

Fundamental of Power Electronics
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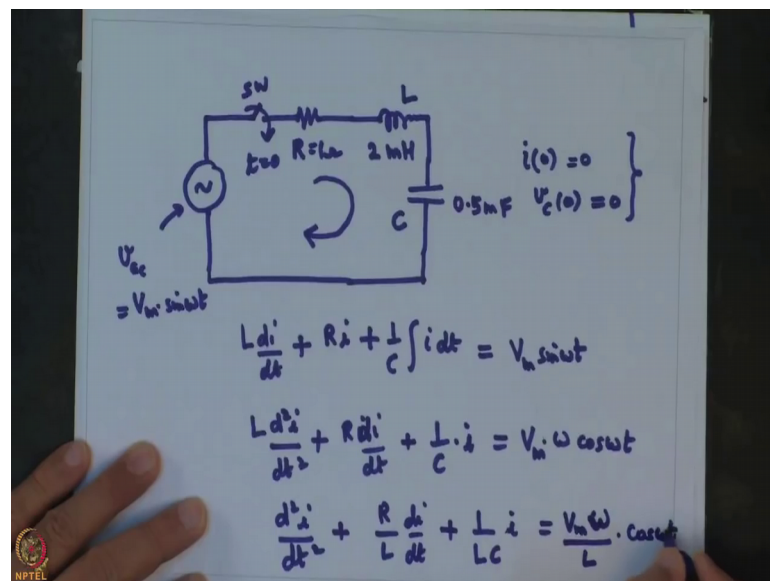
Lecture – 06

Review of Engineering Maths for Power Electronic Circuit Analysis

Hello and welcome back. So far we have seen some examples in which we considered circuits which were fed by DC sources and we represented them by linear differential equations with constant coefficients and we solved them to get the desired response of current or voltage.

Now, let us see an example in which an AC source is feeding the network.

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So, the AC source or the forcing function is supplying through a switch network that consists of R equal to 1 ohm an inductor which is 2 milli henry and a capacitor which is 0.5 milli farad.

Now, when the switch is closed at t is equal to 0, a current results in this network. We are interested in determining the complete response of this current that is for all time after the switch sw has been closed.

Now, the first step as you know is to apply the Kirchhoff's voltage law which will straightaway lead to the following equation $L \frac{di}{dt}$ voltage across the inductor plus Ri the voltage across the resistor and $\frac{1}{C} \int i dt$ the voltage that will develop across the capacitor. I can also say I can just mark or denote this capacitor and inductor by C L respectively.

Now, this whole thing, these three terms summation of these three terms is equal to $V_m \sin \omega t$ where $V_m \sin \omega t$ is nothing, but the AC source that is applied to the network V_{ac} is equal to $V_m \sin \omega t$. Now, if we differentiate this equation on both sides, we can see that it actually turns out to be $L \frac{d^2 i}{dt^2} + R \frac{di}{dt} + \frac{1}{C} i = V_m \omega \cos \omega t$. We are assuming for this circuit that the current in the circuit at time $t = 0$ is 0 and the voltage across the capacitor at time $t = 0$ is also 0. So, these are our initial conditions.

So, taking the derivative this is what we get and by rearranging this term, we can write this in a slightly more organized form as you can see $L \frac{d^2 i}{dt^2} + R \frac{di}{dt} + \frac{1}{C} i = V_m \omega \cos \omega t$. In order to obtain the natural response, we will have to work with the homogeneous equation corresponding to this non homogeneous equation.

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The image shows a whiteboard with handwritten mathematical equations in blue ink. The equations are as follows:

$$\frac{d^2 i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{1}{LC} i = 0$$

$$\zeta = \text{damping factor} = \frac{R}{2} \sqrt{\frac{C}{L}}$$

$$\text{natural freq. } \omega_n = \frac{1}{\sqrt{LC}}$$

$$s_{1,2} = -\zeta \omega_n \pm \omega_n \sqrt{\zeta^2 - 1}$$

Below the equations, there are three lines of text describing the damping factor ζ :

- $\zeta = 1$ critically damped response
- $\zeta > 1$ overdamped response
- $\zeta < 1$ underdamped response

So, we will get $L \frac{d^2 i}{dt^2} + R \frac{di}{dt} + \frac{1}{C} i = 0$. So, this is the corresponding homogeneous equation. Now, many times people use

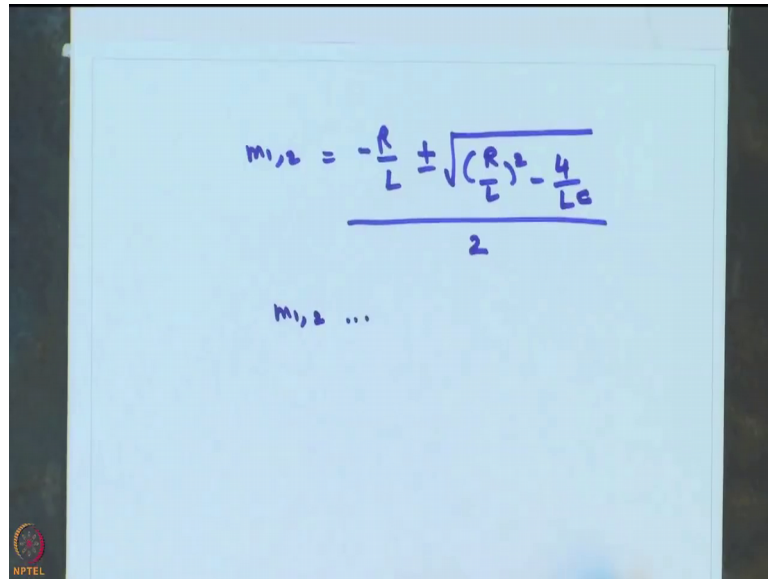
what is called the standard form of the second order system the standard form of the characteristic equation of the second order system which is obtained by using zeta, introducing zeta as a damping factor and define this as $\frac{R}{2\sqrt{C/L}}$.

