Fundamentals of Power Electronics

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Lecture – 05
Analysis of Switched Networks

[FL] Knowledge is supreme.

Welcome back. Now, we have learned so far an interesting fact that power electronic circuits can be represented by a switching matrix. We have also seen another representation of power electronic circuits in terms of the switching impedance which give additional insight into the nature of switches and the desirable characteristics of their operation for the four basic power conversion operations. Indeed several power converter configurations and topologies are available, for each of the basic power conversion configurations and they can all be derived from the general switching matrix. However, besides comprehending the operating principles of the power converters, their analysis is equally important not only for enhanced understanding, but also for the design and practical realization in an optimal manner.

So, obviously, the question arises as to which analysis method and tools we should use, which ones would be the most suitable? To answer this question it is important to have some idea about the nature of the power electronic circuits, at least the general representative circuits. The switching matrix is certainly a good and compact representation of power electronic circuits as we have seen, but it does not provide the field, the physical field of the actual power electronic circuits.

Now, we begin this session by looking at some of the important representative power electronic circuit configurations which are frequently encountered. Their nature and working will give us clues about the appropriate analysis methods that can be used to analyze the power electronic circuits. The presence of one or more switches is what distinguishes a power electronic circuit from non-switched electrical networks. The ON-OFF operation as you know of the switch brings the power electronic circuit in the category of the so called variable structure systems. So, this is what usually the control engineers prefer to call such systems which change their form or configuration while operating.

Now, what exactly does a switching operation in a circuit it achieved, what does it do? So, it may connect an energy source to the network or to the circuit, it may disconnect an energy source, let me take out of the circuit or it may alter or modify the circuit configuration in any desired manner required by the operation. The actual switches in power electronic circuits are the diodes, the BJTs, SCRs, IGBTs and the MOSFETs what which I briefly mentioned in my earlier lectures, but for now it is enough not to really worry about their specifics.

So, let us got worry about the specifics on the switch. Let us assume the switch to be a you know general ON-OFF device with all ideal specifications and characteristics for its use you know in the given circuit. Once we develop a basic understanding we will replace the ON-OFF switches by the actual switching devices and that will be really convenient, and it will help and enhance over understanding. Now, let us just review quickly some of the commonly encountered situations in power electronic circuits.

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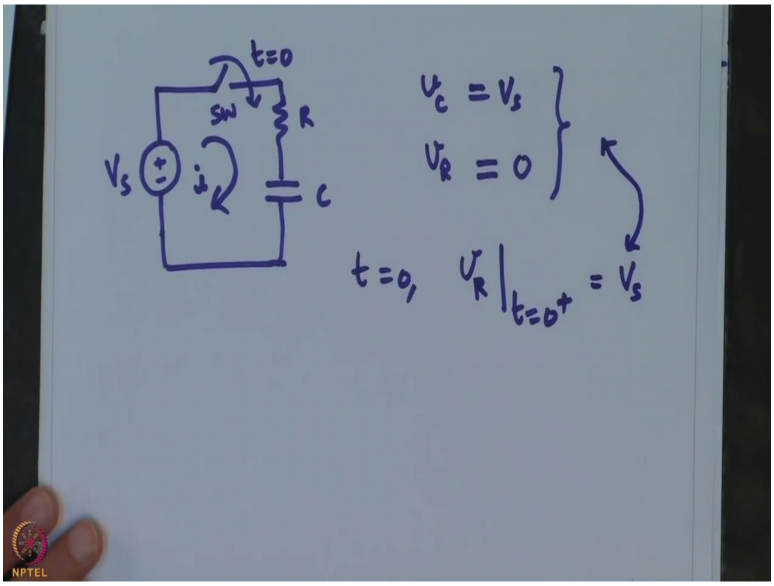

Let us say that there is a DC source V s that is supplying power to an R C network and there is a current that flows whenever the switches turned on or closed, let us say it is done at t is equal to 0. Now, if we analyze this circuit by using the very very fundamental and basic concepts, we can say that a long time after switch has been closed; the capacitor voltage V c will be equal to V s. Now, due to the kvl the Kirchhoff's voltage law the voltage across R during the steady state will be 0. But what we observe is that the steady state solution that you observe here is not really predicting the value of the voltages, the voltage across the resistance for all times after the switch has been closed.

This is because when the switch is closed at t is equal to 0 the voltage across the capacitor cannot rise suddenly. And the Kirchhoff's voltage law demands that the entire source voltage that is applied; therefore must appear across the resistance which means that the initial value of resistance at some point 0 plus immediately after the switch is closed should be equal to V s. But this is not predicted by the steady state solution that we achieved through our basic understanding. There are other circuits also which are important, but we will find that it is difficult to there is a problem in predicting their complete response if they are switched; it is not possible to predict the values correctly for all times after the switch has been closed.

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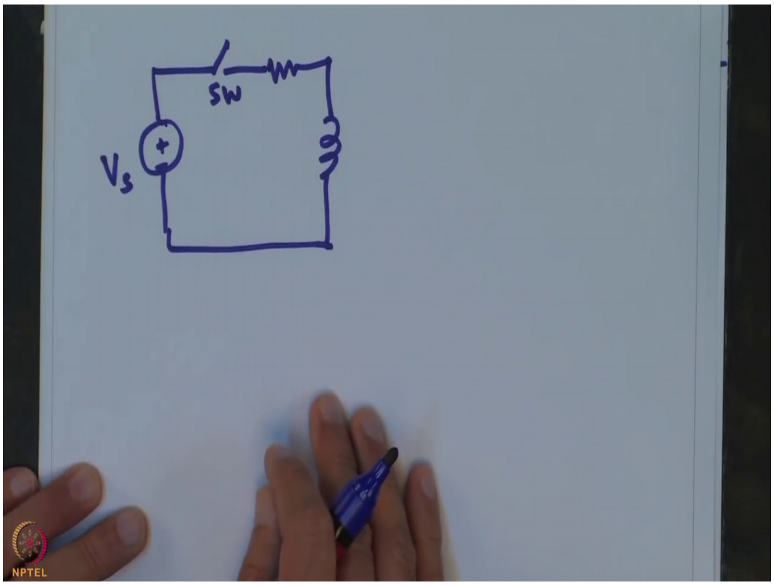

A similar thing can be seen in this example where a DC source V s feeds an R L network. And as we will see later once again if we try to use the our basic understanding about the steady state operation, we are not able to predict the operation of the circuit at all times all instants after closing the switch. This is observed in several other networks also. We can see that the same situation will happen whether it is AC source or DC source whether the network is a simple first order system or it is a second order system, or it is eleven more complicated, we will not be able to we are not able to predict the complete response as it is called by just using the steady state solution.

