Electrical Equipment and Machines: Finite Element Analysis

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Lecture 10

Revisiting EM Concepts: Theory of Eddy Currents

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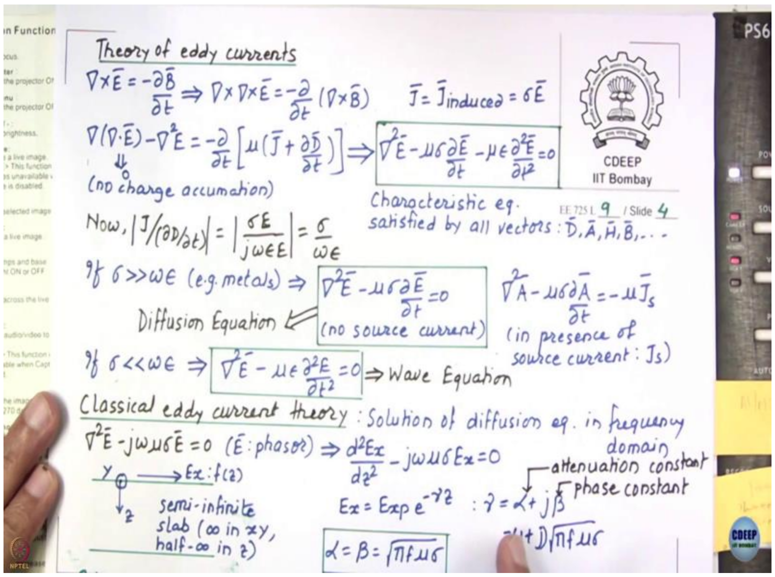
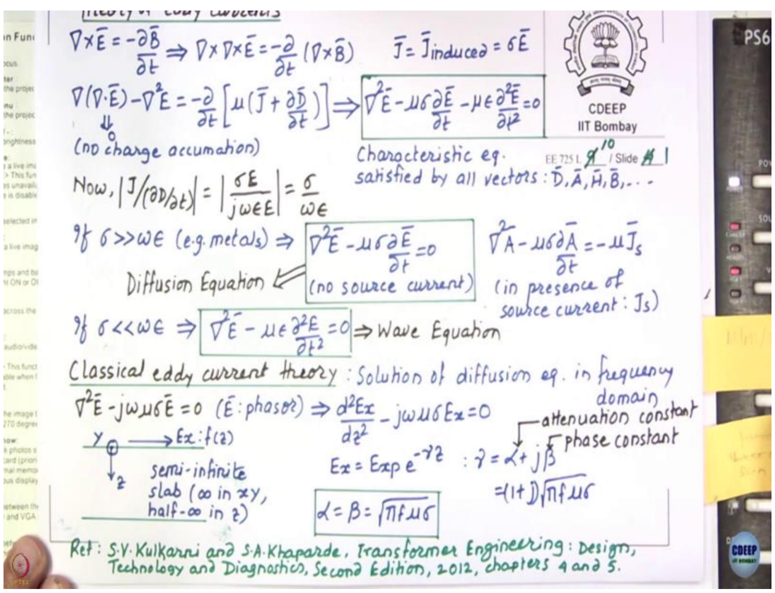
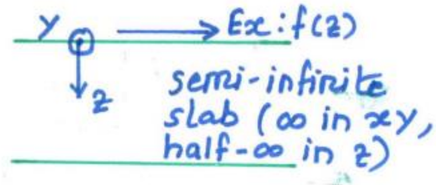




Welcome to the 10th lecture of this course. We stopped at the discussion on the wave equation in the previous lecture. We will start with the Classical Eddy Current Theory in this lecture. So, the above slide will be the first slide of 10th lecture. In classical eddy current theory, we solve the following diffusion equation. Consider a metallic plate (given in the following figure) excited by a surface field intensity which is time varying.

$$\nabla^2 \bar{E} - \mu\sigma \frac{\partial \bar{E}}{\partial t} = 0$$



The x, y, and z directions of the metallic plate are also indicated in the figure. In the above figure, the metallic plate is known as semi-infinite conducting plate. Why it is called a semi-infinite plate? The plate is infinite in x and y-directions and it is half infinite in the z-direction. And then we are analyzing the diffusion of eddy currents and fields into this metal plate.

In the diffusion equation, $\frac{\partial}{\partial t}$ is replaced by $j\omega$. So we are working with phasors now in the frequency domain. Now, if we consider E in x direction, analytical solutions can be derived.

As I have told in one of the first lectures, when you want to derive some simple expressions and understand the theory, you have to do some approximations. Here E is assumed to be only in the x-direction and it is a function of z. So as E diffuses in the z-direction, it changes with z. If that being so, then the above partial differential equation is reduced to the following equation.

$$\nabla^2 \bar{E} - j\omega\mu\sigma\bar{E} = 0 \quad (\bar{E} : \text{phasor}) \Rightarrow \frac{d^2 E_x}{dz^2} - j\omega\mu\sigma E_x = 0$$

The solution that will satisfy the above equation is $E_x = E_{xp}e^{-\gamma z}$. In this equation, E_{xp} is the amplitude and γ is propagation constant which is equal to $\alpha + j\beta$. α is the attenuation constant and β is the phase constant. If you substitute the expression of E_x back into the differential equation, then you will get

$$\alpha = \beta = \sqrt{\pi f \mu \sigma}$$

So the theory of eddy currents and this derivation is available in many books. One of such books is mentioned at the bottom of the slide. Those who are interested to understand a little deeper about this derivation and how this formula can be derived, they can refer any of such books.