Similarly, the natural frequency of oscillations is defined by ω_n and that is nothing, but $\frac{1}{\sqrt{LC}}$. So, I can just write here the natural frequency ok. So, if we use these parameters that we have specially introduced, we can actually get the roots m_1 and m_2 . So, there are two roots minus of zeta ω_n plus minus $\omega_n \sqrt{zeta^2 - 1}$ and of course, we can also write the roots by using the original you know expressions in the terms without making any substitutions which I will write subsequently.

So, now here if you see that this there is a quantity you know which is a square root quantity. So, now depending on whether you get you know inside the root whether you get a positive negative or a 0 quantity, you actually have three different behavior in response of the system. So, if zeta is equal to 1, we call this as a critically damped response. If zeta is greater than 1, we call it over damped response. So, you will not see any oscillatory response in what in any of these two conditions and if you have the zeta less than 1 which means that now this square root quantity would actually become the inside quantity will become negative and hence, you will get imaginary roots.

Now, this as you know because of our experience with Euler's formula ok, we know that this will lead to some sin and cosine terms and hence, we can say that this is going to be an oscillatory response also called as under damped, not damped which means it is going to oscillate. So, under damped response, so coming back to our original equation.

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The image shows a handwritten equation on a light blue background. The equation is
$$m_{1,2} = \frac{-R \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2}$$
 Below the equation, there is a small handwritten note: $m_{1,2} \dots$ In the bottom left corner of the image, there is a small logo for NPTEL.

So, either we can use the roots in this form that we obtain or we can still continue with the same form minus of R by L plus minus R by L whole square minus 4 by LC whole divided by 2 and by putting these values, various values of you know R In C as you know is given in the problem in the example we then find m 1 and m 2 and then, determine the response.

Now, if we want to get the complete response of the system, complete response of the current let me just mark the current L. Here I forgot to mention that i. So, we want the complete response of i of the switch has been closed at t is equal to 0. Now, there are as we know now we know very well that there are two components that will be you know that actually constitute the complete response. One is the forced response which is present long time after the switch has been closed and a natural response which will be present only at the time of switching and the switching takes place and this response remains for some time, but then afterwards it dies down.

So, basically it is a transient response which leaves for a very short time also called the natural response as it depends on the circuit parameters. Various element that are used L R and C, it does not depend on the applied function the forcing function.

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The image shows a whiteboard with handwritten mathematical work. At the top, the roots m are given by the quadratic formula: $m = \frac{-R/L \pm \sqrt{(R/L)^2 - 4/LC}}{2}$. Below this, the roots are identified as p_1 and p_2 , with the value $-250 \pm j968.25$ written next to a brace. The general form of the current response is shown as $i_f(t) = e^{-250t} [A_1 \cos(968.25t) + A_2 \sin(968.25t)]$. Finally, the specific response is given as $i_f(t) = 1.714 \sin(\omega t + 80.11)$. An NPTEL logo is visible in the bottom left corner of the whiteboard.

So, the natural response if you want to determine we know that p_1 or we can just call the general expression m which actually denotes the roots of the quadratic expression that we will get by substituting the right hand side the forcing function equal to 0 in the previous equation. So, basically $V m \omega$ by $L \cos \omega t$ that term vanishes it is 0. So, this becomes the homogeneous equation and then, we are trying to find the roots of this equation. So, there are two roots which are given by this very standard expression and then, we can then write the two roots because we have been using p_1 and p_2 to define the roots.

We can say that p_1 p_2 , they turn out to be minus 250 plus minus. So, one of them is plus the other is minus 2 roots $j 968.25$ approximately. So, these are the two roots we get now depending on what we get in the square root term here you know. So, these could be for example, it could be 0 that we are getting in which case you know we will have the you know the real roots. In one case it might turn out that the left hand term R by L square is greater than 4 by LC in which case we will be getting real roots and as we see in the present example, the term R by L whole square minus 4 of LC turns out to be negative which is what is leading to these imaginary roots two of them.

Now, one of the things that we can immediately try to correlate here is that because they are imaginary roots and from the Euler's formula, we know that something that is having an imaginary component we have seen that how it is related to sin and cos. So, it should

actually give you an idea already that this is going to be an oscillator response. So, it will be an oscillatory natural response that you are going to get.

So, the general form of the natural solution for imaginary rules expressing or actually depicting or showing oscillatory behavior can be given by i_n or t for transient and for natural. So, i of t i of t equal to $\frac{250}{A_1} \cos(968.25 t) + A_2 \sin(968.25 t)$ ok. Now this is the form of the natural response and please remember that the natural response is going to be the same whether we are having a sinusoidal source function or whether we are going we are having a DC forcing function or source function, the natural response is independent of the applied source.