So, what do we observe what is our conclusion from all these samples that I try to give. So, we can say the first observation is it is true that the network may be non-linear in power electronic circuits when we take it as a whole along with the switch. However, between the switchings the network is linear. So, you can see that this is just a combination of R, L and C; you know which are all linear-linear elements? It therefore, follows all the rules of linear systems.

When we say a linear system, a given system is linear when it follows or it obeys what is called the superposition principle and the homogeneity principle. Now, fortunately this is the case with power electronic circuits, and this is what we will see now. Now, one of the a important things about the linear systems is that if you are looking at the response of a linear system if the forcing function f of t is sinusoidal, then we find that the response is also sinusoidal. The only difference that might come is that maybe the magnitude of the response and is different from the input, and maybe it has a phase with respect to the input. Also you know we must note that the linear time invariant systems they produce the same frequency as the source or the forcing function. So, basically the response has the same frequency as a source of the forcing function ok.

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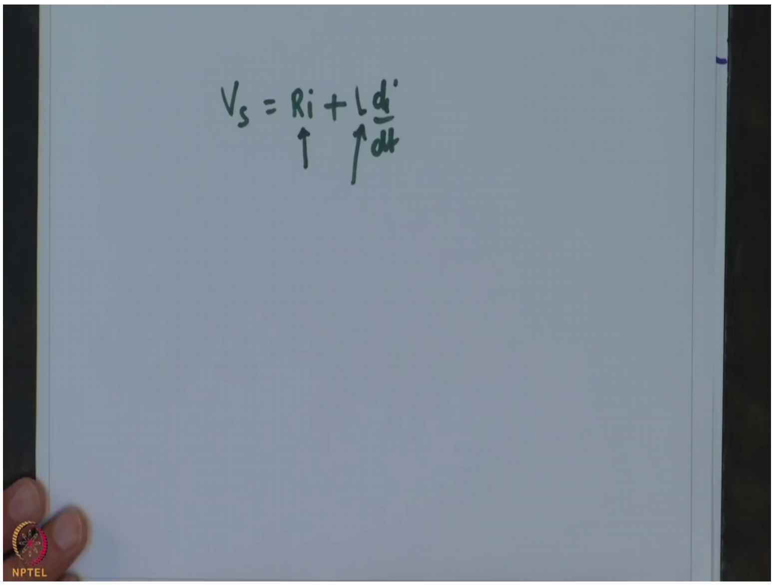

What else we absorb from the general examples we considered of power electronics. So, the general expressions for voltage and current, we observe that they could be written as in terms of linear differential equations with constant coefficients. For example, V s is equal to

R i plus L di by dt. So, here we can see that this is R and L, these are constant coefficients, they do not vary with time. We also observe that the circuit steady state analysis using the basic concepts, it does not lead to a correct solution for all values of t after the switch has been closed. It is expected that some correction factor is required to be added to the steady state solution to get the complete response which will be valid for all values of t after the switch has been closed.

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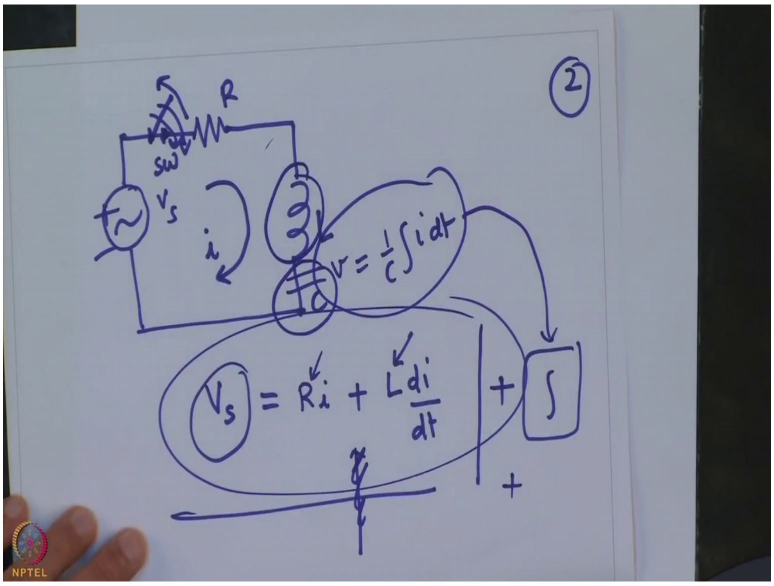

So, let us say I have a source V s, and it is connected to a network of R and L. And there is a current that flows in this system. I am not really right now saying that it is a switch system. To make it a switch system we will see what operations we need to do. So, this could be a DC source or it could be an AC source anything is possible. Now, if I want to know how the nature of current in this circuit what I would do, and how I would actually analyze. So, by using the Kirchhoff's voltage law I can say V s is equal to let us say this resistances R and this inductor value is L is R into i plus L di by dt ok.

Now, as you can see this is you know a differential equation. And we can also see that it is a linear differential equation, but it is a non-homogeneous differential equation because this is not 0. And remember that the coefficient of the various coefficients of this differential equations of various terms, you see R here L here they are all we are assuming that they are time invariant, they do not vary over time. So, basically this qualifies to be a non-homogeneous you know linear differential equation with constant coefficients.

Now, if we look at a switched circuit - a typical power electronic circuit, then I can say that it is going to have a switch here between this point. And let me just denote this switch by sw. Now, a typical problem that would occur because of the continuous ON and OFF of these switches of the switching matrix or the power electronic converter would be the ON and OFF of this kind of a switch. It will continuously switch ON and OFF. Now, we want to see that once the switch is turned on when the switch is closed and my current starts flowing how my current going to look like ok, so that needs basically an analysis and solution of this differential equation. We need to solve this differential equation to do that.

