

**Fundamentals of Power Electronics**  
**Prof. L. Umanand**  
**Department of Electronics Systems Engineering**  
**Indian Institute of Science, Bangaluru**

**Lecture - 65**  
**Inductor area product**


(Refer Slide Time: 00:27)

Area product

1. Core cross section area,  $A_c \text{ m}^2$
2. Window area,  $A_w \text{ m}^2$

$$A_p = A_c \cdot A_w \text{ m}^4$$

↓  
Area product



We will be using the area product approach for designing the magnetics both inductor and the transformer. So, let us look at what area product means. So, there are two important areas for any core, one is the core cross section area  $A_c$  a meter square core cross section area. What is the core cross section area? You see this part here, the central, this is the e core you have seen this e core I have shown it to you before. On the extreme size there are thin limbs and on the inner the central arm is the thicker one. So, this area here which I am showing with the pencil is the core cross section area. And perpendicular to that orthogonal to that there will be flux flow.

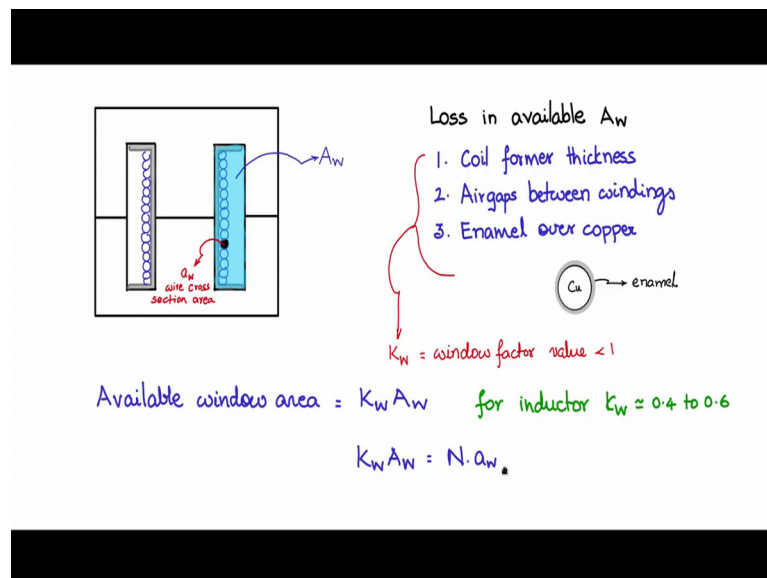
Next let us see another area called the window area. Window area symbol is  $A_w$  meter square and what is the window area In the case of the e core, you see there is this one e core, I will take the other one and place it on top of this like this. This will be the completed e core as you see it here.

Now, this, this gap here what you see here is the window area. This area here is window area not both together. Any one of the area only either it is this area, we have to take or

that, because a turn entering here also come out there. So, it is only one area that we need to consider. So, this is the window area. So, that is what we call as window area.

And now area product  $A_p$  is  $a_w$  into  $A_c$  or  $A_c$  into  $A_w$  and this is meter to the power of 4. Now  $A_c$  into  $A_w$  is called the area product. When we do the core design for inductors and transformers we will be calculating this area product  $A_p$  and checking against the table of core course for the area product that matches or is greater than the value calculated. And we will select such a core which has a greater datasheet value  $A_b$  then what we have calculated and then use it for design. This is an iteration process of course we will now discuss how we will use this area product to design the course for the inductors and the transformers.

(Refer Slide Time: 03:36)



Now, consider a core, let us take an e core and we saw just now how the window, window area looks like this the window area and the this is where you will see the cross section of the wire. So, first let me draw the former, you will have to place the former. There will be a former section which is going to come in here, it will heat up some portion of the window area this is the former section.

Then let us say we put in the turns. So, these are the wire cross section. So, this is one single turn the cross section will appear on this side of the window area and also on this side of the window area. So, I will just put one layer there may they may be many layers which will fill this entire window area. So, this is an e core.

Now, this portion here which I am showing marking on the curve with the cursor is the window area. So, this is the window area  $A_w$ ; however, should understand that the entire window area is not available for you to fill it up with the windings. So, there is some loss in the available window area. What is the loss one is the curve? We just saw that there is this coil former which is eating away a portion of the area.

Then when you are winding the, the cross section of the copper wire is circular in nature and there will be air gaps in between the windings. So, that is going to take away some window area. Thirdly the cross section of the wire has enamel on enamel covering on top of the copper. So, that there is insulation between the windings. So, that there is no short across the windings.

So, this enamel over copper. So, if this the copper cross section wire ANOVA on that over that there is the enamel such that there is an insulation between two neighbouring coils. There will not be a short circuit, but this enamel cover is going to eat up some portion of the window area. So, now, all these are going to reduce the window area by a factor called the window factor  $K_w$ . So, this window factor  $K_w$  has the value less than 1. And the available window area can be written as  $K_w A_w$ . And for the inductor  $K_w$  is between 0.4 to 0.6 should understand that it is not easy to deterministically evaluate all these loss and available window area, because many a times the air gap between the windings depends on the skill with which one winds.

So, therefore, we have a range of  $K_w$ . So, if someone is the no voice who is winding it for the first time who is the new B the  $K_w$  factor will be lower and someone who is very experienced and has very good skills and winding the coils will B around 0.6  $K_w$ .

