# Fundamentals of Power Electronics
### Prof. L. Umanand
### Department of Electronics Systems Engineering
### Indian Institute of Science, Bengaluru

## Lecture - 68
## Transformer design

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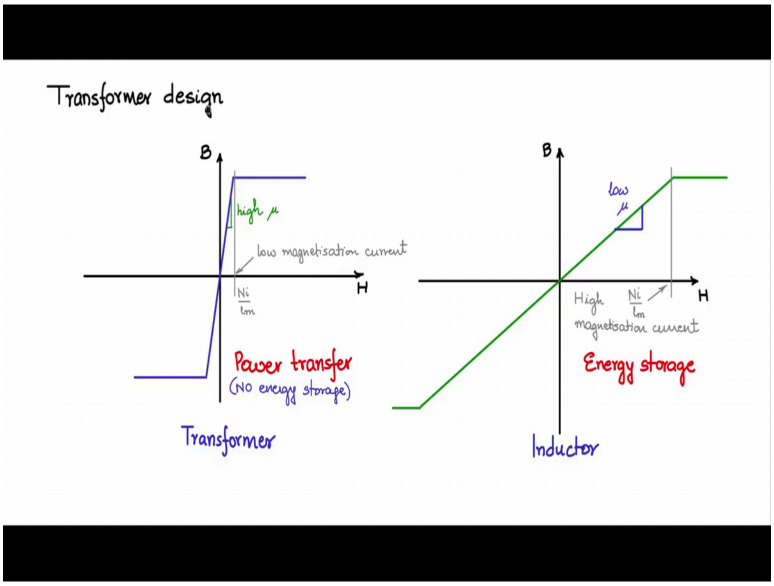

Let us now apply our magnetic design knowledge to the design of transformers. Essentially the transformer and the inductor will appear very similar physically it will also have a core it, will have winding, former all those things. What essentially is the difference is in the BH characteristics. So, let me just point out to you the main differences in the BH characteristic of the transformer that of the inductor.

So, let us say we have a BH graph here for the transformer and let me have another graph here BH graph here for the inductor. So, in the inductor the main focus main job is to store energy. So, energy storage is the main issue in the inductor we want to store as much energy as possible. In the case of the transformer it is power transfer instantaneous power transform of transfer from the primary side to the secondary side. So, this is the main job in the case of the transformer, no energy storage.

So, in the case the inductor, in order to have as high energy storage as possible we discussed that we would like to have the mu the permeability as low as possible. So, it is designed the core is designed to have as low mu as possible. So, that you have very high magnetisation current here. The magnetisation current N i by l m, the i part here should be very high. So, that the energy that is stored in the core maximum energy that is stored in the core is half l i m square. So, more energy get stored because the energy is proportional to the magnetisation current.

Whereas in the case of the transformer we are not talking in terms of energy storage; there is no energy, we do not want energy storage. It is only instantaneous power transfer. So, we would like the mu permeability to be as high as possible. So, if you look at a transformer core BH curve it will be like this very high slope; for the inductor very low slope and because of this high slope you will see that the magnetisation to reach the same B m you will need much lesser current the N i by l m less magneto motive force or less electrical current i or magnetising current i is needed to bring the transformer core to a level operating level B m or even B sat.

So, we have to design the transformer to have low magnetisation current. So, that is the essential difference. A transformer is a power transfer device whereas, an inductor is an energy storage device and when you design we would like to design transformer with low magnetisation current whereas, in the case of inductor we have high magnetisation current so that the energy that is stored within the core is high.

So, with this fundamental and main difference let us look at the transformer again in terms of the area of products and arrive at the design equations.

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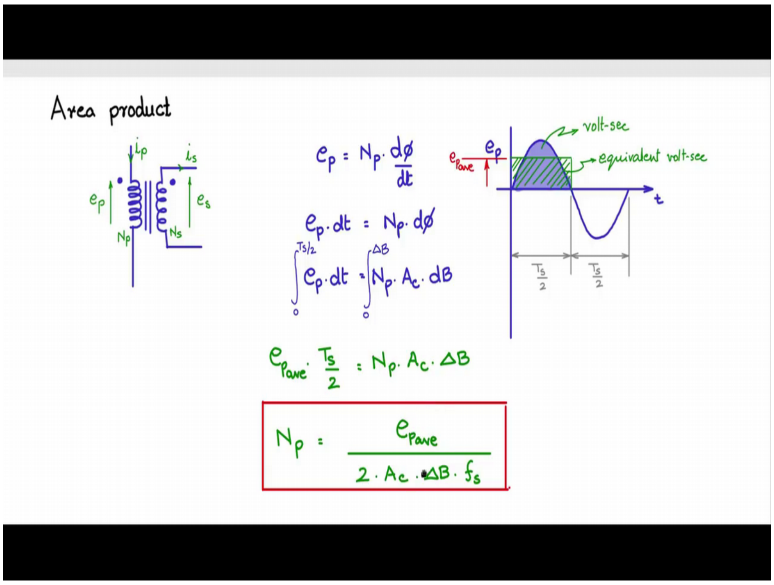

Area product

$$e_p = N_p \cdot \frac{d\phi}{dt}$$

$$e_p \cdot dt = N_p \cdot d\phi$$

$$\int_0^{T_s/2} e_p \cdot dt = \int_0^{\Delta B} N_p \cdot A_c \cdot dB$$

$$e_{P_{ave}} \cdot \frac{T_s}{2} = N_p \cdot A_c \cdot \Delta B$$

$$\boxed{N_p = \frac{e_{P_{ave}}}{2 \cdot A_c \cdot \Delta B \cdot f_s}}$$

Let us now discuss the area product relation for the transformers. Now, consider a general generic transformer which has a primary coil and the secondary coil like this as shown, we have the dot polarities. Primarily, the voltage e p measured in this fashion, secondary potential emf measured e s in this fashion. Primary has N p number of turns, secondary as N s number of turns primary current i p flows in this fashion and secondary current i s as shown like this.