If I just put one C here, you can see that I will have an additional term that will come here that will be having a derivative term actually, because we know we are trying to write voltage. So, voltage across you know what will be the voltage across the capacitor, it will be 1 over C i dt, where C is the value of this capacitor. So, this term will come here. Apart from any initial conditions of an initial charge that might be present on this capacitor or any initial current that might be flowing through this inductor. So, the question is how do we solve such differential equations? And the very good point is that we have very nice and established tools to handle these kinds of systems. Let us just review how they are particularly solved.

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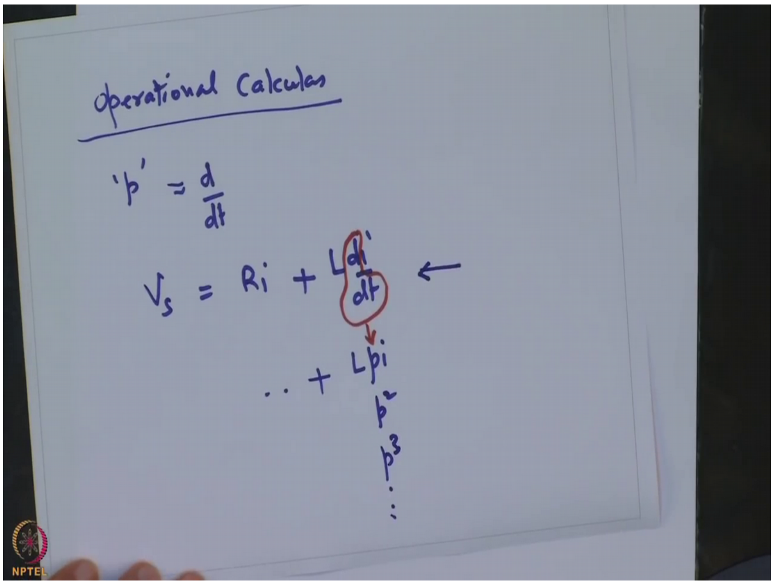

So, use this mean of what is called operational calculus ok. Now, let us try to go or has a step ahead and understand or review what is operational calculus, and how it helps us in analyzing or solving systems you know equations that I just described to you. So, one of the things is that you know there is this concept of an you know an operator which is usually defined by p, and it actually means d by dt So, this is also called a differential operator. We define this.

And if I define this to be equal to d by dt then in my equations in my differential equation wherever I have the presence of a derivative, I can replace that by p. So, what will be the result of that my previous equation which is V s is equal to R i plus L di by dt could be replaced by you know L into p into i, where this d by dt has been replaced by this differential operator p. Now, clearly if this was d square by dt square, then this would become p square. If it was d cube by dt cube, then this will become p cube and it goes on.

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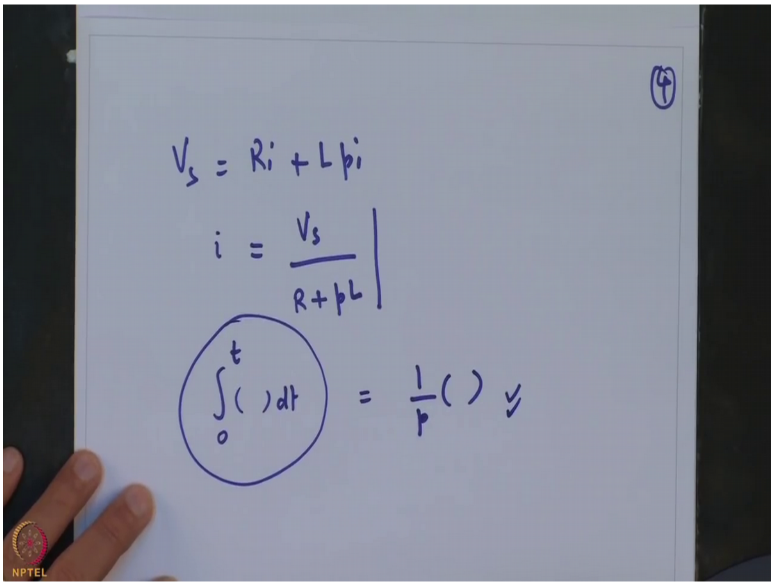

Now, what is the advantage of in this case, coming back to the same equation again what is the advantage of doing this is that now because you are able to write V s is equal to R i plus L p i. Actually you can deal with it as an algebraic equation. So, the introduction of the differential operator p has helped us in converting a differential equation, a linear differential equation into an algebraic equation which can now be solved. So, basically you know if I am trying to look for the solution for i, then i would say it is simply as R plus p L, simply it would be like this. So, you can clearly see that the introduction of p this operator has really helped us in simplifying the differential equation - its solution and as well as its representation. So, it can be represented as an algebraic equation.

Now, not only that you can use p to denote d by dt, but you can also denote the integration or the integral functions using this p. So, if you want to do an operation of this type, all you need to do is that you just do 1 over p multiplication just by 1 by p. And this will actually give you the integration of a function.

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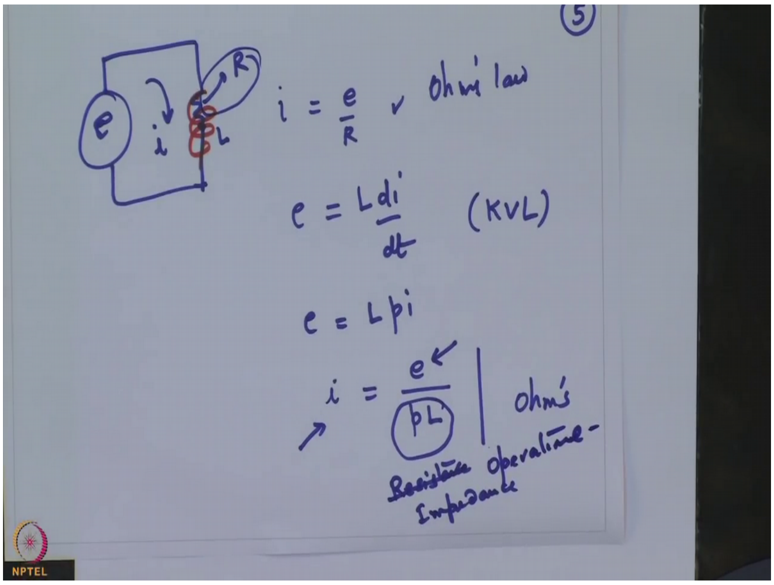

Now, let us say we have a case where a source e you know is applied to a resistor then I can say that the i will be equal to e over R by Ohm's law, there is no problem in this. But let us say that instead of a resistor, this was an inductor, it was an inductor, then we have to see that a e is equal to L di by dt, sorry I should have written in i here ok. And this value of the inductor is L, earlier the resistance I used to values R let us say. Now, we are considering the second case with only inductor being supplied by this excitation e. So, as you can see e is equal to L di by dt is what we will get from the Kirchhoff's voltage law.