So, this natural response is same and we will see you will find this also covered in the previous example, where we consider a dc source based systems. Now, let us find out the first response also for this given network, so that we can then add the natural response and the first response solve for the constants A_1 and A_2 using the boundary conditions and then, substitute back in the expression and get the complete response ok. So, what is the first response in this case? So, i of t the force current response actually turns out to be $1.714 \sin(\omega t) + 80.11$. This is the force response and how we get this? We just got this simply by using the fact that the current, the force current long time after the switch has been closed would be given by the applied voltage v_{ac} divided by the net impedance z and we have just written the expression for complete expression for the voltage v_{ac} which is given a c voltage which is given as z .

So, we see that there is a current which will have a phase with respect to the applied voltage and when we solve this, this is a straightforward process because all the parameters are known to us. Now, what is the next step? Next step is to write the complete response.

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$$\begin{aligned}i_{\text{total}} &= i_t(t) + i_f(t) \\&= e^{-250t} [A_1 \cos(968.25t) + A_2 \sin(968.25t)] \\&\quad + 1.714 \sin(\omega t + 80.22^\circ) \quad (1) \\i(0) &= 0 \\A_1 &= -1.68 \\A_2 &= -0.53 \\i(t) &= e^{-250t} [-1.68 \cos(\dots) - 0.53 \sin(\dots)] \\&\quad + 1.714 \sin(\omega t + 80.11^\circ)\end{aligned}$$

And now we know it very well that i_{total} this is the you know this is the term I have been using to denote the complete response is nothing, but it transient response plus i_f of t the first response and we just add these up using the expression that we already obtained for i_t and i_f e^{-250t} into $A_1 \cos$ of $968.25t$ plus $A_2 \sin$ $968.25t$ and added to this you know the first response which is $1.714 \sin$ of ωt plus 80.22 degrees.

One important thing that you might have noted that because we are considering linear networks and we have said that these are the networks that power electronic circuits have to deal with you know between the switchings that is between the two switchings that take place, we can see that when we apply a sinusoidal forcing function of a response also is sinusoidal and its frequency is same as the applied forcing functions frequency. Only thing is that you will find that its magnitude and the phase would be different from the applied forcing function.

So, these things I would advise that you verify and be confident about this point. Now, we know the boundary conditions already. We are given that i of 0 is equal to 0 and we are also given v_c of 0 is equal to 0 . So, we will be using some of these boundary conditions now to determine the unknowns A_1 and A_2 in the complete response equation. So, if i is equal to 0 or i at 0 is equal to 0 , then if we solve the equation number

1 above, we will get A_1 is equal to minus of 1.68. So, you just substitute it t is equal to 0 and the left side i total is equal to 0 at that time.

So, we just got the values of A_1 , we have all the values of d whatever unknowns are there in the equation 1. Now, we also know that if we differentiate i and we see what is the derivative of the derivative of the current at time t is equal to 0, then we find that that is also equal to 0. You can show this by knowing that the capacitor voltage at the instant 0 is 0. So, this you must verify is very simple and straightforward that you can verify.

So, when you use this condition that is you differentiate equation 1 in both the sides, put the left hand side equal to 0, then you will find that A_2 also stands solved and you get that equal to minus of 0.53 and now even in A_2 we have solved for both and we can then get the complete response which is equal to you know e raise to power minus 250 t and then, we have this minus 1.68 cos of blah blah blah minus 0.53 into sin of blah blah blah. You know this is what you will get plus you will get this term 1.74714 sin of omega t plus 80.11 degrees. This is what would be the final response and you can always try to plot and see also.


So, you can because you can see sin and cosine terms, you can see that this is going to have an oscillatory response as far as the mutual response is concerned and then, afterwards it will steady, it will steady down to an ac response which is given by v by z . So, this is one example that I wanted to do. Now, having seen the classical method ok, now let us look at some other tools that we can use and make life simple for us. Now, about the additional tools I would like to mention the Fourier methods.

Now, one of the things is that an electrical engineer is you know fascinated that if there is a non sinusoidal, but periodic function and if it can be decomposed into sinusoidal components and it is applied to the linear networks as we see in power electronic circuits, then he can get the response which is which is going to be same form that is going to be sinusoidal as well with the same frequency. And then, you can get the response to all the decomposed sinusoidal you know terms and you can get the response to all of them and then, by superposition because the system is linear, they can all be added to get the final response.

So, this actually helps, it actually provides a very nice tool to the electrical engineer and in fact, in various other branches to be able to study the response of a linear system to a

periodic, but non sinusoidal function. Now, non sinusoidal function there are several examples which are there in electrical engineering formally we can actually use what is called Fourier series which is which actually falls in the category of Fourier methods where if you know your function. The applied function is a periodic function with let us say p of t, then the Fourier series representation of you know f of e it can be decomposed into sin and cosine terms which is of the form as you can see on slides.

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Fourier Methods: Fourier series

- An arbitrary periodic function can be expanded into a series/summation of sine and cosine functions. The response of a linear system to this periodic function can be obtained by determining the response to these sine and cosine terms and using superposition.
- If $f(t)$ is a periodic function with *period* T , then the Fourier series representation of $f(t)$ is a trigonometric series of the form:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n \times \pi \times t}{T/2}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n \times \pi \times t}{T/2}\right)$$

where $a_0 = \frac{1}{(T/2)} \int_0^T f(t) dt$

$$a_n = \frac{1}{(T/2)} \int_0^T f(t) \cos\left(\frac{n \times \pi \times t}{T/2}\right) dt, n = 1, 2, 3, \dots$$

$$b_n = \frac{1}{(T/2)} \int_0^T f(t) \sin\left(\frac{n \times \pi \times t}{T/2}\right) dt, n = 1, 2, 3, \dots$$