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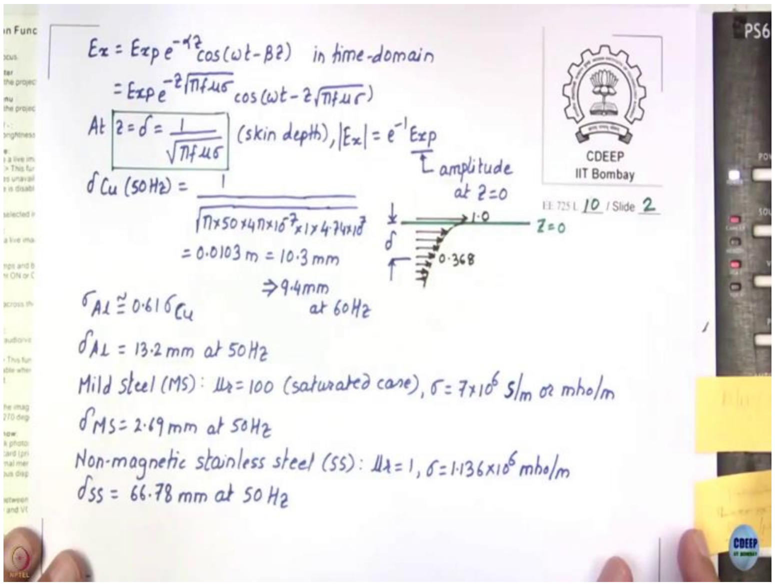


Going further the expression of E_x is written in time domain as

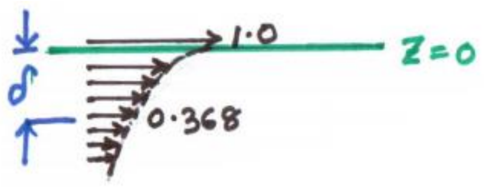
$$E_x = E_{xp} e^{-\alpha z} \cos(\omega t - \beta z) \quad \text{in time-domain}$$

Since β is representing the phase constant, the expression $E_{xp}e^{-\alpha z}$ represents the attenuation and βz will give you the phase. And then if you substitute $\alpha = \beta = \sqrt{\pi f \mu \sigma}$ which we have got from the previous slide, then you get

$$E_x = E_{xp} e^{-\alpha z} \cos(\omega t - \beta z) \quad \text{in time-domain}$$
$$= E_{xp} e^{-z\sqrt{\pi f \mu \sigma}} \cos(\omega t - z\sqrt{\pi f \mu \sigma})$$

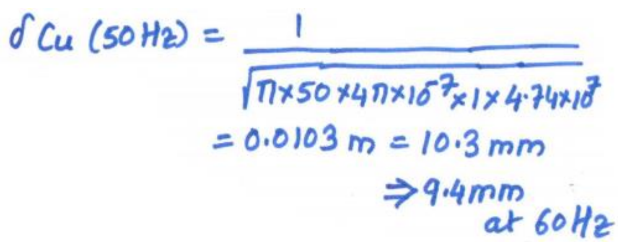
So now if you evaluate E_x at $z = \delta = 1/\sqrt{\pi f \mu \sigma}$ where δ is the skin depth, then the amplitude of E_x will be $E_{xp}e^{-1}$. And then at the surface, the magnitude will be just E_{xp} when $z = 0$, because

$e^0 = 1$ will reduce E_x to just E_{xp}. And here this is normalized with E_{xp}. So here in the following figure, I am just marking the value of E_x on the surface as 1.



Now if you go inside this conducting plate, at one δ (skin depth) of penetration, the magnitude becomes $e^{-1} = 0.368$ times of that at the surface. So the magnitude of E reduces to $0.368E_{xp}$.

Now, what is the importance of all this classical eddy current theory in the context of our course? For example, if you calculate the skin depth for various materials that we encounter in electrical machines and equipment; for copper at 50 Hz if you substitute the parameters ($f = 50\ Hz,\ \mu = \mu_0, \sigma = 4.74 \times 10^7$) of the material in the expression of skin depth, the value of skin depth is calculated as given below.



μ_r for copper is 1 because it is a diamagnetic material and $\mu_r \approx 1$ but little less than 1. For practical purposes we can take it as 1. Similarly for aluminum its value is 1.

At 50 Hz, you get 10.3 mm as the skin depth for copper. Similarly, at 60 Hz it will be 9.4 mm because it is inversely proportional to the square root of frequency. So as frequency goes up, the skin depth will drop. For aluminum, conductivity is 61% of that of copper. So the skin depth for aluminum is 13.2 mm at 50 Hz.

And then for mild steel which is commonly used as a structural element in machines and equipment, μ_r is 100 which corresponds to a saturation case. Why this case is considered as saturation? Because the skin depth is very small and if the flux that is impinging is getting concentrated in the skin depth region, the mild steel material is considered as sort of saturated and the corresponding saturation permeability is taken.

Otherwise, we know generally mild steel is a type of steel with predominant iron, the relative permeability will be around 1000 in its unsaturated case. But for analysis, we take $\mu_r = 100$ and the conductivity of this material is 7×10^6 S/m which is one order less than aluminum and copper. The corresponding skin depth for mild steel is 2.69 mm, which is much smaller than aluminum and copper. And then the thing to note is, usually for mild steel enclosures that we build, the thickness is typically 5 mm, 10 mm etc. You will never find mild steel whose thickness is 1 mm or 0.5 mm, so that it is mechanically stable.

When you use mild steel material almost all the loss will occur in the mild steel because the thickness of the plate is considerably higher than the skin depth. Consider a 12 mm thick mild steel material which is almost 5 times the skin depth. For this expression $E_{xp}e^{-\frac{z}{\delta}}$ which is exponential decay, for a material whose thickness is 5δ the expression reduces to $e^{-5} \approx 0.$ So in 5 skin depths, this value of the field will go almost to 0.

Like in circuit theory, in almost 5 time constants, we get almost final steady state value. Similarly here in about 5 skin depths, the value almost reduces to 0. So we can say that if you use a mild steel materia, almost entire loss will get absorbed in it and to reduce that loss then you may have to use some kind of shielding.