So, if I take the primary induced voltage e p, so, it is given by Faraday's law N p into d phi by dt d phi phi is the flux in the core here. Now, the voltage here it is not only the voltage,it is the volt-second that we have to be bothered about. So, let us take some arbitrary wave shape.  Let us say we have sine wave, this is the sine wave transformer. Let us say the voltage is sinusoidal in nature. So, this is the halfway mark here. This is T s by 2 and this is T s by 2. So, if it is supply frequency 10 milli second and 10 milli second if it is switching frequency whatever the switching frequency that switching period by 2 and switching period by 2.

Now, we are not that much interested in the wave shape as we are interested in the area under the curve. Especially, for design of the transformers we are bothered about the volt-second the area under the voltage profile this one. So, this volt second we need to be worried about. Now, this volt-second and equivalent square wave volt-second here let us say we have equivalent square wave flat top both these will have the same volt- second let us say.  Then what is the; what is this flat top value that will be the average of this particular wave shape.

So, whatever be the wave shape whether be the sine wave, whether it be a square wave whether it be any other or be triangle wave or any other arbitrary wave shape take the average of that and for the half period it will be equivalent to square wave with the flat top and where the height is nothing but e primary average because the volt-second is the same in all these cases. And, this average remember is not the average of the entire cycle the average of the entire cycle is zero because that is very important the average of the entire cycle in the steady state for across any coil will always be zero because volt-second balance will be there you are taking the average for half the cycle. This is the half cycle average keep that in mind.

Now, let us see how we are going to get that dt, we will integrate e p with respect to dt and this also with respect to dt which becomes d phi. Then e p with respect to dt and N p phi can be written as A c into flux density. So, therefore, area of cross section of the core in to dB.;now, integrate for half the period only 0 to T s by 2; 0 to T s by 2. So, in T s by 2 time how much will the flux density vary? It will vary from minus B m to B m in most cases, but there are cases like forward transformer where it will vary from 0 to B m.

We will say 0 to delta B and then we can make delta B has 2 B m or B m accordingly depending upon the topology use. So, let us say some delta B m which is the variable quantity. So, the in T s by 2 in half a period there is a change in the flux from 0 to delta p. Now, on integration you will see that this is a square e p average into T s by 2 and this is N p into A c into delta B.

So, now rearranging you will get N p is nothing but e p average, half cycle average remember that, 2 then you have A c here core cross section area coming down delta B and the T s will be written as f s; T s is in the numerator will make it as f s.

Now, this is the relationship between the turns and the half cycle average of the voltage across the primary or any coil A c delta B and f s.

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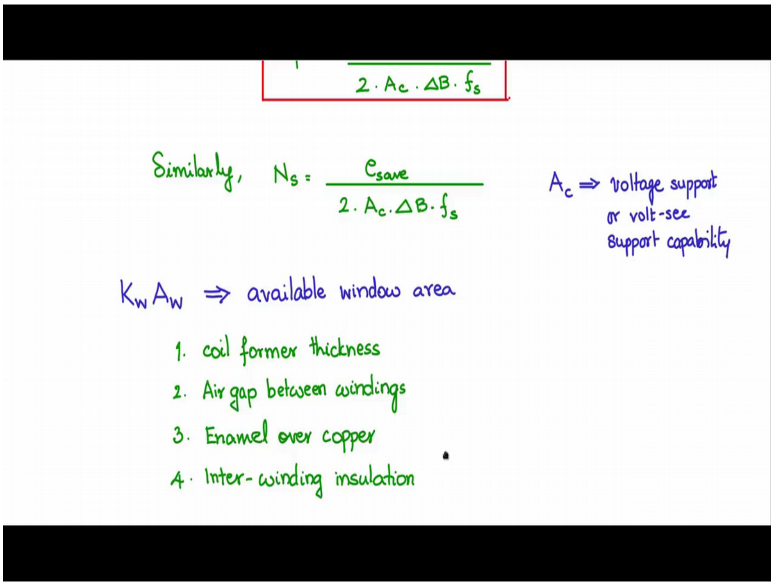

$2 \cdot A_c \cdot \Delta B \cdot f_s$

Similarly, $N_s = \dfrac{e_{s\,ave}}{2 \cdot A_c \cdot \Delta B \cdot f_s}$

$A_c \Rightarrow$ voltage support or volt-sec support capability

$K_w A_w \Rightarrow$ available window area

1. coil former thickness
2. Air gap between windings
3. Enamel over copper
4. Inter-winding insulation

Likewise for the secondary also you can write it down N s is nothing but secondary half cycle average e s average 2 A c delta B within the core into f s. So, you see that there is now a relationship between A c the core cross section area and the volt-second are the voltage. So, A c gives an idea of the voltage supporting capability of the volt-second supporting capability for the transformer.