Now, using our differential operator p as before I can write e is equal to L p into i. And now I can say i is equal to e over p L. Now, what does this equation show, this again you can you know you can see this is ohm's law just like you have ohm's law here. This is also ohm's law, but if it is ohm's law and you know this is applied voltage and this is current, and obviously this is nothing but resistance. But we know that this resistance is not having the same nature as the resistance R that we have employed before this is a very different type of resistance.

And we will see the details probably later, but it is usual to call this one not resistance, but impedance. And it is all it is often called because it actually is using the its also involving this linear of this differential operator p, it is actually called operational impedance, operational impedance. Just way as we saw the example of an inductor, we can see that for a capacitor.

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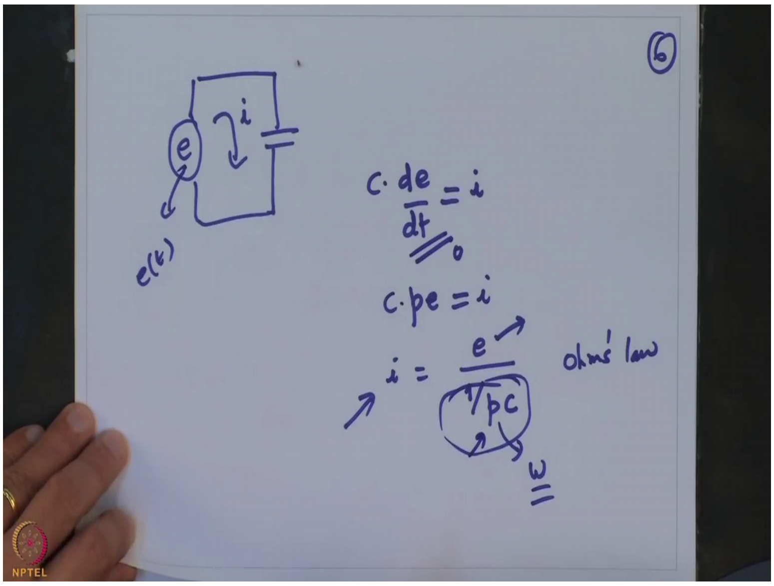

So, if you know that from excitation of e is applied to a capacitor, then we can say C de by dt is equal to i. And then using the operator again we can say that C p e is equal to i. And from here actually we can manipulate this equation a little bit, and we can say i is equal to e over 1 over p C. Again this is ohm's law. Voltage is there; current is there. So, this must be resistance or as we learned in the case of inductor this must be an impedance. And that impedance value is given by 1 over p C.

Now, we will see in due course that whenever the excitation becomes sinusoidal, whether it is this case or the previous case of inductor, or it is any other case or indeed even more complex with any complexity switching circuits, where sinusoidal excitations are involved there this we will see that this p will actually be replaced by omega. Omega is nothing but the angular frequency of the sinusoidal excitation applied to the network. So, you will see that this will actually get replaced by that.

So, if a sinusoidal function is there, so this is a pretty general representation where we have not mentioned what is the form of e actually strictly speaking I should be writing maybe e of t unless it is a DC value ok. In which case of course, you know there will be a 0 coming in here at this point. So, once again 1 over p c is the impedance, and it is also called an operational impedance because it involves p. Now, to distinguish it from the conductive impedance, the case previous case where inductor was involved, this is called an inductive operational impedance. And the one in this case where there is a capacitance, this is called a capacitive operational impedance. Let me show you one example.

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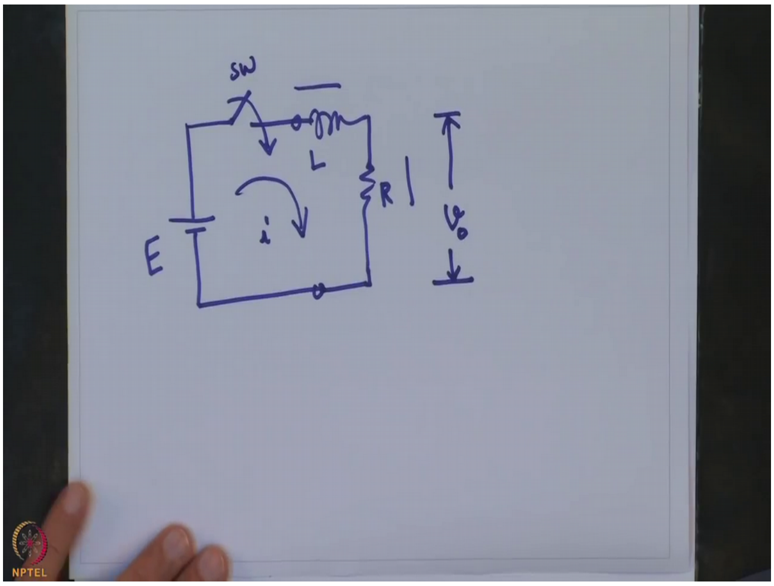

So, let us say we have a circuit we have a network which is supplied from a source e and let us say it has a switch now. So, talking about the switch circuits in power electronics, let me just bring in this switch here. And let us say there is an inductor L and a resistance R. Now, the moment this switch is closed, the applied source function e which is in this case a battery e gets applied to this network which consists of R in L, and it results in certain current and; obviously, it will result in some voltage across the inductor and the resistance.