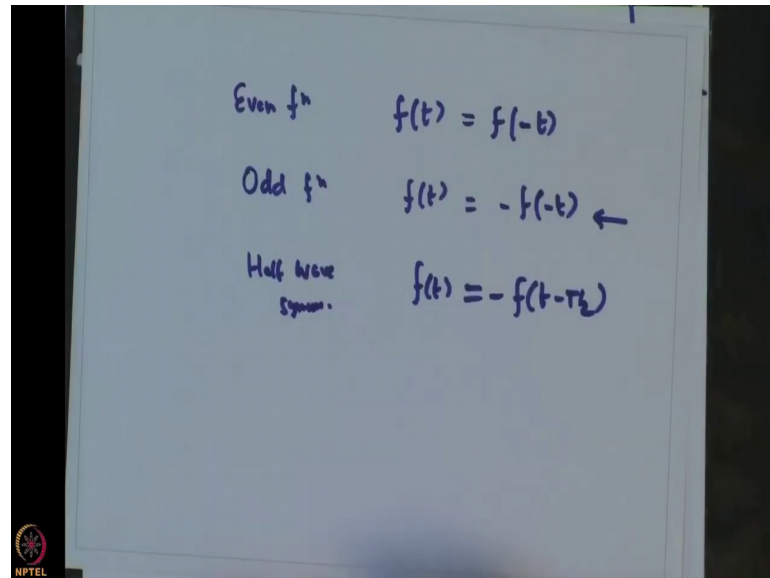
So, you can see that f of t can be shown equal to some term a_0 by 2 which actually represents the DC or the average value or content in the function f of t plus it is you can see a summation that goes from n is equal to 1 to infinity of you know cos and sin terms which are added and they both have a_n and b_n as their coefficients where these coefficients a_n and b_n they vary and a_n and b_n , they vary with the value of n.

So, you get various values of you can get for a_n by putting various values of n and similarly for radius values of you know n for you can get for b_n , the expressions the general expressions for obtaining the a_n and b_n coefficients is also given on this slide T by 2 is actually the half time period of the periodic function which we are trying to decompose into sin and cosine terms. Now, before we move on with one quick example on Fourier series.

I would like to mention that there are some ways in which we can actually simplify the Fourier series analysis ok. These rules are they make use of the symmetry of the given

periodic waveform ok. So, there is something called uneven symmetry or if the function is even, you can then use this even symmetry property. Now, if a function is even, a given function f of t is having an even is an even function.

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Then, basically what we mean is that I just write here and show you. So, even function, so a given function is even if f of t is equal to f of minus t and the example for even function is a cos function.

So, we know that \cos of θ is equal to \cos of minus θ . So, this is the even function. Similarly if a function is odd for an odd function f of t is equal to minus of f of minus t ok. So, if that is the condition which a function is actually satisfying, then it is an odd function. Similarly we also have situations you know which are referred to as half wave symmetry and the quarter wave symmetry.

In the half wave symmetry we have f of t equal to minus of f of t minus $T/2$ and you will find that the wave forms which have a half wave symmetry, they do not contain even harmonics and an example of this is the inverter output. Similarly, we have the quarter wave symmetry which exhibits both even symmetry and half wave symmetry or odd symmetry and half wave symmetry.

So, we can make a similar simplifications as a matter of rule when you have this kind of a situation. So, using these various conditions it is possible to simplify the Fourier series

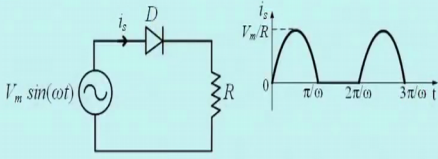
analysis. So, as I mentioned as far as power electronics is concerned, Fourier series will help us to decompose the voltage and current waveforms which may be non sinusoidal into the corresponding sin and cosine terms summation of the corresponding sin and cosine terms and as you will see later with several examples, they are very useful for the harmonic analysis of power electronics systems.

Now, here you see a very typical example which I would like to leave as an exercise to the participant. You can see that this is a half wave rectifier. When the input AC supply is positive, the diode d will conduct. When it is negative, the diode does not conduct because it gets reverse biased during that time. So, what you will see the voltage across the resistance r will be you know this half wave rectified. So, you have these alternate loops which appear from the input AC supply.

Now, somebody would want to know that what is the decomposition of this output, this half wave rectified output? How do you decompose it? How does it look when you are when you want sin and cosine terms to represent this? So, obviously, this is a periodic function, but this is not a sinusoidal function. So, if you can actually do this you know analysis further, then you will be able to get the term that is shown right at the bottom of the slide.

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Example



$$i_s(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n \times \pi \times t}{T}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n \times \pi \times t}{T}\right)$$

$$i_s(t) = \frac{V_m}{\pi R} + \frac{V_m}{2R} \sin(\omega t) - \frac{2V_m}{\pi R} \left(\frac{\cos(2\omega t)}{3} + \frac{\cos(4\omega t)}{15} \right) + \dots$$

So, this will actually need some simple calculations and manipulations which I expect you to do to learn this and also I encourage you to do many other examples. So, there are

several other sources which are available which talk about Fourier series expansion which you must try.