Next, let us talk about the window area K w A w is the available window area; just like in the case of the inductor we have loss of window area due to coil former thickness. We have to insert coil former into the core and then the thickness of the coil former that amount will not; will go unavailable for the windings. So, that much amount decrease in the window area is there. Not only that when you have circular cross section coils one there is always air gap in between the coil sections and that will eat away some area.

Apart from that we have seen that enamel over the copper; copper cannot be just pure copper because copper will come in contact with the next turn and there will be short circuiting of the turn therefore, you need to insulate and therefore, copper is normally available as enamelled copper there is insulation which acts as an insulation and avoid short circuit between coils.

Now, this enamel has a finite area and that will also eat into the available window area. Now, in the case of the transformer there is another additional loss of area and that is due to inter winding insulation. Normally transformers at least minimum two windings are there; one primary, one secondary and in some other applications you may have multiple secondarys in which case for every winding there has to be an insulation between that winding and the one previous previously wound.

So, this inter winding insulation can eat up a significant amount of area. So, the available window area is much lesser than that for an inductor.

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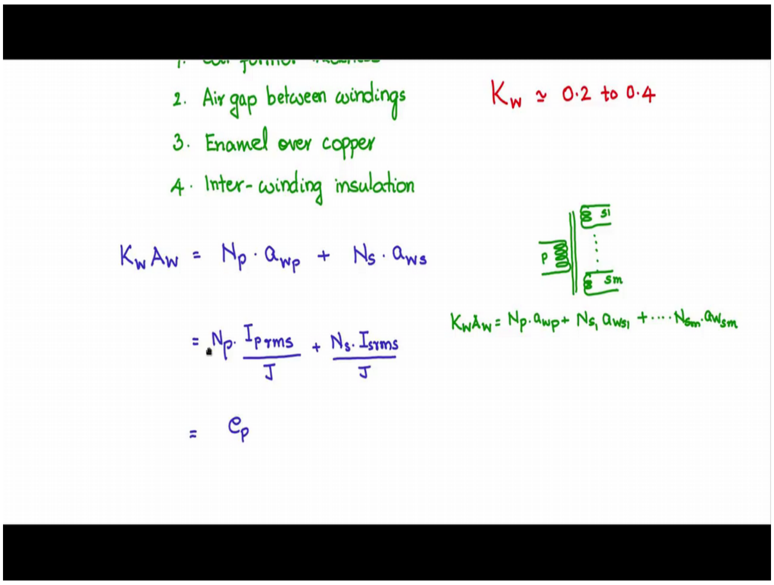

2. Air gap between windings
3. Enamel over copper
4. Inter-winding insulation

$K_w \simeq 0.2$ to $0.4$

$$K_w A_w = N_p \cdot a_{wp} + N_s \cdot a_{ws}$$

$$= N_p \cdot \frac{I_{prms}}{J} + \frac{N_s \cdot I_{srms}}{J}$$

$$= e_p$$

$$K_w A_w = N_p \cdot a_{wp} + N_{s_1} a_{ws_1} + \cdots N_{sm} \cdot a_{wsm}$$

So, K w is will vary between 0.2 to 0.4. In the case of the inductor K w vary between 0.4 and 0.6, but in the case of transformer because of the additional loss of area the K w or the window factor vary between 0.2 to 0.4 and no wise unskilled winder should use 0.2 and very skilled winder will use 0.4.

So, therefore, the available window area has to be used to accommodate N p primary number of turns into the wire cross section of the primary wire plus N s number of secondary turns plus wire cross section of the secondary wires; if it is just a primary and the secondary, but in the general case there can be multiple secondarys. You can have a primary, secondary 1, secondary 2 so on to secondary m. In such a case K w A w should accommodate N p wire cross section area of primary plus N s 1 wire cross section of secondary 1 plus N s m into wire cross secondary m so on.

So, that is something that let us now take just these two primary and secondary and we see that N p and wire cross section of the wire can be calculated from the rms current flowing through that let us say I p rms by the current density J, just like as we discussed in the case of the inductor windings. And, secondary you can calculate N s into I s rms by J.

So, now N p we saw N p and N s we have a relationships here with respect to the half half cycle primary average voltage and half cycle primary secondary average voltage with respect to the other parameters core cross section area. We will replace it here accordingly.

