

Fundamentals of Power Electronics
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Lecture - 64
Inductor value and energy storage

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INDUCTOR VALUE

$e_c = L \frac{di_L}{dt}$

$\phi = \lambda \cdot \text{mmf} \quad (\text{A} \cdot \text{C} \cdot \text{V})$

$\phi = \lambda \cdot N i_L$

$N \frac{d\phi}{dt} = \lambda \cdot N^2 \frac{di_L}{dt}$

$e_c = L \frac{di_L}{dt}$

$L = \lambda \cdot N^2$

Consider an inductor which is carrying a current i_L through it, inductor of value L and there is a voltage across the coil e_c and we have discussed while discussing converters how to find the value of L using the Faradays law. The faradays law related to electric currents e_c is equal to $L \frac{di_L}{dt}$. So, the rate of change of current into the inductance will give you the voltage across the coil.

Now is it possible to get the value of inductance L from the other parameters physical parameters and magnetic parameters of the inductor? Now that is what we will try to find. See typically this is a circuit symbol what we have written here. In reality you will see that there is a core as I said ferrite core or CRGO core or any of the magnetic core materials on which you will wind some turns.

And it will have these two terminals just like here and then a current will go in through and come out; this is i_L and voltage across the terminal will be the voltage across the coil e_c and this coil may have N turns and within the core it will set up a flux ϕ and this core will have some predesigned value of permeance λ shown here. Now

with respect to such a physical fabricated inductor what is the value of the L or how is L related to these core parameters and physical parameters?

So, let us back go back to the fundamental equation phi is equal to permeance into mmf. Recall for as a comparison you can jog your memory using charge Q is equal to C V; phi flux is equivalent to charge Q, permeance is equivalent to C, mmf is equivalent to the potential and there is what this equation is. Now phi is equal to permeance into mmf can be written as $N i L$. Let me now differentiate this equation; $d \phi$ by $d t$ equals permeance constant N constant $d i$ L by $d t$.

Now, let me multiply both sides with N, so $N d \phi$ by $d t$ is equal to permeance of N square $d i$ L by $d t$. Now if you look at this is nothing but Faradays law e_c which is equal to $N d \phi$ $d t$ voltage across the coil and what is this should also be equal to e_c and we see here the Faradays law from the electrical perspective which is $L d i$ L by $d t$.

So, this is the form $L d i$ by $d t$ and therefore, this L must be equal to this permeance into N square. So, the inductance is equal to permeance into N square, see that inductance is related to the permeance magnetic properties in the physical properties and this is the important relationship will be using it in design of the inductor remember this.

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$e_c = L \frac{di}{dt}$
 $\phi = \lambda \cdot \text{mmf} \quad (Q = C \cdot V)$
 $\phi = \lambda \cdot N i_L$
 $N \frac{d\phi}{dt} = \lambda \cdot N^2 \frac{di}{dt}$
 $e_c = L \frac{di}{dt}$
 $L = \lambda \cdot N^2$
 $L = \frac{\mu_0 \mu_r \cdot A_c \cdot N^2}{L_m}$

Now, let us expand permeance; L is equal to permeance is mu naught into mu r into A c core cross section area into N square here and divided by L m, the mean magnetic path

length. So, you see that the inductance is related to the permeability, the core cross section area, the number of turns, mean magnetic path length all the physical and the magnetic properties.

So, this is one way of getting the L from the physical magnetic properties in the physical properties. The other way is to get from $L = \frac{d\Phi}{di}$ which we obtained from the electrical requirements of the convertor. So, these two we have to map and match to fabricate the inductor. Of course let us see how we will go about do it, some more fundamentals and then we can design the inductor.

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ENERGY IN INDUCTOR

$$E_L = \frac{1}{2} \cdot L \cdot i^2 \quad \Rightarrow \text{from electrical perspective}$$

$$= \frac{1}{2} \cdot L \cdot \left(\frac{Ni}{N}\right)^2 = \frac{1}{2} \cdot L \cdot \frac{mmf^2}{N^2}$$

$$= \frac{1}{2} \cdot \mu \cdot mmf^2 \quad (\Rightarrow \frac{1}{2} \cdot c \cdot v^2)$$

Let us now discuss the energy that an inductor can handle. From the electrical perspective we know that E_L , let us say E_L is the energy in the inductor is half $L i^2$, i is the current at a given instant, so this is the instantaneous energy at an instance when the current is taking some value i . So, half $L i^2$ is the energy in the inductor and this is coming from the electrical perspective by virtue of the motion of the electrons.