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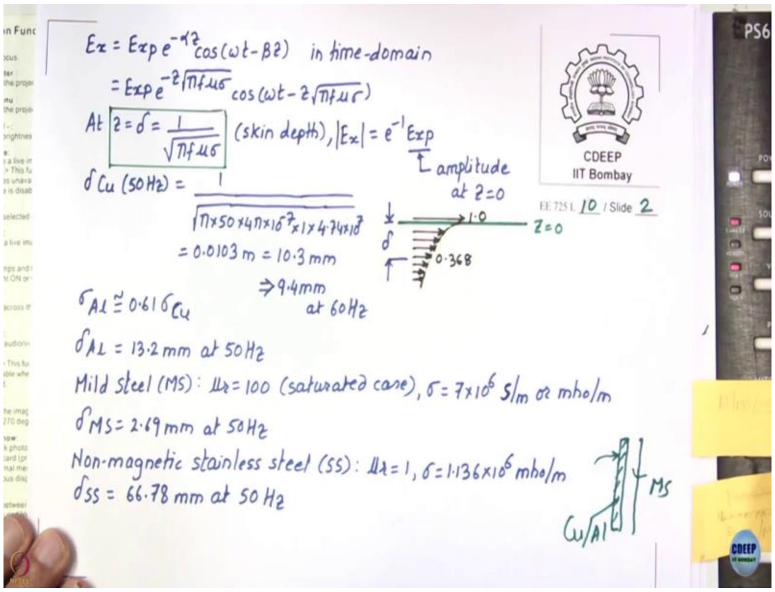

For example, if you have a mild steel plate in the vicinity of some source, which is producing field and that field is impinging on the plate causing eddy current losses in the plate. What you could do to reduce the losses? You could shield the mild steel by using aluminum or copper plate, if you use copper, it should be at least 5 mm thick. If it is aluminum, it should be around 10 mm thick. Why it should be like that? I will tell in the next slide.

The aluminum or copper plate will shield the mild steel material. This is called as electromagnetic shielding. So more about it little later. And what will happen in these materials? The eddy currents induced in these aluminum or copper plates will repel the incident field and the mild steel plate will be shielded from the incident field. So this is a practical application of such materials.

One may also use non-magnetic stainless steel in some applications. For these materials, $\mu_r = 1$ and conductivity is 1.13×10^6 S/m which is even lower than the mild steel because it is a high resistance material.

Now the skin depth for stainless steel can be calculated as 67 mm for 50 Hz by using the same formula. This is something important. That means if you shield some electromagnetic device, say transformer or any other device by enclosing it in a stainless steel tank. That stainless tank is not going to have a thickness of 70 mm. Typically the thickness will be some 10 mm, 5 mm, 15 mm. So the field is going to come out of that. Because in 1 or 2 skin depths, the field is not reducing to

0. So, field will come out of that stainless steel enclosure. If you are required to design an electromagnetically shielded product wherein your customer specifies that no field should come out of that device, you cannot use stainless steel material as enclosure because the field will come out for the practical thicknesses that are used. This concept is having a lot of practical significance. So what I am trying to say is this theory of eddy currents is very important while designing machines and equipment.

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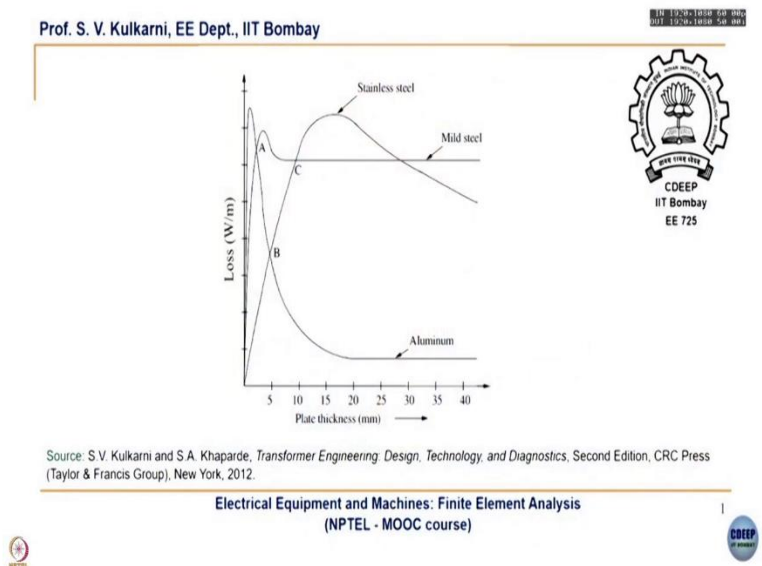

We will discuss the above slide. The figure in this slide shows the eddy loss as a function of material thickness. Now the 3 materials shown in the above slide are commonly used. Among these materials, aluminum is representing non-magnetic but highly conductive material, mild steel is conducting material but magnetic and stainless steel is high resistive non-magnetic material. So, 3 materials are representing 3 different types of characteristics. And now you can also see here in this slide the pattern of variation of loss with plate thickness is considerably different in these 3 materials.

Now, I am not going into details of this, but this has considerable influence in deciding which type of material to be used for shielding purposes, and what should be the thickness of those materials and all that. For those who are interested further in understanding the theory of eddy currents and their applications, they can refer to the book that is mentioned here at the bottom of this slide.