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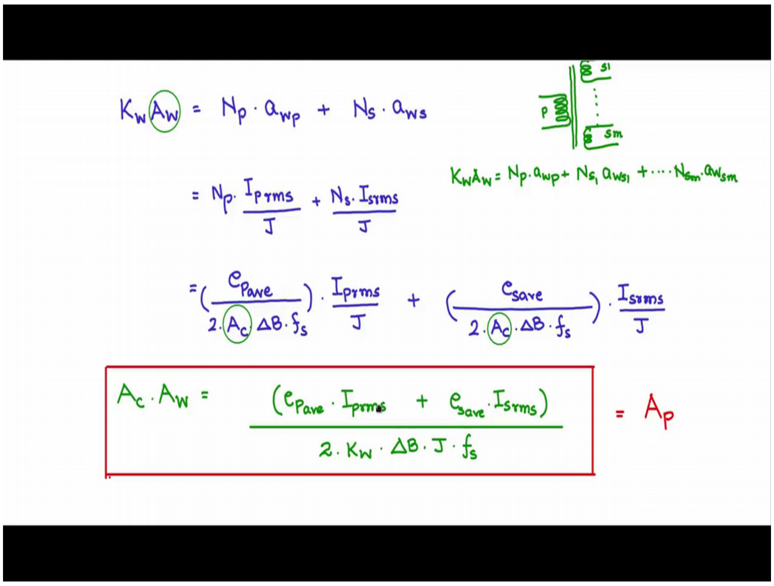

$$K_w A_w = N_p \cdot a_{wp} + N_s \cdot a_{ws}$$

$$= N_p \cdot \frac{I_{prms}}{J} + N_s \cdot \frac{I_{srms}}{J}$$

$$= \left(\frac{e_{Pave}}{2 \cdot A_c \, \Delta B \cdot f_s}\right) \cdot \frac{I_{prms}}{J} + \left(\frac{e_{save}}{2 \cdot A_c \cdot \Delta B \cdot f_s}\right) \cdot \frac{I_{srms}}{J}$$

$$K_w A_w = N_p \cdot a_{wp} + N_{s_1} a_{ws_1} + \cdots N_{sm} \cdot a_{wsm}$$

$$\boxed{A_c \cdot A_w = \frac{(e_{Pave} \cdot I_{prms} + e_{save} \cdot I_{srms})}{2 \cdot K_w \cdot \Delta B \cdot J \cdot f_s}} = A_p$$

So, e p average by 2 A c delta B f s I p rms by J plus for N s I will put e s average half cycle average 2 A c dealt B f s into I secondary rms by J.

So, now you see here A w, A c, A c; now let me bring A w and A c as a product to one side and then put all the remaining terms. So, you see that the denominator here 2 A c delta B f s J, 2 A c delta B f s J, take it out common. A c and A w we keep it on one side, K w bring it down. So, you will see 2 and K w I am bringing it down delta B J f s all this come in the denominator in the numerator summation of the half cycle averages and the rms currents.

So, e p average is the half cycle average into I p rms e s average half cycle average into I s rms. So, this is the generic equation for the area product. If there are multiple secondarys like here so on it will the half cycle average of that across that particular winding and the rms current through that winding will come depending upon as many windings that are there.

Now, this is an important relationship a general area product relationship for any given transformer and we just have to calculate e p average and I p rms and delta B for specific application of the transformers and you will get the area product for that specific application.

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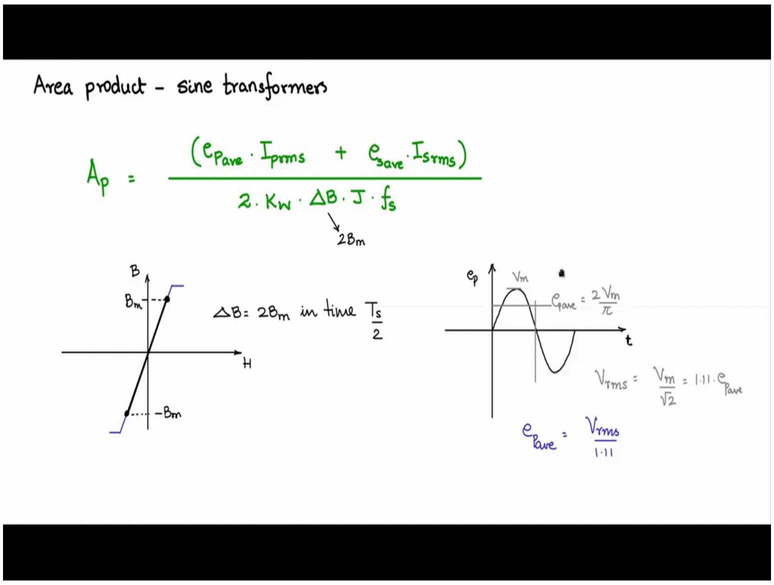

Area product - sine transformers

$$A_p = \frac{(e_{P_{ave}} \cdot I_{prms} + e_{S_{ave}} \cdot I_{srms})}{2 \cdot K_W \cdot \underset{\searrow 2B_m}{\Delta B} \cdot J \cdot f_s}$$

$\Delta B = 2B_m$ in time $\frac{T_s}{2}$

$e_{P_{ave}} = \frac{2V_m}{\pi}$

$V_{rms} = \frac{V_m}{\sqrt{2}} = 1.11 \cdot e_{P_{ave}}$

$e_{P_{ave}} = \frac{V_{rms}}{1.11}$

Let us find out what is the area product for sine transformers from the generic area product relationship that we just developed. The area product generic area product is given in this fashion we have here on the primary side e p average half cycle average I p rms, e secondary half cycle average I s rms and the denominator portion given by the winding factor delta B J and f s. We need to find out for sine transformers what is the e p average e s average and delta B.