Now, let us see we can get the energy in the inductor from the magnetic perspective. Now half $L i^2$ can be written as half $L i^2$ can be written as $N i$ by N whole square which is equal to half L ; $N i$ is nothing but mmf, so mmf square by N^2 . Now what is L by N^2 ? L by N^2 is nothing but permeance because we know that L is equal to permeance into N^2 . So, this nothing but permeance, so we can

write half permeance into mmf square. So, this is the magnetic perspective for the energy in the inductor half permeance mmf square, this is half $L i^2$ in the electrical angle.

Consider the electrical equivalent potential storage, how does it look like? We know permeance is equivalent to capacitance. So, half C and potential mmf is nothing but voltage. So, this will translate into an equivalent as half $C V^2$, so we can be satisfied that the equivalence works.

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The image shows a handwritten derivation of the energy stored in an inductor, E_L . The steps are as follows:

$$= \frac{1}{2} \cdot \lambda \cdot \text{mmf}^2 \quad (\Rightarrow \frac{1}{2} \cdot C \cdot V^2)$$

$$= \frac{1}{2} \cdot (\lambda \cdot \text{mmf}) \cdot \text{mmf}$$

$$= \frac{1}{2} \cdot \phi \cdot \text{mmf} \quad (\Rightarrow \frac{1}{2} \cdot Q \cdot V)$$

$$= \frac{1}{2} \cdot A_c \cdot \frac{\phi}{A_c} \cdot \text{mmf}$$

$$E_L = \frac{1}{2} \cdot A_c \cdot B \cdot \text{mmf}$$

for a given core size & B
 $E_L \propto \text{mmf} \propto i^2$
 Greater current, greater energy

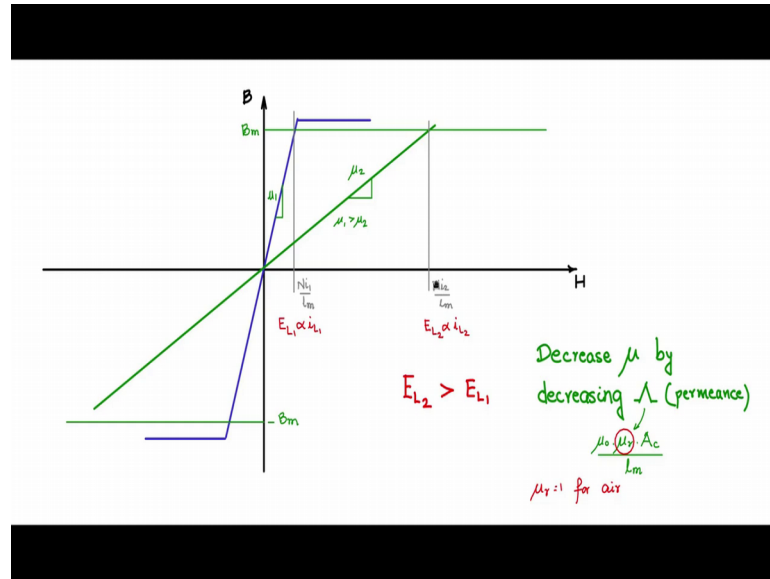
Now, let me further introduce the small modification permeance into mmf; I will take that together into mmf. Now permeance into mmf is nothing but phi flux half phi into mmf, this again from the equivalence you can say it is half $Q V$, so here again equivalence works. So, you see that half phi mmf is still energy, now phi I want to express it in terms of flux density B .

So, let us say half into A_c into phi by A_c phi by A_c is the flux density B to mmf. So, I can write that one as half into A_c into flux density B this one and into mmf. So, this becomes the energy that is stored in the inductor from the magnetic perspective.

Now, let us say A_c for a given core; A_c is fixed and for a given core let us say ferrite, B_m the maximum operating flux density B_m can be fixed, as I told for ferrite we can fix it at around 0.25 tesla. So, like that if I know the core type core material you can fix this B

m, so these two are fixed constants. So, then energy is directly proportional to mmf or current because mmf is N into i ; N is again a fixed quantity, energy is directly proportional to the current. So, greater the current greater will be the energy that can be stored in the inductor. So, how do I increase the storage capacity of the inductor? Let us see that one.

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It is of interest for us to see that for a given size of the core and flux density we would like the inductor to handle more and more energy. So, let us understand this by drawing the BH axis and let us take a typical core where you have the BH curve like this; this is the saturation, negative saturation and this has slope of permeability μ or μ_1 in this case.

So, let me fix the operating flux density, so this is the operating flux density on the positive side we will call that one as B_m , likewise operating maximum flux density on the negative side minus B_m . So, we know that the flux density in the core will swing only between minus B_m to plus B_m , within that and never crossing that.

So, if we limit the flux density to this value, what is the maximum energy that this inductor can store? So, if you take the intercept here; here the field will be $N_1 I_1$ by L_m , this is the H field at this point. So, when a current i_1 is flowing then we know that the energy is stored in the inductor E_{L1} is proportional to $i_1^2 L_1$. So, this particular core having this μ_1 as the permeability can store this particular amount of current for a

current i_1 flowing through that.

