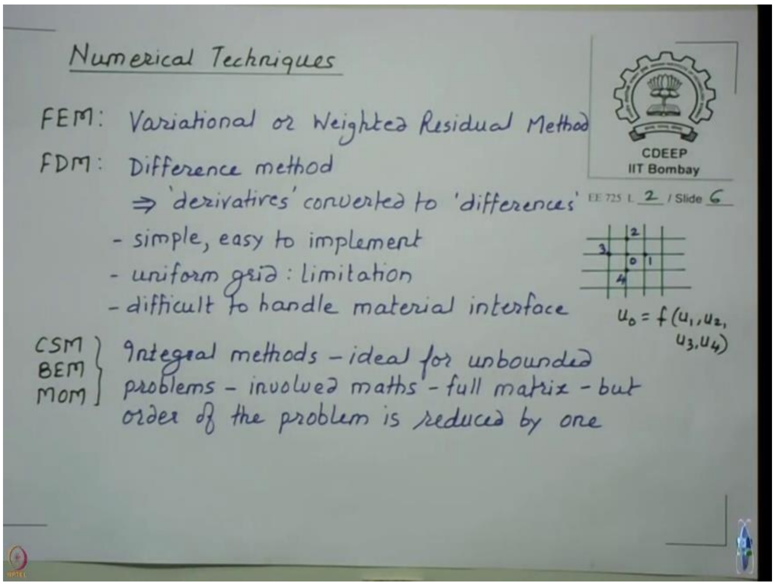

So, we are going to study in this course only finite element method which is based on either variational approach or weighted residual approach. Variational approach is based on energy minimization whereas, weighted residual approach is error minimization approach. So, we will see both these methods, but we will see more of variational approach because it is easy to understand and it is more closely related to the physics. Then next method is finite difference method, it basically has a procedure in which you have the whole geometry converted into a uniform grid. And that is in fact, one of its main limitations that you need to have uniform grid, there is no such requirement in case of finite element method. But, it is easy to implement because derivatives in partial differential equations are converted into differences, so this method is conceptually very simple, but it becomes difficult to handle material interfaces in this method.

One of the simplest applications for example Laplace's equation, the potential at this point O is expressed as a function of the surrounding four nodes and in case of Laplace's equation, the u_0 is just $\frac{u_1+u_2+u_3+u_4}{4}$ that is average of surrounding four potentials. So, it is a very intuitive and simple technique. But as I said for practical problems, it has some limitations.

Then there are some integral equation methods like charge simulation method, boundary element method or method of moments, as I mentioned previously in the first lecture that these methods are useful for open boundary problems. And the involved maths to derive the final set of equation is high and they lead to a full matrix, whereas finite element method, as we will see later, results into a highly sparse matrix and you can exploit that sparsity to gain computational advantage, but the advantage of these integral equation methods is the order of the problem reduces by one.

And so, depending upon the problem type you either have to use integral equation method or finite element method for your application. So, we will see more of these methods, particularly finite element method in the next set of lectures. What we are going to do is, now from next lecture onwards, we will see some basics of low frequency electromagnetics, as relevant to finite element method and analysis of electrical equipment and machines. Having done that for about eight lectures of half an hour approximately we will get into the finite element method theory and then applications. Thank you.

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