So, let us look at delta B first. If you take the BH characteristic and let me say that a typical BH characteristic something like that one this is plus B sat minus be sat and let us say the transformer flux. The flux in the transformer course swings from positive B m value to a negative B m value minus B m and that is plus B m.

So, which means total swing, so, for every cycle it will go minus B m to plus B m and then back minus B m which means in a T, T s by 2 half the cycle it would traverse minus B m to plus B m or plus B m to minus B m. So, at delta B of 2B m B m minus of minus B m 2B m swing it will undergo in T s by 2 time. So, this portion gets replaced with 2B m.

Next so, that is 2B m. Next for the averages here if you take a sine wave typically if you take off think of the half cycle; the half cycle average if this is V m; V m sine omega T average e p average for the half cycle will be V m by pi and we know the rms value is V m by root 2 and the relationship between rms and the half cycle average will be 1.11

times the average. So, therefore, e p average will replaced by V rms by 1.11. Likewise the secondary average by V secondary rms by 1.11, so, we can take that out common.

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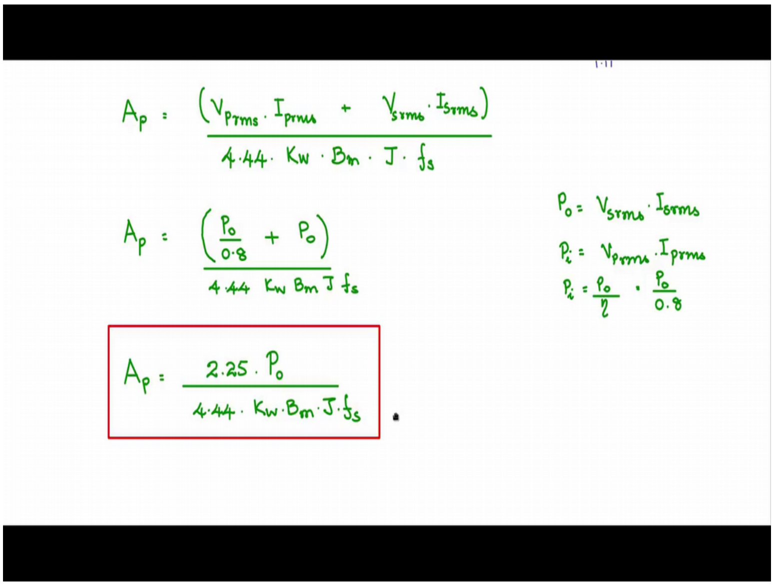

1·11

$$A_p = \frac{\left(V_{Prms} \cdot I_{prms} + V_{srms} \cdot I_{srms}\right)}{4{\cdot}44 \cdot K_W \cdot B_m \cdot J \cdot f_s}$$

$$A_p = \frac{\left(\frac{P_o}{0{\cdot}8} + P_o\right)}{4{\cdot}44\ K_W\ B_m\ J\ f_s}$$

$$\boxed{A_p = \frac{2.25 \cdot P_o}{4{\cdot}44 \cdot K_W \cdot B_m \cdot J \cdot f_s}}$$

$$P_o = V_{srms} \cdot I_{srms}$$

$$P_i = V_{prms} \cdot I_{prms}$$

$$P_i = \frac{P_o}{\eta} \cdot \frac{P_o}{0{\cdot}8}$$

So, let us make that substitutions. So, A p is V primary rms into I primary rms plus V secondary rms into I secondary rms divided by 4.44. How does 4.44 comes is 1.11 because this is V rms by 1.11. This is another V secondary rms by 1.11 that will get taken common come down which will come 2.22. delta B is having 2B m, so, it becomes 4.44 and K w, this is B m J f s.

Now, V primary rms and V secondary V primary rms into I primary rms is the power on the primary side. V secondary rms I secondary rms is the power on the secondary side, we will call this power has P naught. So, for the transformer this is the P naught. V secondary rms into I secondary rms all sinusoidal quantities and P i is V primary rms and I primary rms, P i we can say is P naught by efficiency.

And, generally efficiency of transformers are much higher than 80 percent; if we take a conservative value worst case value of 80 percent then we will be rating the primary for P naught by 0.8 and that is very very conservative. Generally the efficiency of the transformers are much higher and therefore, if we design for a primary side V primary rms and I primary rms of P naught by 0.8 we will definitely be on the safe side.

So, therefore, A p will become P naught by 0.8 plus P naught divided by the denominator

K w B m J f s and if you simplify that becomes 2.25 into P naught divided by 4.44 into K w B m f s. Now, that becomes the area product for sinusoidal transformers. Can be used for low frequency operation for where you are using for rectifier capacitor filter power supplies, where you have to step down 230 volts to 18 volts. You can use this relationship for the design of those transformers because they are sinusoidal operated where f s will become 50 hertz you have to use CRGO – Cold Rolled Grain Oriented silicon steel as the core material which has a B m of one 1.1 Tesla.