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Fourier Series representation in exponential form

Compared to the trigonometric form, exponential form is mathematically easier to handle with.

$$\sin(n\omega t) = \frac{(e^{jn\omega t} - e^{-jn\omega t})}{2j}$$

$$\cos(n\omega t) = \frac{(e^{jn\omega t} + e^{-jn\omega t})}{2}$$

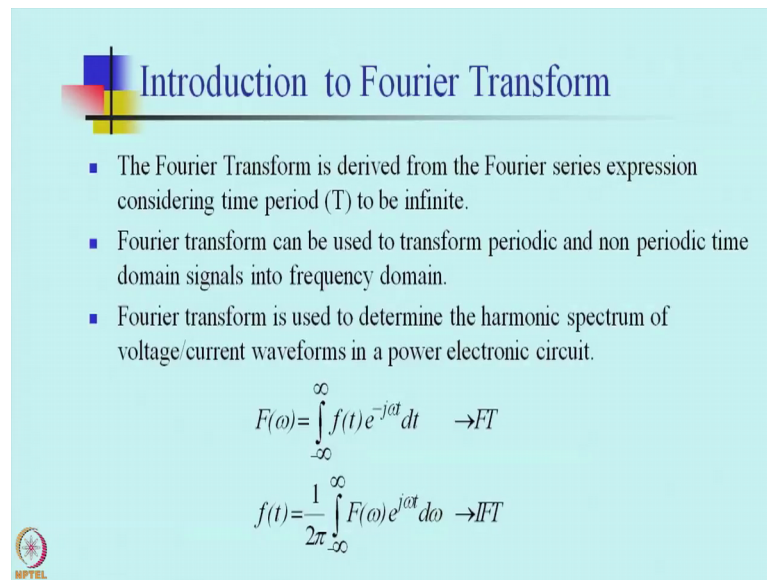
$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega t}$$

$$C_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-jn\omega t} dt$$


These are what you see on the screen are the you know Fourier series expansion. So, you have actually you know in this case the function is actually broken into a Fourier series, but now instead of an and bn you have only one coefficient which is Cn and you can see the Cn expression is written at the bottom on the right side in terms of the time period of the periodic function. So, this is a much more compact form and we have made use of the fact that sin of n omega t can always be written as e raised to power you know j n omega t minus of e raise to power minus j n omega t whole divided by 2 j. So, these things actually follow from the Euler's formula and everybody knows there. So, there is an expression for cos and omega t also which you can see which is given on the left side. Now, there are many functions which are obviously not periodic.

Fourier transform is a tool which has been derived from the Fourier series expression which considers the time period of such functions to be infinite and with this assumption, the Fourier transform method it can be used to transform periodic or non periodic time domain signals into frequency domain signals.

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The slide features a light blue background with a decorative graphic of overlapping colored squares (blue, red, yellow) and a black crosshair in the top left corner. The title "Introduction to Fourier Transform" is centered at the top in a dark blue font. Below the title, there are three bullet points. The first bullet point states that the Fourier Transform is derived from the Fourier series expression by considering the time period (T) to be infinite. The second bullet point states that the Fourier transform can be used to transform periodic and non-periodic time domain signals into the frequency domain. The third bullet point states that the Fourier transform is used to determine the harmonic spectrum of voltage/current waveforms in a power electronic circuit. In the center of the slide, two mathematical formulas are presented: the forward Fourier Transform $F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t} dt \rightarrow FT$ and the inverse Fourier Transform $f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)e^{j\omega t} d\omega \rightarrow IFT$. A small circular logo with the text "MPTEL" is located in the bottom left corner of the slide.

Introduction to Fourier Transform

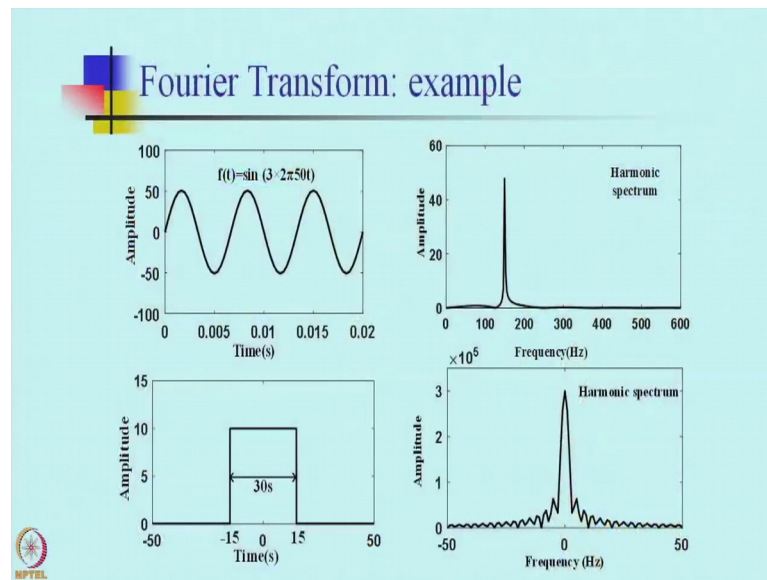
- The Fourier Transform is derived from the Fourier series expression considering time period (T) to be infinite.
- Fourier transform can be used to transform periodic and non periodic time domain signals into frequency domain.
- Fourier transform is used to determine the harmonic spectrum of voltage/current waveforms in a power electronic circuit.

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t} dt \rightarrow FT$$
$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)e^{j\omega t} d\omega \rightarrow IFT$$

So, it is definitely a more powerful thing than the just Fourier series expansion. Fourier transform can be used to determine you know the harmonic spectrum of the voltage and current wave forms in a power electronic circuit ok. This is what it does and you can see that there are the corresponding formulas which are given. So, f of t is a given function which we have represented you know as you can see you know the bottom and you can also see the Fourier transform of that which is a capital F of ω and you can see the integral limit going from minus infinity to infinity f of t e raise to power minus j ω t dt .