Now, can I increase the energy handling capability of this core which means that, I should be capable of handling higher currents higher mmf which means it should go further on into the x axis. But if you start going further on into the x axis, the core will reach saturation it will flatten out and there is no induction and therefore, permeability is 0. So, how do you still keep the flux in operation without saturating the core and still be able to store larger energy?

So, let us see what happens when you reduce the permeability μ ? So, let us do that. Let me draw another line here, so which is having a lower slope and this intercept here. You see for the same B_m , the intercept here for this lower permeability core where μ_1 is greater than μ_2 , this is $N i_2$ by L_m and here also let say the energy E_{L2} is proportional to $i L^2$

So, we see here that this the energy at this point for this particular current being handled for this curve, for this core the green line is E_{L2} is greater than the energy that can be maximum energy that can be handled by this core indicated by the blue line. So, if I reduce the permeability for the same flux B_m this will go to 0 and meet the saturation much later, in the much later in the field axis as shown here.

So which means that it is capable of handling higher currents larger currents and therefore larger energy. So, how do you achieve that? You have to decrease permeability. How do you decrease permeability? By decreasing the permeance and what is permeance? Permeance we saw is $\mu_{naught} \mu_r A_c$ by L_m the magnetic path length.

Look at this μ_r ; μ_r is the relative permeability which is of the order of 2000 to 2500 in the case of ferrites, let us say you make it equal to 1. What is it μ_{naught} permeance becomes $\mu_{naught} A_c$ by L_m which is for free space, so μ_r is equal to 1 for air.

So, if you start introducing air gaps, then my relative permeability value will start coming down for the same core and the effective permeability μ_{naught} into μ_r will start reducing and the slope decreases and as the slope decreases the intercept at the B_m maximum operating flux density will move farther onwards the farther on away into the x axis and the core will be able to handle higher currents and therefore, the inductor is capable of storing higher energy.

So, this way we try to maximise the energy storing capability of the inductor. See one primary important feature of the inductor that it should be capable of storing energy. So, that is why we try to design the inductor which is capable of carrying higher and higher currents.

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Summarising electrical and magnetic equivalence

Parameter	Electrical	Magnetic
potential	voltage, emf	mmf
kinetic	current, i	$\frac{d\phi}{dt}$
power	(emf \cdot i)	(mmf \cdot $\frac{d\phi}{dt}$)
Capacitance	C	Λ
Elastance	$\frac{1}{C}$	Reluctance, $R = \frac{1}{\Lambda}$
Charge	Q	ϕ
Energy	$\frac{1}{2} \cdot C \cdot V^2$	$\frac{1}{2} \cdot \Lambda \cdot \text{mmf}^2$

In the discussions still now we have made some equivalence between electrical and magnetic parameters let me summarise them. So, that we understand the electrical and magnetic properties well equally well.

So, the parameters potential, potential and electrical is voltage v or emf electromotive force, in the magnetic domain it is the magneto motive force. The kinetic parameter; the kinetic parameter or the flow parameter and the electrical domain is current i amps and the magnetic domain it is $d\phi$ by dt . Power in the electrical domain is emf voltage into current or potential into the kinetic variable, here also it is the same thing potential variable mmf into $d\phi$ by dt and both have units of power which is watts, this watts and this is also watts.

Capacitance in electrical domain is C and the magnetic domain it is permeance. Elastance in the electrical domain it is 1 by C and the magnetic domain it is called the reluctance, reluctance R which is 1 by permeance. Charge in the electrical domain Q which is integral of i dt here it is ϕ flux. In energy you have half C V square half C V square is replaced by half permeance mmf square or you can club C V and call it as half

Q V and here also half phi into mmf.

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Parameter	Electrical	Magnetic
potential	voltage, emf	mmf
kinetic	current, i	$\frac{d\phi}{dt}$
power	(emf \cdot i)	(mmf \cdot $\frac{d\phi}{dt}$)
Capacitance	C	Λ
Elastance	$\frac{1}{C}$	Reluctance, $R = \frac{1}{\Lambda}$
Charge	Q	ϕ
Energy	$\frac{1}{2} \cdot C \cdot V^2$ $\frac{1}{2} \cdot Q \cdot V$	$\frac{1}{2} \cdot \Lambda \cdot \text{mmf}^2$ $\frac{1}{2} \cdot \phi \cdot \text{mmf}$

So, these are some of the equivalence that we saw. You can write many more equivalence you will see that because the energy is the same across the domains, there is a very nice strong equivalence between the electrical and the magnetic parameters. Use this equivalence to remember the equations on the magnetic domain.