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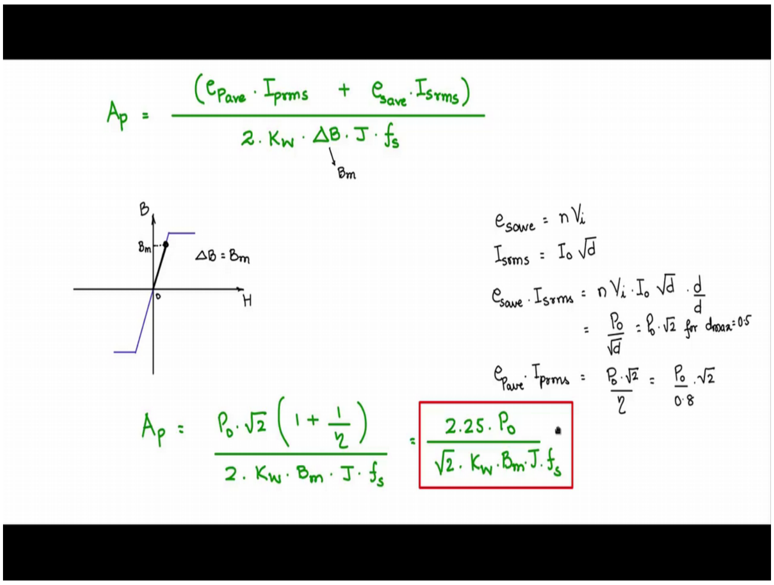

$$A_p = \frac{(e_{Pave} \cdot I_{prms} + e_{save} \cdot I_{srms})}{2 \cdot K_W \cdot \underset{\substack{\downarrow \\ B_m}}{\Delta B} \cdot J \cdot f_s}$$

$$e_{save} = n V_i$$

$$I_{srms} = I_o \sqrt{d}$$

$$e_{save} \cdot I_{srms} = n V_i \cdot I_o \sqrt{d} \cdot \frac{d}{d}$$

$$= \frac{P_o}{\sqrt{d}} = P_o \cdot \sqrt{2} \text{ for } d_{max} = 0.5$$

$$e_{Pave} \cdot I_{prms} = \frac{P_o \cdot \sqrt{2}}{\eta} = \frac{P_o}{0.8} \cdot \sqrt{2}$$

$$A_p = \frac{P_o \cdot \sqrt{2}\left(1 + \frac{1}{\eta}\right)}{2 \cdot K_W \cdot B_m \cdot J \cdot f_s} = \boxed{\frac{2.25 \cdot P_o}{\sqrt{2} \cdot K_W \cdot B_m \cdot J \cdot f_s}}$$

Now, let us take another example and that is of a switched mode DC-DC converter. We have studied and discussed the forward converter. Let us try to find the area product of the forward converter transformer. We use the generic A p formula; A p e primary average half cycle I p rms plus e secondary average half cycle I secondary rms by 2 K w delta B J and f s.

First delta B; in the case of the forward transformer we know from the BH alright. The BH curve. So, you have the typical BH curve of the core. The swing in the B is from 0 to B m only because the case of the forward converter the forcing voltage is applied only in one direction, unidirectional and therefore, it will swing from 0 to B m and B m to 0. So, in T s by 2 time maximum it will swing from 0 to B m and therefore, delta B is nothing, but B m. So, this will be B m.

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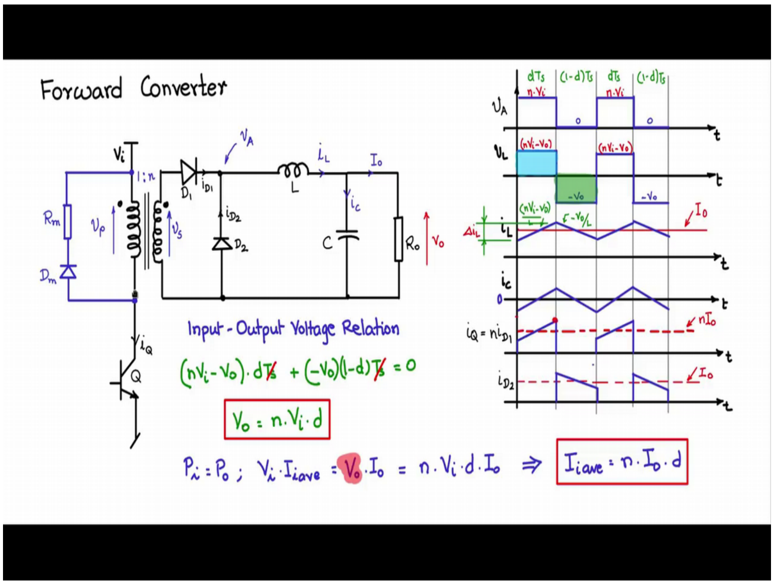

Forward Converter

Input - Output Voltage Relation

$(nV_i - V_o) \cdot d\cancel{T_s} + (-V_o)(1-d)\cancel{T_s} = 0$

$\boxed{V_o = n \cdot V_i \cdot d}$

$P_i = P_o \; ; \; V_i \cdot I_{iave} = V_o \cdot I_o = n \cdot V_i \cdot d \cdot I_o \Rightarrow \boxed{I_{iave} = n \cdot I_o \cdot d}$

So, let me show you again the way from that we had drawn for the forward converter, let us recall that. Observe this is the forward converter also the voltage across the coils forward converter transfer primary and the secondary. When the switch is on the voltage across V p is V i flat and across the secondary it will be n times V i and when the switch is off of course, the freewheeling power portion will come and the area will be same as V i into dT s during the secondary during the portion of the time 1 minus dT s.