So, it is basically an integral of FT along with e raise to power minus j ω t when we do you know Fourier transform of a sinusoidal function as expected you get in the frequency domain.

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The right side plot you see a peak which is there at the fundamental frequency corresponding to the frequency of the given sinusoidal function. So, you see a peak there and rest all the frequency components are 0.

Likewise, if you see the bottom example it is just a single pulse which has been given which is now non periodic and if you actually do a Fourier transform using the formulas given before, you actually end up with what you see on the right side. You know that kind of a harmonic spectrum that is what you will get by using Fourier transform. The problem however is Fourier transform is good, but it does not exist for those signals which increase with time. So, for example a ramp signal. So, you will find that the Fourier integral in this case you know the Fourier integral is the integral which was involved in the Fourier transform expression. The two expressions which I showed you a short while ago that Fourier integral is not finite. Now, it is you know as a solution of this because you know ramp signal and similarly such many signals are very important in power electronic domain.

So, obviously if we cannot analyze signals such as a ramp signal, then Fourier transform has a very big drawback ok. It actually has a lacuna which must be overcome, we must be able to overcome this drawback. Now, Laplace transform is actually a modification of the Fourier transform method which now ensures that even the signals which are rising

with time which are increasing with time such as a ramp signal, they also would converge. So, Laplace transform it uses the finite Fourier integral ok.

So, basically you know the limit from minus infinity to infinity is changed to 0 to infinity. So, there are no restrictions when we use the Laplace transform and therefore, Laplace transform is the one which is used extensively.

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Laplace Transform


- Fourier transform does not exist for those signals which increase with time (example: ramp signal) as the Fourier Integral is not finite.
- Using Laplace transform finite Fourier integral can be obtained. This is done by multiplying the signal $f(t)$ with a decaying factor $e^{-\sigma t}$.

$$L\{f(t)\}=F(s)$$

$$= \int_0^{\infty} f(t)e^{-st} dt; t \geq 0$$

where $s=\sigma+j\omega$, and $f(t)$ is defined for $t \geq 0$

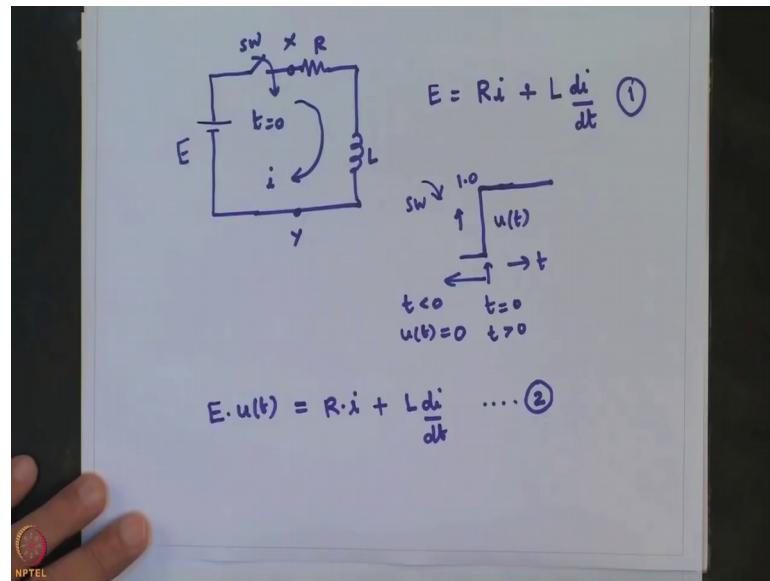
$$f(t)=L^{-1}\{F(s)\}(t)$$

$$= \frac{1}{2\pi j} \lim_{T \rightarrow \infty} \int_{\sigma-jT}^{\sigma+jT} e^{st} F(s) ds$$


So, you can see that if there is a function f of t and if you are doing the Laplace transform of this function, then you can say that it is denoted by capital F of s and just below that you can see the expression which says it is integral of 0 to infinity f of t e to power minus s t dt with t greater than 0. And here you will find that this function always converge and after you have used this transformation, you have actually solved your problem, the time domain problem by converting everything into the frequency domain using the Laplace transform and you have come to a point where you have got the Laplace transform of the solution that you are looking for you can actually do a Laplace inverse. So, on the right side you see how you actually get the function f of t in time domain back from a given Laplace transform F of s .

So, we will look at some example of this, so that this becomes clear.

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Let us again consider our earlier example of a switched RL network, where we have a battery source, a DC source which is applied through a switch sw to a network consists of R and L assume and this which is let us say closed at time t is equal to 0 and that is the time when this applied this dc source is applied to terminals x and y that is right across the network. Now, once the switch is closed by using the Kirchhoff's Voltage Law KVL, I can straight away write that the applied source function e will be balanced by you know the voltage that will develop across the resistance R plus this expression you know the Faradays law which relates the voltage that will develop across the inductor, and gets the voltage that will develop across the inductor which is given by this expression ok.

So, this is how the governing equation will look like once a switch is closed and a current begins to flow here. Now, because there is a switch in the system and we have seen in one of the earlier lectures that you know when you are closing the switch, when you are closing the switch it is like applying the applied source or the source function e in a stepped manner ok. So, if I just denote this kind of a function, so this is the time axis and this is the let us say the magnitude and if I denote this by u of e , then this is called the unit step function.