So, now I have shaded one wire cross section. So, that area which is shown black is called a  $w_s$  lower case w which is the wire cross section area. It is the cross section area of the copper wire, it is only of the cross section area of the copper wire inside no excluding the enamel, because the enamel portion is taken within  $K_w$ .

So, if there are  $N$  such turns  $N$  such cross sections will be there. So,  $K_w A_w$  should be equal to  $N$  into a area of cross section of the wire.

(Refer Slide Time: 08:04)

Available window area =  $K_W A_W$  for inductor  $K_W = 0.4$  to  $0.6$

$$K_W A_W = N \cdot A_w$$
$$= N \cdot \frac{I_{rms}}{J}$$

Crest factor,  $K_c = \frac{I_m}{I_{rms}}$

current density,  $A/m^2$   
default value =  $3 A/mm^2$   
 $= 3 \times 10^6 A/m^2$

$$\therefore K_W A_W = \frac{N \cdot I_m}{K_c \cdot J}$$

$$I_m = \frac{K_W A_W \cdot K_c \cdot J}{N}$$

 ①

Now area of cross section of the wire, we will write it as  $I_{rms}$  there is a rms current that flows through the cross section of the wire and a permissible current density. So, there is a permissible current density  $J$  and this for copper is at a default value of around three ampere mm square or 3 into 10 to the power of 6 ampere meter square. So, this is an amps, this is ampere meter square and therefore, the units of  $K_W A_W$  will become meter square.

So, this is the starting value of the default value the  $J$  varies in practice from 2 ampere per mm square to even 5 ampere mm square in very low power systems. As the systems become higher and higher in power the  $J$  value becomes 2 ampere mm square 3 ampere mm square is a reasonable value to take for most of our design purposes.

Let me represent the rms current here in terms of max current. So, let me introduce a term crest factor crest factor is  $K_c$  which is  $I_{max}$  or  $I_{peak}$  divided by  $I_{Rms}$ . And therefore, using this relation here you have  $K_W A_W$  equals  $N$  times  $I_m$  by crest factor into  $J$  or  $I_m$  can be written as  $K_W A_W K_c J$  divided by  $N$  number of turns.

So, this is now is relationship we will mark it as one the current the max current through the inductor is expressed in terms of a number of turns the window area or the window factor and the current density  $J$ .

(Refer Slide Time: 10:23)

$$\therefore K_w A_w = \frac{N \cdot I_m}{K_c \cdot J}$$
$$I_m = \frac{K_w A_w \cdot K_c \cdot J}{N} \quad \text{--- ①}$$
$$e_c : L \cdot \frac{di}{dt} = N \frac{d\phi}{dt} = N \cdot A_c \cdot \frac{dB}{dt}$$
$$L \cdot I_m = N \cdot A_c \cdot B_m \quad \text{--- ②}$$

Next we go back to faradays law, the voltage across the coil induced is equal to  $L \frac{di}{dt}$  which is also equal to  $N \frac{d\phi}{dt}$ . Now  $\phi$  can be written as  $A_c B$   $A_c$  is a constant. So,  $N A_c \frac{dB}{dt}$ .

So, consider these two, if the voltage is constant the rate of rise of the current is linear. Therefore, rate of rise of the flux, flux density should also be linear. In a given time, let us say the current rises to  $I_m$  from 0 to  $I_m$   $L I_m$  and that same time the flux density would have reached  $B_m$ ; so,  $N A_c B_m$ . So, this is another relationship where  $I_m$  current is related to the magnetic properties and the core cross section area. So, this is relation 2.

(Refer Slide Time: 11:39)

$$\begin{aligned}
 E_L &= \frac{1}{2} \cdot L \cdot I_m^2 \\
 &= \frac{1}{2} \cdot (L \cdot I_m) \cdot I_m \\
 &= \frac{1}{2} \cdot (\cancel{N} \cdot A_c \cdot B_m) \cdot \frac{K_w \cdot A_w \cdot K_c \cdot J}{\cancel{N}}
 \end{aligned}$$

$$A_c \cdot A_w = \frac{2 \cdot E_L}{K_w \cdot K_c \cdot J \cdot B_m} = A_p$$

Now, let us evaluate the energy stored in the inductor. So, what is the maximum energy stored in the inductor half  $L I_m^2$  max current square. So, this I can write it as half  $L I_m$  I will group this together into  $I_m$ . And we have developed relation for  $L I_m$  wherein  $L I_m$  is related to  $A_c$  and we have also relation for  $I_m$  which is related to  $A_w$ , we will substitute these two relations here and rewrite. So,  $L I_m$ , I am writing it as  $N A_c B_m$  and then for  $I_m$  here  $K_w A_w K_c J$  by  $N$ .

So, let us remove these two  $N$ s and I will keep  $A_c$  and  $K_w$  to one side their product, this is the area product  $A_c$  into  $A_w$  which is equal to 2 times  $E_L$  divided by  $K_w K_c J$  and  $B_m$ . So, this is area product  $A_c A_w$  to the power of 4 and this is the area product equation. And this is the equation that you will be calculating and checking it in table of a cores and comparing it with the area products of the cores. And you will pick out that core which is having an area product greater than that which is calculated.

And that is what we will be using for the inductor design. And later on for the transformer design too, but therefore, the transformer design the area product equation will be different, we will come to that later at that point in time.