So, I will use that equation that during the time when the switch is on and let us say for the forward converter the maximum on time or maximum dT s or d can be 0.5. So, during that time T s by 2 when this is on you will see the voltage across the primary as V i, voltage across the ah secondary will be n times V i and that would be the half cycle average whether it will be during the on time or during the off time.

So, going back here let us write it down e secondary half cycle average n V i during T s by 2; I secondary rms the current that is flowing through is I naught during that time and rms value is into root d. So, e secondary average into I s rms is nothing, but n V i into I naught into root d. Let me let me multiply numerator and denominator by d then n V i I naught n V i into d is nothing but V naught, n V i d is V naught into I naught that is P naught and root d and this root d will cancel. So, you will have in the denominator root d.

And, if you consider the maximum operating duty cycle it will d will be 0.5 and therefore, you will have it as P naught into root 2 for d max as 0.5. Likewise for the e primary half cycle average into I primary rms current you will find that it is P naught by P naught into root 2 by efficiency and if I take worst case efficiency 80 percent P naught by 0.8 into root 2 will be the e primary average and I primary average.

Let us replace it there and see what is that we get. So, P this would be; P naught root 2 I will take it out common both the cases. We will have P naught root 2 P naught root 2 in both the cases, let us take that out common and it will be 1 by efficiency plus 1 which is 2.25 and the denominator 2 K w delta B J f s. So, this portion 1 plus 1 by 0.8 is 2.25 into P naught root 2 and this 2, so, I will replace in the denominator root 2 into K w B m J f s. So, this would be the area product of a forward converter.

So, let us use this area product and do an example.

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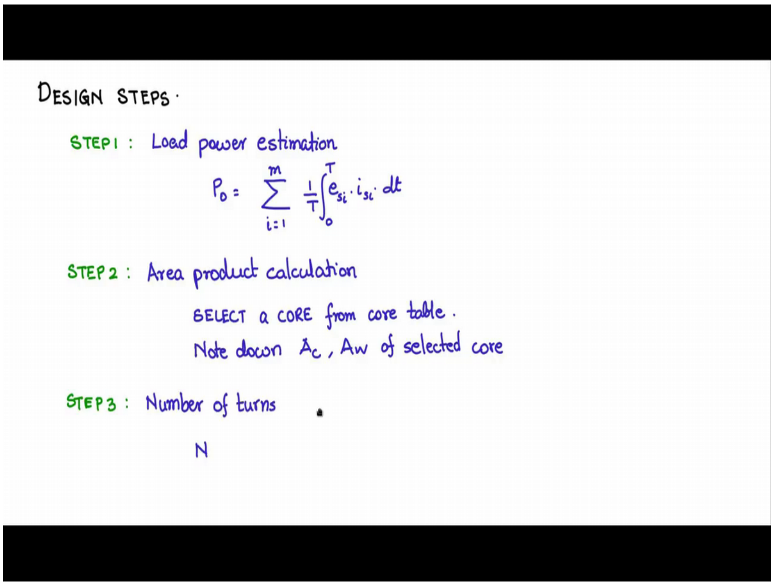

DESIGN STEPS ·

STEP 1 : Load power estimation

$$P_o = \sum_{i=1}^{m} \frac{1}{T}\int_0^T e_{si} \cdot i_{si} \cdot dt$$

STEP 2 : Area product calculation

SELECT a CORE from core table .

Note down $A_c$ , $A_w$ of selected core

STEP 3 : Number of turns

N

Before we workout any example for transformer design let us write down the design steps. There were many equations that we discussed and developed. Let us sequentially list down all the equations relations that are necessary to design the transformer and step by step write them down.

Step 1: First let us estimate the load power P naught; estimation of P naught this load power of load for the transformer is all the secondaryss put together. So, P naught should reflect all the power in all the secondaryss if it is the multiple secondary transformer. So, let us say in a more generic sense if there are m secondaryss, I is equal to 1 to m, 1 by T integral of 0 to T voltage across every secondary i th secondary current through i th secondary dt. So, this will give you the average power of every winding, will give you the average power of the i th winding and i th rating I from 1 to m all the winding is average power is obtained and they are summed up to give you P naught.

Next, step 2: Area product because the area product needs the P naught value. We need to use the P naught of step one to find out the area product for the particular application. Then after you find the area product using the area product for the particular application select a core from the core table just like we did for the inductor after the area product you have to go to the core table and then select a core which is having an area product higher than the calculated area product then after having selected note down the core cross section area A c and window area A w of the selected core.

Next, step 3: So, in step 3 we have to evaluate the number of turns.