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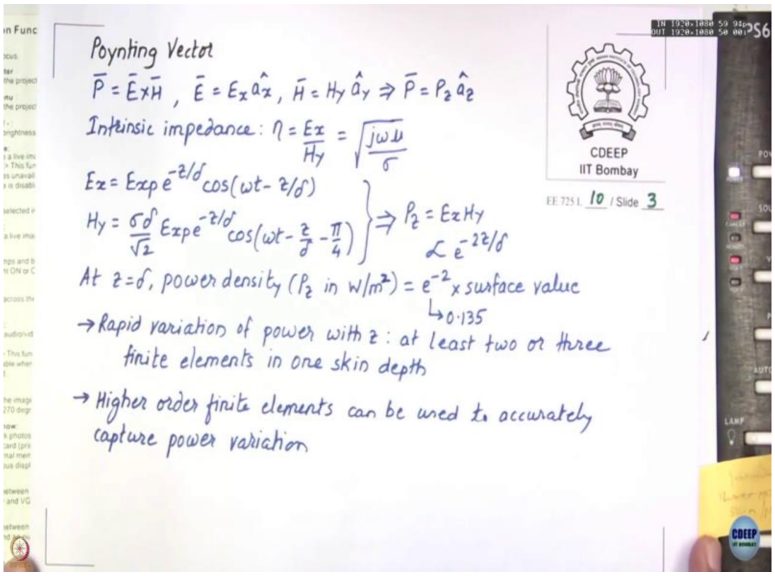

Let us go to the next slide. Now, we will talk about Poynting vector which is another important vector in electromagnetics. And it has a good application in field computations that we will do as part of this course and it is useful for even high frequency electromagnetics when we are considering wave propagation.

Poynting vector **P** is given by cross product of **E** and **H**. Now in the case when we are analyzing E fields diffusing into the metallic plate in one of the previous slides, we assumed that **E** is only in the x-direction. And if we use one of the Maxwell's curl equations, we can easily derive the solution if **E** is only in the x-direction and is a function of z.

If that **E** is substituted in one of the curl equations ($\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$), you can easily derive **H** in the y-direction as a function of z. So **H** will be $H_y\hat{\mathbf{a}}_y$ and $\mathbf{P} = \mathbf{E} \times \mathbf{H} = E_x\hat{\mathbf{a}}_x \times H_y\hat{\mathbf{a}}_y = E_x H_y \hat{\mathbf{a}}_z$**.** Then, you will get **P** in the z-direction. So basically this z is the direction of diffusion of power into the conductor.

Going further, here we also need to understand what is the definition of intrinsic impedance of a conducting medium. Generally, intrinsic impedance is defined as $\frac{\mathbf{E}}{\mathbf{H}}$. But here for the considered case of diffusion problem, it will be simply $\frac{E_x}{H_y}$ because E is in x-direction and H is in y-direction.

And then if you use the expression that we derived in the previous slide and if you refer standard books on eddy current theory, you can easily derive the following expression of intrinsic impedance as

$$\text{Intrinsic impedance: } \eta = \frac{E_x}{H_y} = \sqrt{\frac{j\omega\mu}{\sigma}}$$

And then the expressions for E_x and H_y are given below.

$$E_x = E_{xp}\, e^{-z/\delta} \cos(\omega t - z/\delta)$$
$$H_y = \frac{\sigma\delta}{\sqrt{2}} E_{xp}\, e^{-z/\delta} \cos\left(\omega t - \frac{z}{\delta} - \frac{\pi}{4}\right)$$

The expression of E_x is already derived in one of the previous slides and H_y can be easily determined by using this intrinsic impedance formula.

Similar to **E**, **H** also varies exponentially with z as given in the above equation, but there is phase difference between E_x and H_y. In case of wave propagation problems, for example, lossless case, E_x and H_y will be in phase. Although they are orthogonal in space, they will be in time phase. But for lossy medium case, there is going to be a phase difference between **E** and **H**.

Now when you calculate $\mathbf{P} = \mathbf{E} \times \mathbf{H}$ by multiplying both E and H (as cross product), the power will be proportional to $e^{-2z/\delta}$. So what is this power calculated by using the Poynting vector? This is representing eddy current loss in the metallic plate. What this variation is telling? **E** and **H** are varying with a single exponential $\left(e^{-z/\delta}\right)$, the power is varying with double exponential $\left(e^{-2z/\delta}\right)$. So for $z = \delta$, you will have power as e^{-2} times that of the surface value and $e^{-2} = 0.135$ which means almost 86% of the power will be lost in one skin depth.

So this inference tells us that there is more rapid variation of power as compared to E and H fields and that is important. If you want to capture that variation, then we have to use a very fine FE mesh. Eventually from the next lecture onwards we are going to see the finite element theory.

We will again discuss the same point when we solve a diffusion problem. So we need to have a sufficient number of elements in one skin depth, at least two or three layers of elements, if you are using linear elements. If you want to use less number of finite elements, then you will have to use quadratic approximation that is second order approximation, when you use finite element formulation.

Till now we saw basic theory of eddy currents and we derived the solution of the diffusion equation and based on that we determined the expression for skin depth and all that. Now, we will go further and we understand how do we compute eddy current losses in conductors. Now this is commonly used in electrical machines and equipment.

You have either conductors of copper or aluminum subjected to an alternating field or you may have field because of currents and leakage fields in the windings. Because of the eddy currents induced by these fields, heating of the structural parts occurs. This situation is common in static as well as rotating machines.

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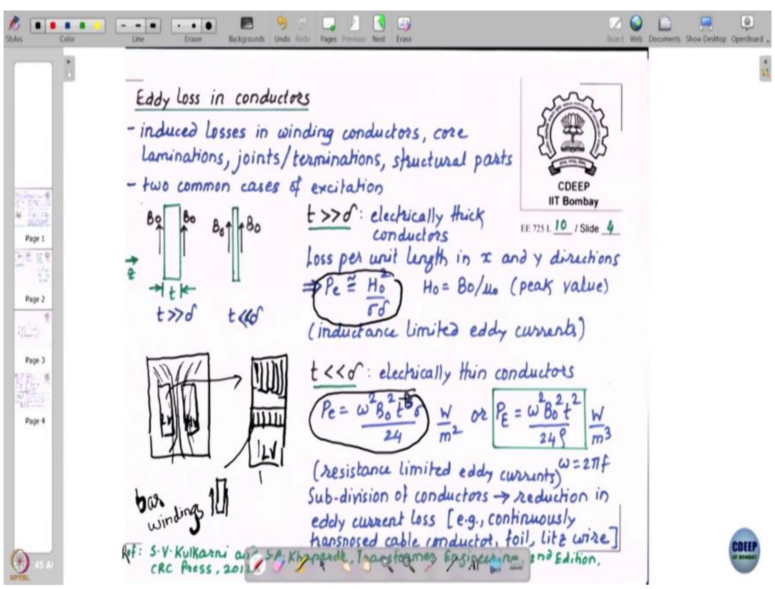
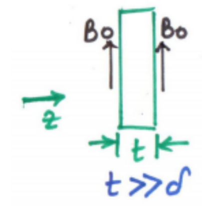