Now, what we want to do is we want to determine let us say the voltage V o across the resistance R after the switch has been closed. So, let us say the switches SW. We want to determine in this output voltage after the switch has been closed. Let us say we are interested in knowing how the output voltage V o cross R varies as a function of e.

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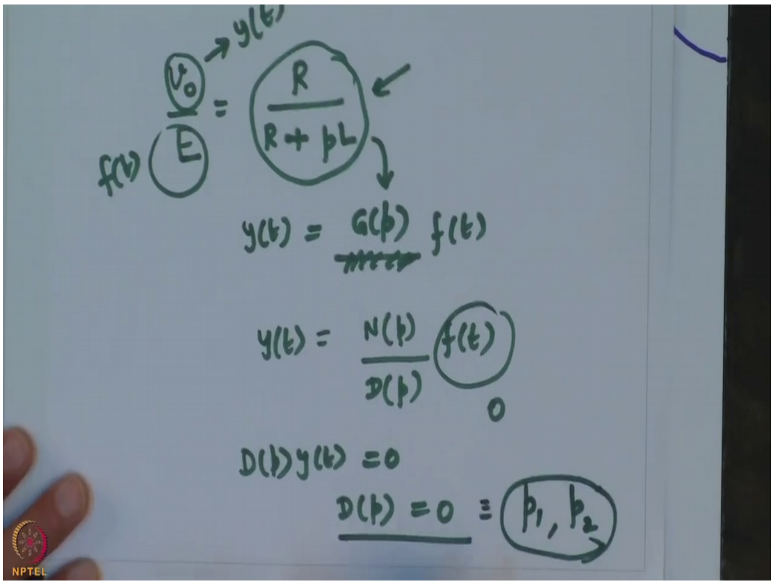

Using the potential divider i can straight away write Vo by E is equal to R over R plus p L, where p L represents the inductive impedance of L. Now, in this expression, we can actually say that E is nothing but the forcing function and we may denote it by f of t. V o may be considered as the response and we may denote it as y of t. Then we can say that y of t is equal to some function of p into f of t. Now, it is this function which actually comes here which is denoted by G of p, because we can see that this expression is a function of p. G of p is called the network function or the transfer function.

When we talk about the response y of t, it actually would consist of two components; one of them is the force response component, and the other we will see would be the natural response component. The first response component is actually obtained by using the non-homogeneous linear differential equation; it is called the force response because it is it remains as long as the forcing function remains. However, it becomes prominent in the steady state. In the initial transient state, after the disturbance has come like the closing of the switch, it actually is dominated by the natural response.

The natural response which is the second component dies down with time. And the solution or the response y of t it actually becomes equal to the steady state response a long time after the switch has been closed in the steady state. Now, G of p can be represented by N of p divided by D of p you know these are two polynomials in p. So, if we can see this general network which will obviously, get more and more complex, if we see more complex networks which will not just be a simple first order system where p you can see is only having a power of 1, it may have many more. So, it will be a general polynomial in n. So, y of t can be written as N of p by D of p into f of t.

Now, the determination of the natural response is done by considering the homogeneous equation corresponding to the original non-homogeneous will be a linear differential equation representing the system. So, basically what we do is we make f of t is 0. And when we do that we actually end up with D of p into y of t is equal to 0. And since we are talking about a non-trivial response or solution we can say that this actually means D of p equal to 0. Now, when we solve this polynomial D of p which is equal to 0 this equation, D of p is polynomial equal to 0 this equation is also called the characteristic equation of the system. And it actually governs the behavior of the natural response of the system as we will see in examples that will follow now.

Now, depending on the order of the system which is actually determined by the number of energy storage elements like how many L s and C s are there, the order of this characteristic equation is determined by that. So, if you have one c and one L, the order is 2. If you have only one L and there is no C, then the order is 1. And if you have two Ls and one c, then you have an order of 3. Depending on the nature of the roots that this equation gives basically which actually has come down to D of p equal to 0. The solution that it gives let us say it gives for a second order system 2 roots p 1 and p 2. So, it is the nature of these roots p 1 and p 2 that determines the response the nature of the response, the nature of the natural response of the system. We will see these things in details with the help of some examples in due course.

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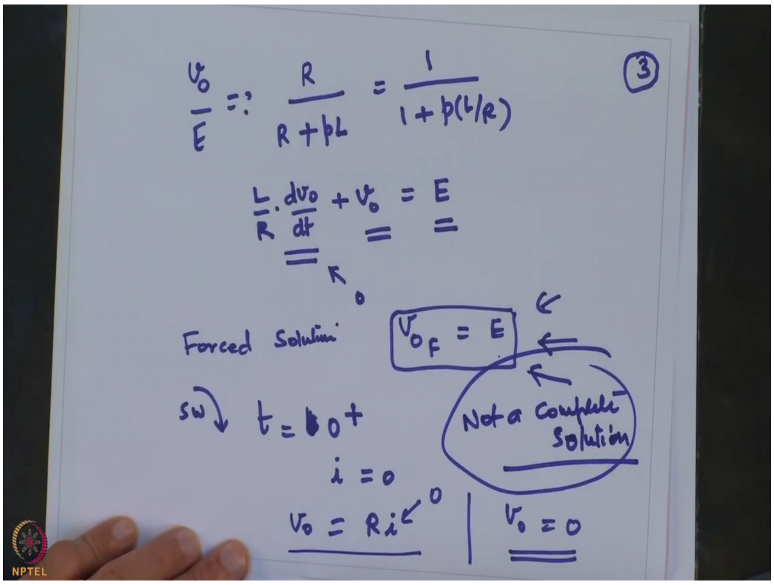

Now getting back to our original problem that we wanted to see how V o varies with e or we wanted to determine V o by E we can write the following expression again. What is V o over E using the potential dividers and using the differential operator p we can straight away write that V o is nothing but V o by E that ratio is equal to R over R plus p L which I can just manipulate a little bit and write it in a different form, this way. Now, this gives me L by R into d V o by dt plus V o is equal to E.

Now, in this steady state in a long time has passed, long time has passed after the switch sw has been closed, then we can we know that our derivative term will go to 0. So, we can say that this is 0. So, I can say that my V o is equal to e long time after switch has been closed. Now, this is what is called the forced solution. So, basically by putting the derivative term equal to 0 for a DC excitation source, what we mean is that in the steady state, we do not expect any time variation you know in the response. And that is why if we take that term if you make that term 0, it actually means that we are making p is equal to 0.