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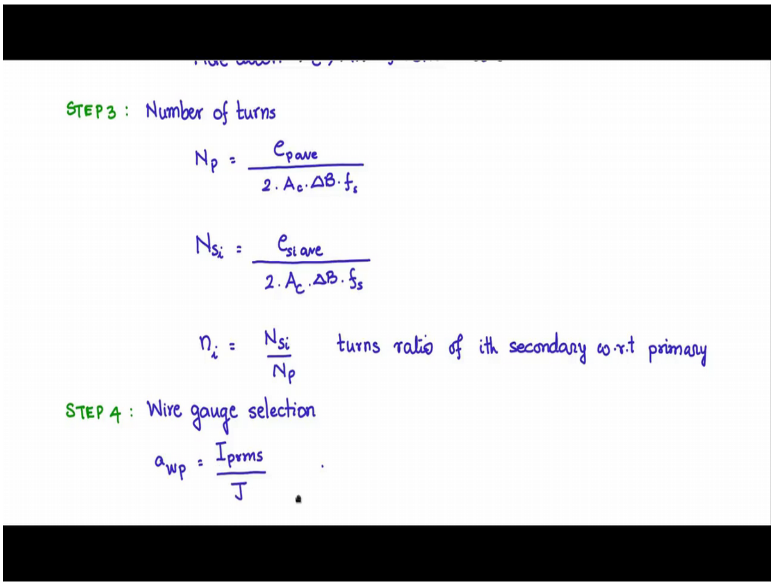

STEP 3 : Number of turns

$$N_p = \frac{e_{p\,ave}}{2 \cdot A_c \cdot \Delta B \cdot f_s}$$

$$N_{s_i} = \frac{e_{si\,ave}}{2 \cdot A_c \cdot \Delta B \cdot f_s}$$

$$n_i = \frac{N_{si}}{N_p}$$ turns ratio of ith secondary w.r.t primary

STEP 4 : Wire gauge selection

$$a_{wp} = \frac{I_{prms}}{J}$$

So, N P we know we have this relationship e primary voltage average half cycle average divided by 2 A c delta B into f s. Secondary windings N s the i th winding is given by the voltage of the secondary of the i th winding average half cycle divided by 2 A c delta B f s and the turns ration of the i th winding i th secondary winding with respect to the primary is given by N si by N P. So, this is the turns ratio of the i th secondary winding with respect to the primary.

Step 4: Next, we find the wire gauge, we have the turns ratio, we have the number of turns to wind, we have the core, we now need to choose the wire; what is the thickness of the wire. So, wire gauge selection like in the inductor the area of cross section of the wire the primary is given by the I p rms; rms current flowing through the primary by J the current density.

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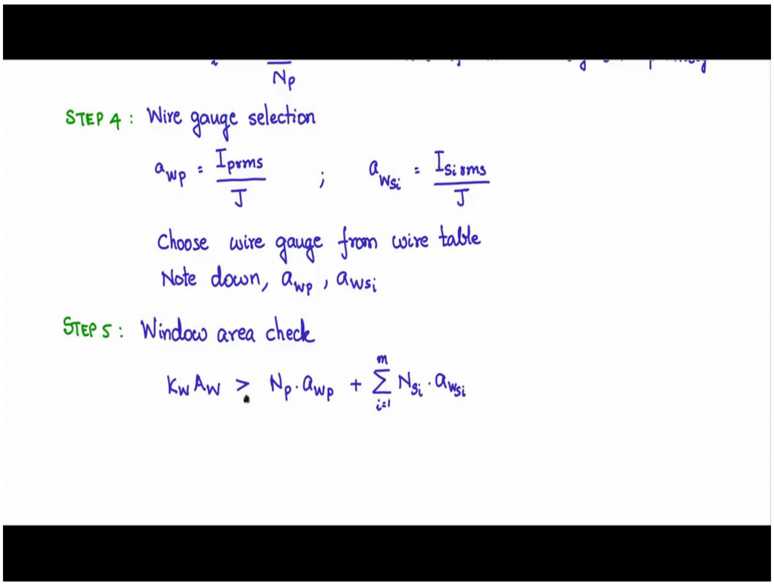

$N_p$

STEP 4 : Wire gauge selection

$$a_{wp} = \frac{I_{prms}}{J} \quad ; \quad a_{wsi} = \frac{I_{si\,rms}}{J}$$

Choose wire gauge from wire table
Note down, $a_{wp}$ , $a_{wsi}$

STEP 5 : Window area check

$$K_W A_W \geq N_p \cdot a_{wp} + \sum_{i=1}^{m} N_{si} \cdot a_{wsi}$$

And, the area of the wire cross section of the wire of the secondary of the i th winding will be the rms current of the i th secondary winding divided by J. So, like that for every winding you can find it is rms current and divide by J you will get calculated value of a w wire cross section area. J what you are going to use just like in the inductor we start with 3 amp per mm square or 3 into 10 to the power of 6 amp per meter square.

After having calculated a wp and a w all the secondarys wire cross section area go to the wire table and choose the wire gauge from the wire table. After having chosen the particular SWG Standard Wire Gauge; note down the values of the actual wire cross section area of the primary and wire cross section area of the secondarys. This would complete the design, but there is one more step needed to complete one cross check step just like we did in the inductor design.

Window area cross check. K w A w that is available window area or should be greater than N P primary number of turns into the wire cross section area of the primary plus all apply for every secondary winding secondary number of turns of the i th winding into area of cross section, wire cross section of the i th secondary wire and add them all up. All these turns into area should add up and fit into the available K w A w, only then the design will be successful.

If it; if this inequality is not satisfied you go back to step 2 select a core another core having a higher area product and then repeat all these calculations for step 3, step 4 again, till step 5 inequality is satisfied, then the design is successfully completed.