So, there is a this pulse, it rises to a magnitude of 1.0 at this point and we said that this point is exactly 0 and then, this pulse remains 1 for all times with greater than 0, for all times, then 0 that is in this direction that is e less than 0. This function u of p is 0 we

know this. So, a complete representation you know this equation that we wrote let us say this is equation number 1. A complete representation or more complete representation of this governing equation should have this kind of an expression E into u of t is equal to R into i plus L di by dt .

Now, let us see how we solve this. Let us see how we solve this and what difference it will make you know to bring the Laplace transforms into this solution. So, now let us begin, let us introduce the Laplace transforms and let us see how it makes our life easy. We have to solve the you know the this linear differential equation which is given by two. So, you know to begin with let us take the Laplace transform of both sides of equation 2.

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Handwritten notes on a whiteboard showing the Laplace transform of a differential equation. The top equation is $E \cdot \frac{d[u(t)]}{dt} = R \cdot i(t) + L \cdot \frac{d[i(t)]}{dt}$, with terms circled and numbered. Below it, the Laplace transform definition is given as $F(s) = \int_0^{\infty} f(t) e^{-st} dt$. Further down, the transformed equation is $\frac{E}{s} = R \cdot I(s) + L \cdot [sI(s) - i(0^-)]$, with terms circled and numbered.

So, on the left side we will get E and the Laplace transform of u of t we already know how u of t looks like. We have drawn. It is equal to R times the Laplace transform of i of the response that we are interested in looking at or understanding plus L times the Laplace transform of d by dt of i of t the derivative term of the response function i of t .

Now, this old style capital L is used here to denote the Laplace transform. So, when we write the old style capital L of u of t or of i of t , we basically mean that we are doing the Laplace transformation of u of t or i of t or the derivative of i of t . Now, by definition if I want to do the Laplace transform, if I want to do if I want Laplace transform of a function f of t , I will say that the Laplace transform of the function f of t would be given

by an integral of the integration actually is the definite integration which goes from 0 to infinity and integration of 0 integration of the function f of t from 0 to infinity into e raised to power minus s of t dt .

So, this is how we get the Laplace transform of our time domain function f of t . So, basically you just multiply by this exponential term e raised to power minus s t the function whose Laplace transform you want to determine and then, do an integration from 0 to infinity and then, what you get is the Laplace transform.

So, coming back to our equation what we were solving using the fact that the Laplace transform of u of t can be shown to be equal to 1 by s . The Laplace transform of i of t let us say it is I of s if t is what we are trying to determine. So, I of s we do not know, we do not know i of t and that is why we are doing all this. That is why we are solving this differential equation and similarly let us denote so ok.

So, this is what we use this notation is what we use and this is the you know the Laplace transform of the unit step function. Then, using the this back in equation 3, I can write down E over s is equal to R time. So, E over s 1 over s is nothing, but the Laplace transform of u of t r into Laplace transform of the current I of s plus I . Write another term I times. Now, this is important. Now, what we have done is here we had this derivative term. This derivative term of the current the response function in this case. So, we need to write the Laplace transformation, we need to write the last for Laplace transform of the derivative of i of t .

Now, this of course we have seen before will be given by s times I of s minus i of 0 minus and just back it closed. So, this just this is nothing, but the Laplace transform of this derivative term and you can see that it involves the initial condition. So, when we will use the Laplace transform, you will find that it to really simplify the three step solution. The classical solution into a much simpler one, it into you know almost one or two step process more straightforward it, but of course you will also have you will have to still you know work with the initial conditions and as you will see later you know also with the roots of the characteristic equations.

So, these are several things that we will have to compute apart from the other things that we are using the Laplace transform for ok.

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Using I.C. | $i(0^-) = ?$
 $i(0^-) = i(0^+) = 0$ (jw)

$$\frac{E}{s} = I(s) \cdot R + L s I(s) \dots (5)$$
$$I(s) = \frac{E/L}{s(s + R/L)} \dots (6)$$

Now, the question is before I solve this further, I need to know what I put for I of 0 minus in this case. So, this is you know a 0 minus which is shown just to kind of denote just to kind of you know highlight that we are talking about a time just before we close the switch. Now, there was an inductor in the circuit before the switch was closed. We know that there was no current in the circuit and after the switch has been closed also because of the presence of the inductor, there would not be any current.

So, basically i of 0 minus is equal to i of 0 plus there is nothing, but 0 in this case. So, using this you know in the previous expression which I can show you, so this is the last expression. We got equation 4, I can write. So, using this using our initial conditions we can say that E by s will be equal to we can write I of s into R plus L s of I of s Laplace transform of i of t Laplace transform of L of s and this s is the transformation that we are doing. So, from time domain we have gone to s domain and as we have seen before this s is nothing, but j omega.

So, basically you can say that from time domain in a way we have just gone to the frequency domain ok. So, this is number 5, equation number 5 and we can now just kind of rearrange this you know just like an algebraic equation and we can just write we can get L of s as E by L divided by s into s plus R by L ok. This is what we get ok. So, very nice we got an expression in s domain, we got an expression for the current or I should say the Laplace transform of current we got an expression for that.

Now, I do not understand this language. I do not know what this means I am used to working in the time domain, I am not used to working in s domain. So, I must get this thing back or inverted into a time domain expression and then, I will be able to understand what it means.