In this slide, we will derive the basic formula for induced losses in windings conductors, core laminations, joints and structural parts. There are two common cases of excitation. In the first case, the conductor is very thick and it is excited on both sides by tangential flux density (B_0) as shown in the following figure.



Here the conductor is electrically thick, since the thickness (t) is much greater than skin depth (δ). In the second case, conductors are electrically thin, because the thickness is less than skin depth.

For electrically thick conductors, loss per unit length in x and y directions with t in z-direction can be calculated by using the following expression.

$$P_e \cong \frac{H_o^2}{\sigma \delta}$$

Now the derivation of this expression is given in the reference given in the bottom part of the slide. Those who are interested can refer to this book for the derivation of this formula.

Similarly, the formula to calculate losses for electrically thin conductor is given below and the corresponding derivation is given in the same reference.

$$P_e = \frac{\omega^2 B_o^2 t^3 \sigma}{24} \quad \frac{W}{m^2}$$

In the previous equation, H_o is the peak value of magnetic field intensity which is B_0/μ_0. This will be a case of inductance limited eddy currents because the thickness is high and eddy currents will be induced. Because when the thickness is small, the conductor will have high resistance and that high resistance will limit eddy currents. But when the thickness is large, resistance is small, but inductance will be more.

So that is why the first case $(t \gg \delta)$, is the case of inductance limited eddy currents. The second case, electrically thin conductor $(t \ll \delta)$, the loss per meter square is given by the above formula. Here, σ is the conductivity and δ is the skin depth that we have already studied.

Generally in electrical machines and transformers, you would finally want eddy loss expression in terms of loss per unit volume (in W/m^3) because generally, it is easier to calculate the volume of the winding conductor or core. So that is why you further divide the above formula by t in z-direction which results in the following formula. The previous formula is in W/m^2 with per unit length in x and y.

$$P_E = \frac{\omega^2 B_0^2 t^2}{24 \rho} \ \frac{W}{m^3}$$

So when you further divide the previous equation by the dimension in the z-direction, you will get the famous formula for eddy loss per unit volume which is given by the above equation. So this is as I mentioned earlier, as the thickness of the conductor is small the resistance will be high and this will be a case of resistance limited eddy currents.

The eddy current loss can be reduced by sub-dividing the conductor. So you can divide the conductor and eventually reduce the thickness to such a value which is possible from the point of mechanical considerations. Because lesser the thickness you make, there could be other design considerations like mechanical strength and all that. So as long as the mechanical considerations permit, you can reduce the thickness, and of course, that conductor with that small thickness should be commercially available. That is why at very high frequencies, you generally do not have single strip conductors in very thin dimensions. For high frequencies, we use conductor in the form of foil, for example, a very thin foil of aluminum or copper, which will be of quite small thickness say, in the order of 0.5 mm or 1 mm.

Also, at very high frequency, for some power electronics application where frequency is something like more than 10 kHz or 50 kHz, Litz wire conductors are used. In that Litz wire, you have thousands of small strands which are continuously transposed, to reduce the circulating current loss.

See because when you use a number of parallel conductors, eddy current loss can be reduced. But if those parallel conductors do not link the same flux, then there will be circulating current loss between these parallel conductors because the inductance and impedance of these parallel conductors will not get equalized.

Therefore whenever you use a large number of conductors in parallel to reduce the eddy current loss, you have to transpose them continuously to reduce the circulating current loss.

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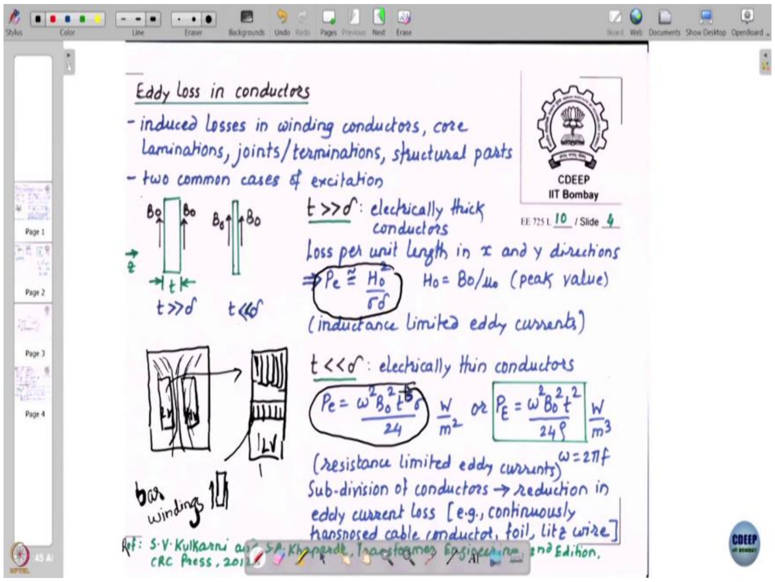
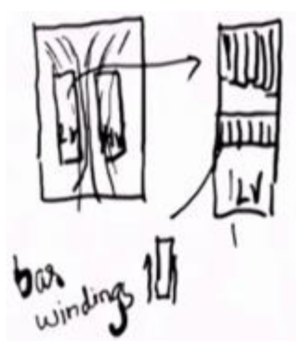


Now before going further, let us discuss the conductor configurations which are practically possible. Suppose consider the case of a transformer shown in the following figure.