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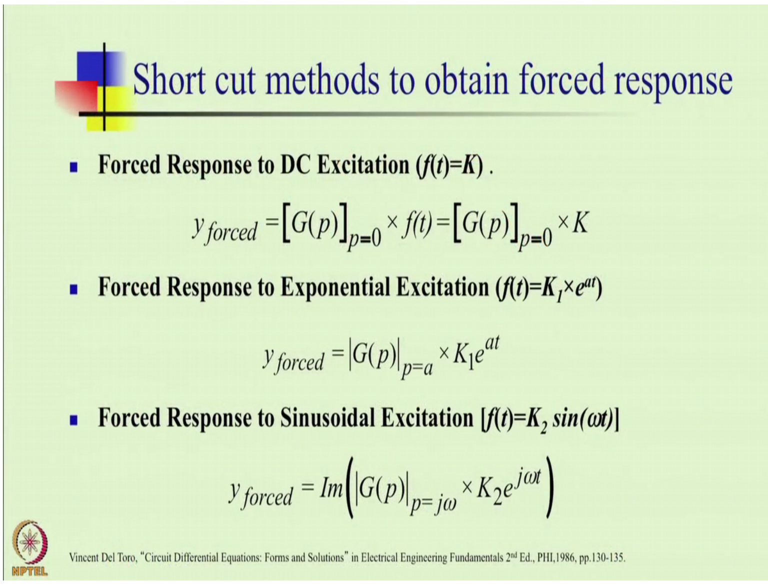

Now, similar shortcut methods are applicable when other types of excitations or sourcing functions are used. For the DC excitation, if we put p is equal to 0 in the network function as you can see in the slide the first force response method that we have put there, now we can straight away get the force response to a DC excitation. Similarly, for an exponential excitation, we have written that how a force response can be obtained by putting p is equal to the coefficient of the exponential power in the network function. Similarly, we can obtain the force response to this sinusoidal excitation also. So, these shortcut methods are very handy in quickly getting the force response.

So, V o after a long time the switch has been closed is equal to E, and we are saying there is called the force solution. And let me just say that V o F for the force part is equal to E, let me just write this ok. Now, the question is that when the switch was closed, when switch was closed, immediately after that immediately after that let us say at some time p is equal to 0 plus what would be the current in the system, the current i in the system i will be 0, because there is an inductor in the circuit and we all know that the current through the inductor cannot change suddenly.

So, initially if there was no current in the inductor, it will remain like that even after closing the switch at a very small time at every infinity similar time after the switch has been closed at t is equal to 0. So, t is equal to 0 plus also you know your i is 0. Now, if that happens if i is equal to 0 immediately after the switch has been closed, then what will be V o. Now, we know that V o is nothing but R into i. And if i is 0, that means, V o is 0 also. So, the voltage across the resistance which we are saying is output voltage is 0, because the current through the inductor cannot change suddenly which means that the solution which we just obtained you know we just obtain this solution by which we said is long time of the switch has been closed is no longer valid.

As per this solution what we have seen before V o f should be E. But as per the solution that we have seen now by doing this simple you know explanation, because there is an inductor the current is 0 by a simple analysis and understanding, we found that V o is equal to 0. So, basically what we are saying is that this solution is not a complete solution is not a complete solution. There must be something else in this solution which will make it valid for all times starting from t is equal to 0 plus all the way up to the you know a long time which is actually called the forced state or it is called steady state. Now, everything has steadied after a long time, everything is in a stable steady form now and we call its steady state.

So, clearly the solution we got what we called as a forced solution is not working out when we try to analyze the circuit for a very short time after the switch has been closed. So, what has gone wrong, what is the problem? So, problem is that we have not considered the response of the elements of the circuit themselves, we have not considered the solution that we have got the first solution that we got, we did not consider the property of inductor L. And for getting a complete response or to get a complete solution which will be valid for all times after the switch has been closed, we must actually do something to obtain or include the response of the inductor and the resistor also in the solution.

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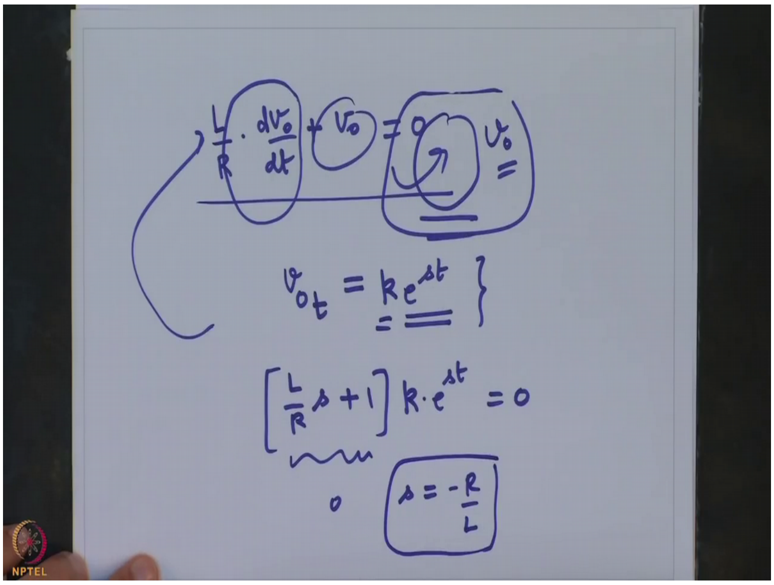

Now, how this can be done? Now, this is very simple complementary function represents the natural response of the system. So, to get this complementary solution complementary the second part of the solution as I mentioned some time back, you just do L by R d V naught by dt plus V o equal to 0. And this is a corresponding homogeneous equation, but the original non-homogeneous equation that we derived at the very beginning ok. Now, how can we solve this equation? As you can see that it gives us some ideas as you can see that if you know I take this V o onto this side some and also I take L by R term onto the right side, it will be like saying that you know there is some constant term and some variable which you are getting after doing the time derivative of that quantity.