So, it is already, so solution is right there in front of us, but it is in s domain. Now, how do we do this? As you can see that you know there is a product of you know these expressions which both have s and the way to do this is by using what is called the partial fraction expansion. So, we use, so let me just call this is expression 6 ok.

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$$I(s) = \frac{E/L}{s(s+R/L)} = \frac{k_0}{s} + \frac{k_1}{s+R/L}$$

$$k_0 = \left[\frac{E/L}{s+R/L} \right]_{s=0} = \frac{E}{R}$$

$$k_1 = \left[\frac{E/L}{s} \right]_{s=-R/L} = -\frac{E}{R}$$

$$I(s) = \frac{E}{R} \left[\frac{1}{s} - \frac{1}{s+R/L} \right]$$

$$i(t) = ? \quad \frac{E}{R} \cdot 1 - \frac{E}{R} \cdot e^{-R/Lt}$$

$$f(t) = \frac{1}{s}$$

So, now solving it using the partial fraction method that we have briefly seen before we can say that the trick is to say that I of s is equal to E by L divided by s into s plus R by L. You know I just write this you know as a sum of two expressions or two partial expressions k 0 by s plus k 1 s by s plus R by L. Look at this expression.

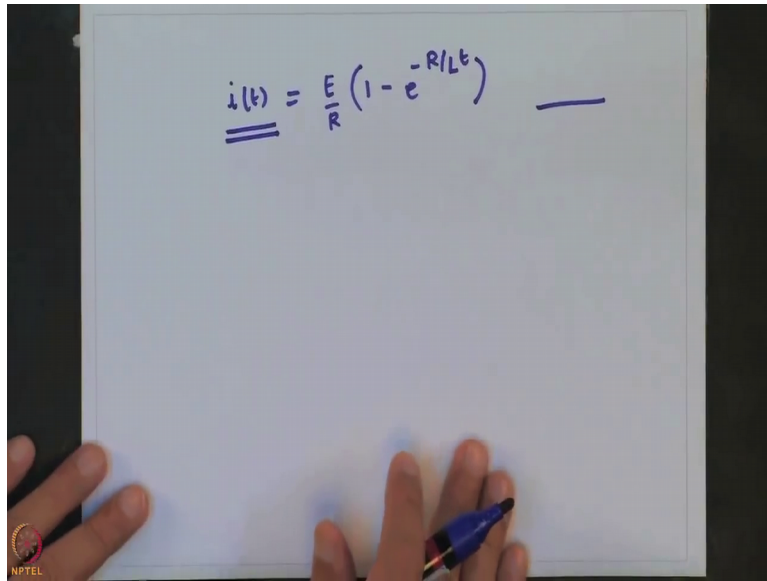
So, we have actually written this in this manner. So, we say that this actually is representable in this form. The only thing is I do not know what k 0 is and what k 1 is. It is very simple. As we have seen before that we can straight away find the expression for k 0 which will be nothing, but you know our this expression ok. So, because we are finding out k 0, we will take the term on this side which has this particular factor over there.

So, this means that I just do s is equal to 0 and I will take just this factor out of this you know this product here and this will straightaway give me E by R . If I just if L substitute s is equal to 0, I will get an E by R \ln and it will cancel. Likewise I can get the value of k_1 which is nothing, but E by L whole divided by s . So, I have now taken you know this term because I am trying to determine k_1 and I will try to I will just substitute s is equal to minus R by L .

You know as is the method is the rule that we use to determine the partial fractions, these coefficients k_0 and k_1 , so you get this and therefore, the moment I put this E by R for k_0 and minus E by R for this k_1 , I can go back and write the expression that we have obtained before you know now as a very nice clear factored sums. So, I of s will be equal to E by R and the first term would be 1 by s and the second term would be minus of minus 1 over s plus R by L . This will be the term, ok. So, coming back to you know our objective that we want to find out from here what is I of t . So, all I need to do is now I have now these two expressions and these are very nicely identifiable with a set of you know such similar expressions that are available in standard tables. You know usually what people do, they will just find out the Laplace transforms and the inverse Laplace transforms of some of the very common functions and they will actually put them in a table. So, here I will show you such a table.

So, all you can do is now you can identify what is the inverse Laplace transform of this term and this term. So, it can be very easily verified that the Laplace transform, the inverse Laplace transform I am sorry of 1 over s is nothing, but 1 , because if your f of t is 1 and if you do the Laplace transform of that you actually get 1 over s . So, it is just 1 and you have another term e by minus of E by R and you will see that this is nothing, but the Laplace transform of R by L into t . So, basically 1 over s plus R by L is the Laplace transform of this number, this expression. Here this can be directly identified or is identified in general by looking at this some of this standard and very often used frequently used Laplace transforms of common functions and their inverse Laplace transforms.

(Refer Slide Time: 54:26)



The image shows a whiteboard with the equation $i(t) = \frac{E}{R} (1 - e^{-R/Lt})$ written in blue marker. The equation is underlined. A hand holding a blue marker is visible at the bottom of the whiteboard. In the bottom left corner, there is a small NPTEL logo.

So, basically what you now get complete expression is i of t equal to E by R into 1 minus E raise to power minus R by L into t . Now, this is a time domain expression, this equation is a time domain equation and in fact, we can see that we have got how the response function i of t the current in the system would behave. We note that the classical solution is a three step process while the Laplace transform is just a one step process to get the complete response. I hope that the tools we have learnt would be useful for analyzing the power electronic circuits that are to follow in this course.

I thank you very much for your attention.