In the figure on the left side, the outer boundary is the boundary of a core window, also you have 2 windings (LV and HV) and leakage field indicated with black lines as shown in the figure.

Now, you zoom the LV winding as shown in the figure on the right hand side. It is made up of conductors or turns as shown in the figure . The conductors are stacked in a disc form. Now, if you consider one particular turn of a disc as shown in the bottom of the figure and the leakage field is as directed as indicated in the figure. Now, you can see this leakage field is incident tangentially on both surfaces and this is the case what we are analyzing in this slide.

So when the thickness of the conductor is very small compared to skin depth, then you have to use the expression for the thin conductor case. Whereas when the conductor is very thick for example, in case of bar windings wherein winding is made up of very thick conductor which cannot be easily bent. Then that becomes a case of an electrically thick conductor. And then the formula to be used is that for electrically thick conductors.

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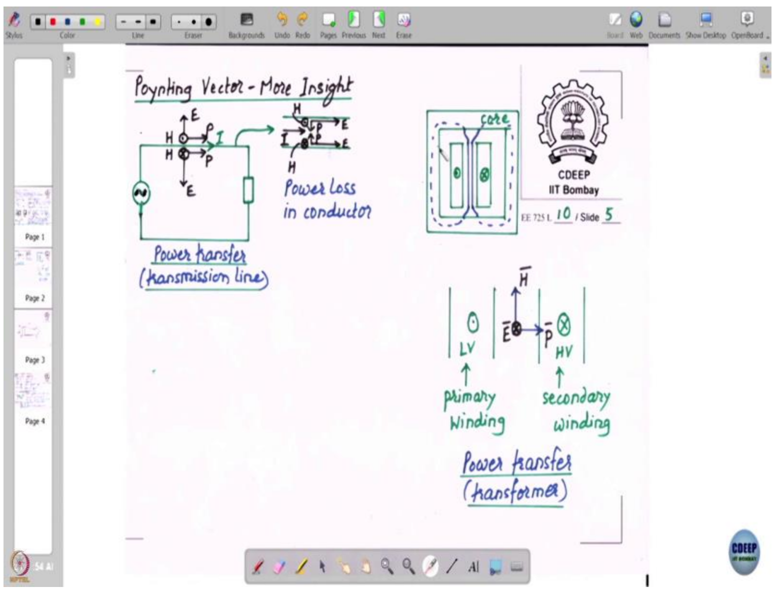
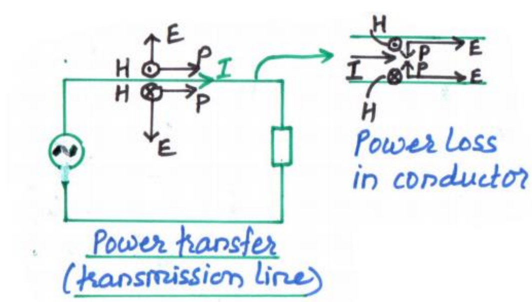


Till now we have more or less covered many topics in basics of electromagnetics. I wanted to explain Poynting vector so that some of the very common doubts that we have can be cleared. Consider a transmission line as shown in the following figure.



In transmission lines, we generally say the power flows through a conductor. The source and load are connected to the transmission line as shown in the figure. But power is not carried by the conductor, it is carried by the surrounding field generated by the transmission line. The power flows in two directions. One is along the line from source to load. But that power is carried by the E and H fields indicated in the figure on the left side. In this case, there is an E field vertically down between the two conductors of transmission line and the H field is shown by a dot and a cross because current is flowing in the direction as indicated in the figure.

Now since, E is vertically down, and H is into the plane the vector $\mathbf{P}$ $(= \mathbf{E} \times \mathbf{H})$ will be in the direction as indicated in the figure. In the second case, E is vertically up and it will be normal to the conductor because E is always normal to conductor surface and this we have seen earlier. So at a point on the conductor surface, E will be normal and its direction changes immediately to reach the ground plane.

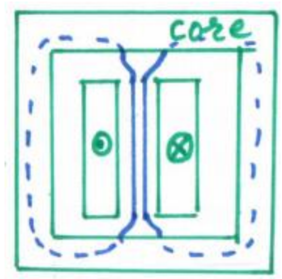
So now E and H reversed their directions. So, H is coming out of the plane and E is vertically up. Again the direction of $P(= \mathbf{E} \times \mathbf{H})$ which can be determined by applying right hand rule is again from source to load. So here, who is carrying the energy and the corresponding power? It is basically carried by the E and H fields which is all in this space around the transmission line. So the E and H fields together with the corresponding cross product (= **P**) is carrying the energy. So the fields carry the energy.

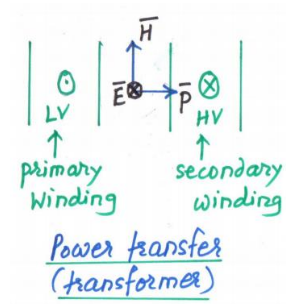
Now, you may wonder then what is this current doing inside the conductor. So, let us zoom the conductor as shown in the inset of the above figure. Now, here the E field is in the horizontal direction which is due to the voltage drop across the conductor because of its finite conductivity. And then there will be corresponding electric field intensity. So that electric field intensity is now in the direction opposite to the voltage drop (direction of current flow). And the current is still in the same direction and the corresponding H fields within the conductor are shown in the inset. Earlier we were considering H fields outside the conductor, in its vicinity. Here H fields are considered inside the conductor.