So, you do a time derivative of V o, you get some constant and V o itself. Now, if we all know that there is only one function, the exponential function which has the capability to satisfy this; that is if you do the derivative of an exponential function, it will actually give you the exponential function itself. So, therefore, it is very customary to find V o to assume a form for this solution to be V o. And let us mention t here. The earlier one I mentioned V o F for forced, now I am saying V o transient. So, let us mention this t to indicate that this is a transient solution or the natural solution the due to the presence of the elements L and R in the circuit that solution trying to find out.

And let us assume by the explanation that I just gave you that this has the form k times e raised to power s t. So, this is I am assuming that it is some exponential function with some constant ok. Now, if we substitute this assumed value into this equation that we have the homogenous equation, I can say L by R into s plus 1 k e raise to power st is equal to 0, this is what I would get. And from here I can say that you know this term must be 0 which means that s must be equal to minus R over L ok. Now, in a moment we will see the significance of this particular value i. So, this actually is very very important relation, it is a very very important you know analysis that we are doing.

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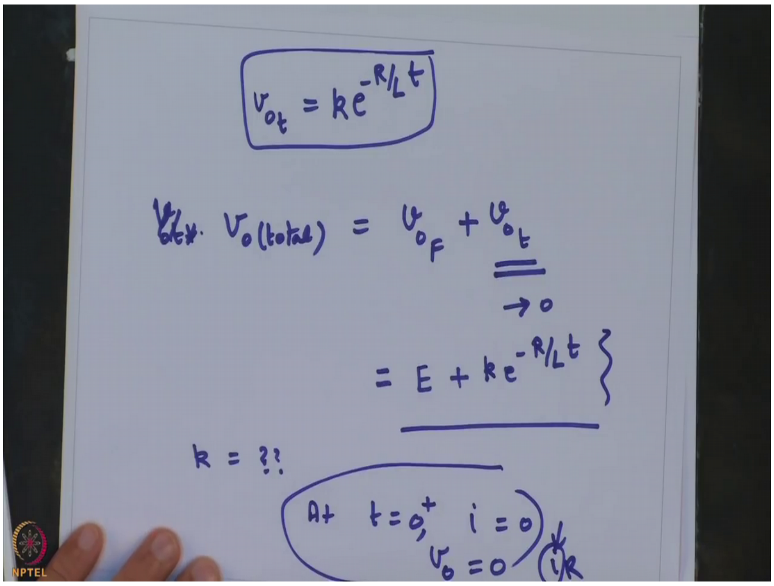

So, basically you know if I get s is equal to minus R by L then I can say that my assumed natural or the transient solution, I can now write as k times e raised to power minus R over L into t, this is what I can do. And considering our earlier discussion and our conclusion, we can say that the total solution, therefore the V o total, V o total is nothing but V o forced plus V o transient. And this particular term is a transient term which is due to the natural response of the system it goes to 0 with time.

So, as a time a long time elapses after the switch has been closed, this term will go to 0. So, what you by putting the values of V o F and V o t that we have obtained we know that V o F is nothing but E, and V o t is k times e raised to power minus R by L into t. So, we got this as a solution, the total solution. Now, the question is k is still unknown in this expression. So, how we can find this value of k, how we will find this particular you know expression for k? We note that we use the initial condition. We note that at t is equal to 0 plus i is equal to 0. And therefore, my V o is 0 also because V o is nothing but i into R. And if i is 0, my V o is 0, so V o is 0. Using this initial condition; using this condition because this equation the total solution must satisfy and under all values of time after switch has been closed.

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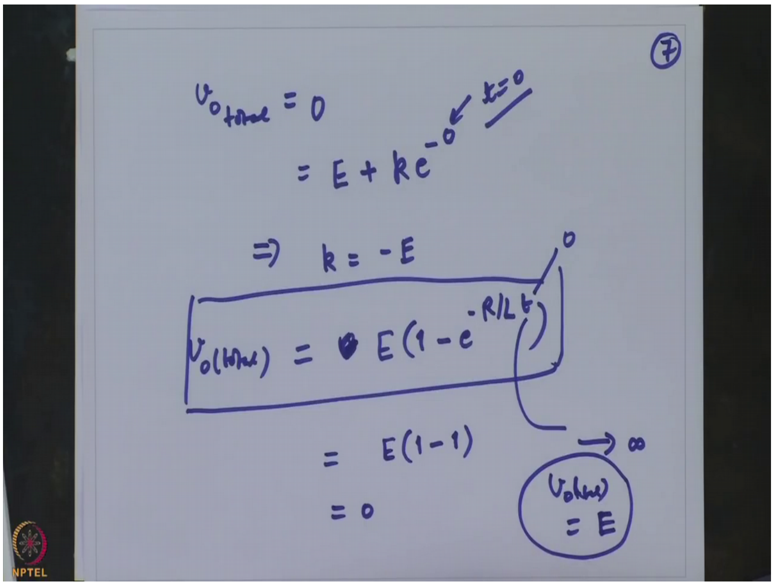

So, considering this particular initial condition, I can say V o total is equal to 0 is equal to E plus k e raise to power minus 0, because I have just said t is equal to 0 here. So, at t is equal to 0, we are using this condition that the output voltage is 0. And I have just substituted t is equal to 0 in the rest of the expression on the right side. This yields k as minus E. And hence I can write down now, I can write down the total solution to be equal to E into 1 minus e raise to power minus R over L into t. This is what I can write down.

Now, if we use this expression rather than the force solution you know the very beginning, if you use this expression, and we say that what is the output voltage when the time is close to 0 just after the switch has been closed t is equal to 0 plus, we just put this equal to 0. And you can see that this voltage will become E into 1 minus 1 equal to 0 which is as desired. And likewise when the t tends to infinity, you can say that the V o total is equal to E the forced solution. So, long time of the switch has been closed only the forced solution which is just because of the applied excitation function will remain the transient term has actually decayed to 0 ok.

So, now let me just generalize this ok. So, how do we get the total solution? We first will get the forced solution or the steady state solution. And then we will get the natural solution or the transient solution. And then we will add this transient solution to the force solution, and there will be a constant. Now, then how many constants will be there in the previous example there was only one k. But if you have more number of elements which can store energy like you have more number of Ls and Cs, then you will find that the number of constants which we need to determine they will also increase. And in fact, the number of constants that we will have to determine is directly equal to the number of storage elements used in a network which we are analyzing ok.