Now again, if you calculate $\mathbf{P} = \mathbf{E} \times \mathbf{H}$, its direction is vertically down for the horizontal component of E and H which is coming out of the plane, P will be down. And for the other two components of E and H, P will be vertically up. So these two vertical components of P are representing conductor loss because of its finite conductivity.

So what we understood here is the 2 sets of components of E. One set of E field is vertical because of the potential difference between the top conductor and the bottom conductor. And the other E field will be considered along the conductor and it is representing the voltage drop.

And this clearly explains that the power and corresponding energy is carried by the fields in the space surrounding this conductor. So, the space surrounding the transmission line is carrying the power from the source to the load. Let us see another example of a transformer shown in the following figure.





Now this is a transformer having LV and HV windings with current directions coming into and out of the plane and the corresponding leakage field will be as shown the above figure.

Now actually you zoom a part of the region between the two windings as shown in the figure on the right side. The leakage field in the gap between the two windings is vertically up as shown in the figure. So H field is vertically up in the gap between the LV and HV windings. E in the region near the HV winding is into the paper because the direction of current is coming out of the plane.

And remember that E is not always necessary to be associated with a conductor. By Maxwell's equation $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ there will be curling of E field in the whole space surrounding the $\frac{\partial \mathbf{B}}{\partial t}$ due to the leakage field. So that is why there will be E field in the winding space between the 2 windings.

So near the LV winding, E is into the paper and H is vertically up. If you calculate $\mathbf{P} = \mathbf{E} \times \mathbf{H}$ for this case, the power flows from LV to HV. Transformer core is responsible for electromagnetic induction. Core is efficiently inducing voltages in LV and HV windings because without core also you would have voltage induced in the secondary winding. Only thing is it will not be efficient.

Both E and H fields are responsible for power flow. We can understand E field using Faraday's law $\left(\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}\right)$. The corresponding H field in the gap will come into picture if current starts flowing in the secondary and then only power will be transferred to the secondary from the primary.

So, moment the secondary is loaded, you have the corresponding leakage field here as shown in the above figure. And then in the whole space of the windings wherever this leakage field is there, $\mathbf{E} \times \mathbf{H}$ will result in power flow from primary to secondary.

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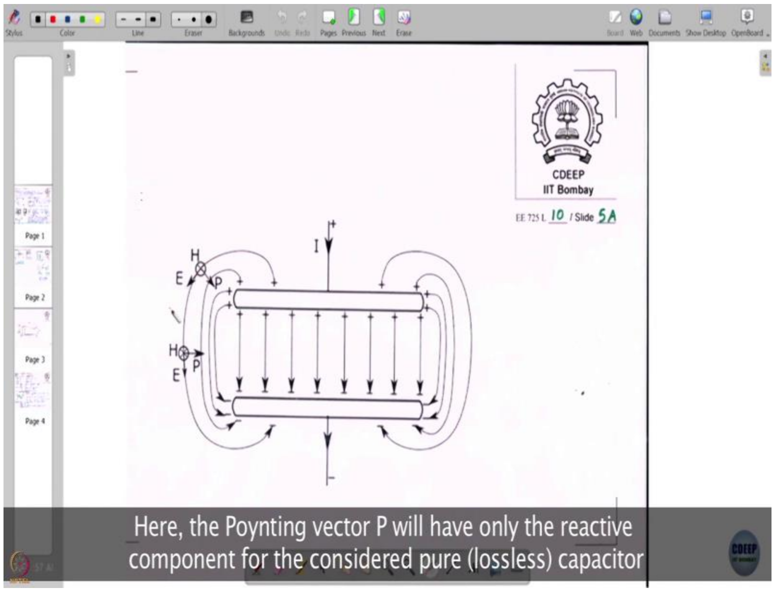

Now we will see another example to understarding the power flow while charging a capacitor using Poynting vector. Now the capacitor shown in the above slide is charged by some DC source. And then capacitor plates get charged as shown in the slide. Now carefully observe. Earlier also I told, the charges are marked with specific logic. Here you can see all the charges are more or less equally spaced in the middle region.

At the corners of the plates, they are concentrated because electric field intensity is high at the ends because of sharp corners. On this right side of the top plate also there will be charges as shown in the slide. We have been showing capacitor plate as a thin line and we show charges only on one side. But practically charges will be on the top surface as well as the bottom surface of each plate. But on the top surface they will be sparsely placed because you have more columbic force in the inner region of the capacitor (between the plates).

That is why the positive and negative charges will be concentrated on the inner surfaces of the two plates. Here only few charges are shown on the top surface of the positively charged plate and the bottom surface of the negatively charged plate. And there will be E field lines from the top surface of the upper plate to the bottom surface of the lower plate. Now, you consider that the capacitor is being charged. We may tend to say that the power and the energy is flowing into the capacitor in the direction of current (vertical direction). But actually the direction of power flow can be analysed by using the Poynting vector concept.

Consider a point in the above figure which is outside the capacitor and E is vertically down, H is into the paper for the indicated direction of current. This gives the direction of P as going into the space between the plates. So energy is being fed into the capacitor through the fields in the whole space in the vicinity of the capacitor. Now on the right side of the capacitor, E is in the same direction and the direction of H is reversed. So here H will be the coming out of the plane and that is why power direction will be reversed but still the power is directed into the region between the capacitor plates. I hope this makes the things clear and we will go further.

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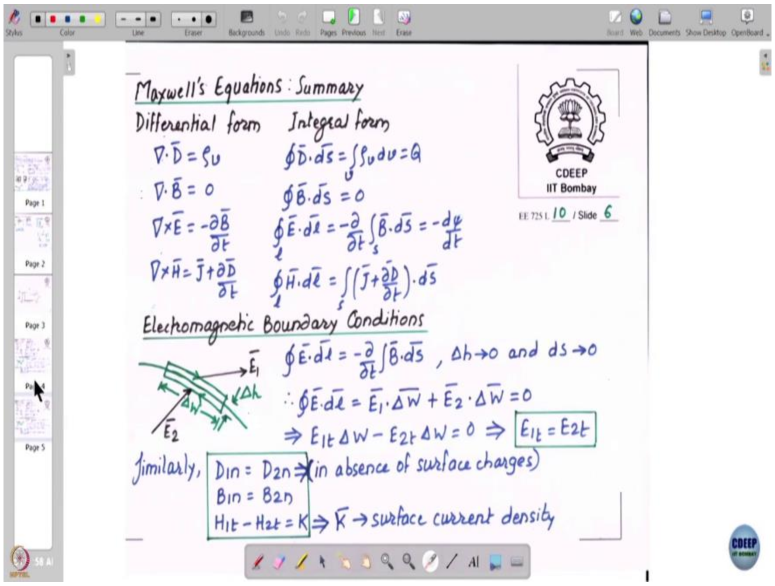
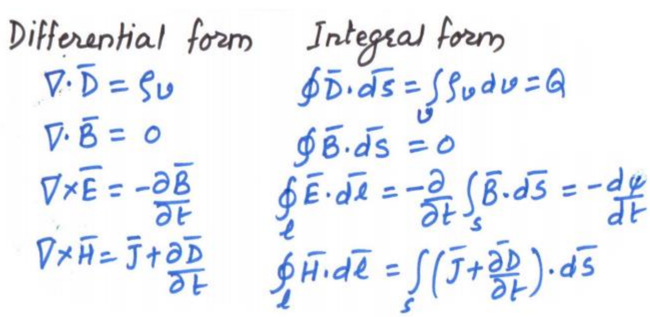


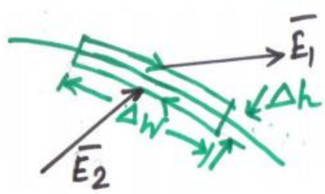
The above slide is the last slide of basics of electromagnetics module. Here in this slide, we are summarizing Maxwell's equations. Maxwell's equations are written in differential as well as in integral form as shown below.



From time to time we converted the differential form of equations into integral form as listed above.

Now one question about electromagnetic boundary conditions and the time varying fields may come into our mind. We initially saw boundary conditions for electrostatic fields and magnetostatic fields. Now, we will see what happens to boundary conditions when time-varying terms like $\frac{d\psi}{dt}$ or $\frac{\partial D}{\partial t}$ terms come into picture.

Although we did not derive any of the boundary conditions earlier, for understanding the contribution of the time-varying terms, I will just derive the boundary condition for tangential components of the electrical field intensity. Now, if you refer any standard textbook on electromagnetics, in that a boundary between the two materials will be considered as shown in the following figure.



Consider a contour at the interface as shown in the figure and you take that contour such that Δh is very small. The corresponding area $d\mathbf{S}$ enclosed by the contour will be in the plane of the paper.

And since $\Delta h \to 0$, $d\mathbf{S}$ also will tend to 0 because if you assume the contour as a rectangle, Δh is one of its sides. So, as $\Delta h \to 0$, $d\mathbf{S}$ also will tend to 0. Also, we have seen earlier that boundary conditions are derived by using the integral form of the equation.

Differential form of equations would not help because they model what is happening at a point in space. But here we need to integrate something along the surface, so you have to use the integral form. Now consider the integral form of Faraday's law which is given below.

$$\oint \bar{E}\cdot d\bar{l} = -\frac{\partial}{\partial t}\int \bar{B}\cdot d\bar{s} \quad , \Delta h \to 0 \text{ and } ds \to 0$$

Now here $d\mathbf{S} \rightarrow 0$, so that is why the time directive term on the right hand side of the equation does not contribute to boundary condition. The above contour integral will reduce to the following equation.

$$\therefore \oint \bar{E}\cdot d\bar{l} = \bar{E}_1\cdot\Delta\bar{W} + \bar{E}_2\cdot\Delta\bar{W} = 0$$
$$\Rightarrow E_{1t}\Delta W - E_{2t}\Delta W = 0 \Rightarrow \boxed{E_{1t} = E_{2t}}$$

So you get $E_{1t} = E_{2t}$ which is derived as shown above.

The negative sign in the above equation is because tangential component of $\mathbf{E}_2$ is opposite to the $\Delta\mathbf{W}$ and E1 is along $\Delta\mathbf{W}$. So that is why you get $-E_{2t}\Delta W$. So that is how you finally get $E_{1t} =$

E_{2t}. Just for the sake of completeness, I derived this, so that you can also understand that the time-varying term does not contribute in deciding the boundary condition.

And then using the same concepts, the condition $D_{1n} = D_{2n}$ can be derived by using pillowbox at the interface and then you do the surface integration and all that. For this, you can refer any standard textbook on electromagnetics. And remember this is in absence of surface charges. Similarly, the following equations can be derived, which were stated previously when we studied magnetostatics.

$D_{1n} = D_{2n}$ ⇒ (in absence of surface charges)

$B_{1n} = B_{2n}$

$H_{1t} - H_{2t} = K$ ⇒ $\bar{K}$ → surface current density

In magnetostatics, we equated $K = 0$ because that time the surface currents were not considered as they will generally appear at high frequencies. Now, since we have studied the skin depth and theory of eddy current, now we know that as the frequency increases, the skin depth will become smaller and smaller and in the limit, it will tend to be a surface current.

So in such cases it is very much relevant to consider surface currents by K and in that case, $H_{1t} - H_{2t} = K$. And here K is the only magnitude because this H_{1t} and H_{2t} are magnitudes. So I think with this, we have completed the basics of electromagnetics. In about 4 hours we have completed the basics of electromagnetics.

And as you might have noticed they are not just simple basics as described in any textbook but we understood the relevance of the covered theory in terms of finite element analysis. In general, the relevance was there in most of the cases. So I think we are all set to get into finite element analysis of machines and equipment from the next lecture. Thank you.

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