Now let us graphically see how these two will response the force response and the natural response looked like, and what the addition of the two leads to. So, what is the total response looks like.

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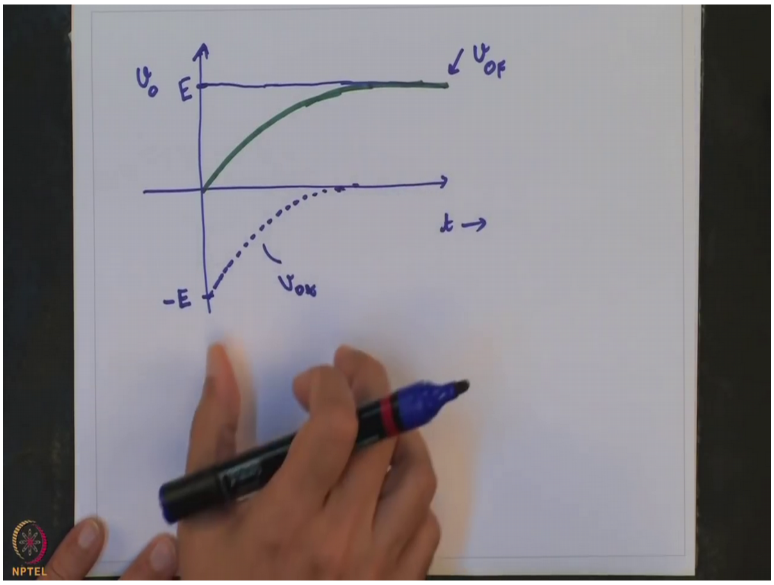

So, as you can see if I was to just plot these on a graph so let us say this is my time axis and this is the axis on which I am plotting the output voltage of interest in the circuit that appears across R. Then let us say if this is the point E in terms of magnitude, then I can say that this is the forced response. And if I say that this is the minus E point the graph, then I can say that this is the natural response. And when I combine these two that is so I can actually mark this as V o t the transient response, and if I combine these two, if I add these two, I can say that I get nice response like this shown by green. And it actually becomes same as E, because you can see that after this point here my transient response has become 0. So, after some time you know after an initial phase, the switch has been closed here at this point. So, this is in the time, this is the instant at which the switch was closed.

So, after some time the natural response has died down, and what remains is only the forced response. Now, this we will find this kind of an action, this kind of process, this kind of a phenomenon you know happening all the times in power electronic circuits. Power electronic circuits making use of several switches, they do go into these kinds of transitions. And depending on what is the network that we are considering there we will get different response.

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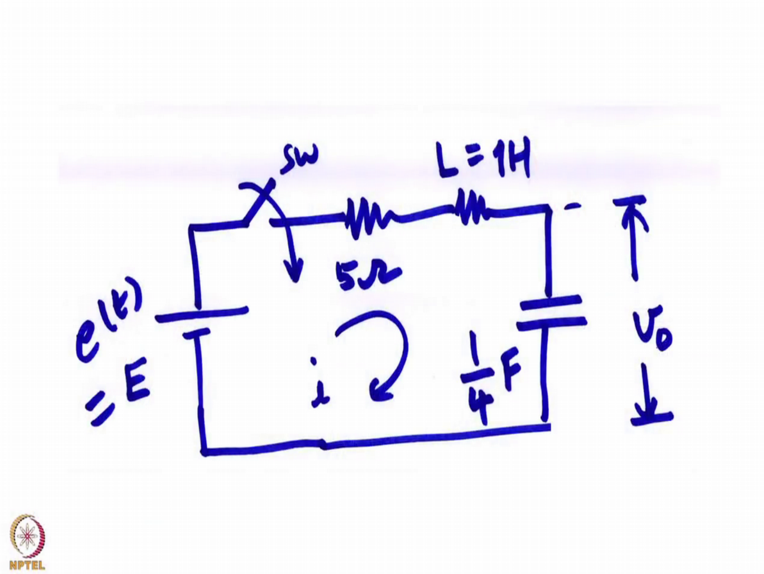

With the concepts learned so far, can you try to solve the above example, can you try to find out the complete response of the network shown above? Just to give you a hint as this is a second order system we expect that it will involve two constants whose values will have to be determined by using the boundary conditions. So, I can actually provide you the solution and I would like you to verify that indeed this solution satisfies.

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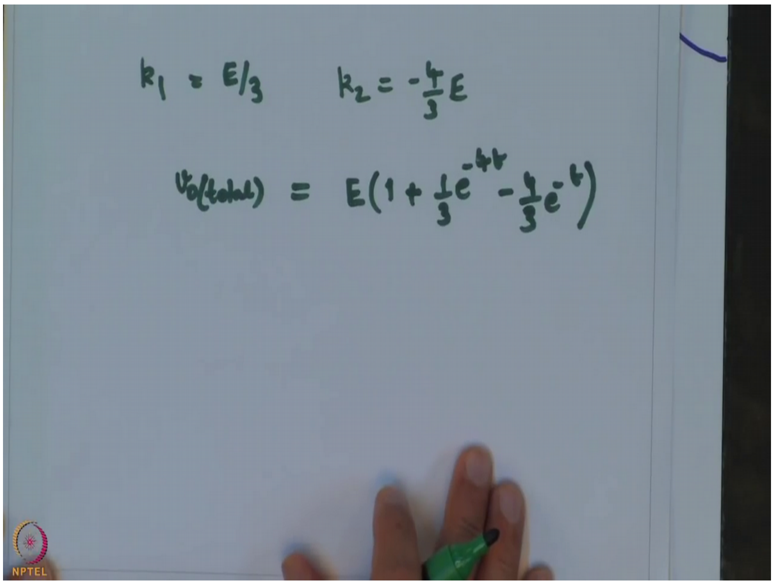

So, you will find that k 1 will be equal to E by 3, and k 2 is equal to minus 4 by 3 into E. The complete response of the voltage V o let us say total will actually turn out to be E into 1 plus 1 by 3 e raise to power minus 4 t minus 4 by 3 e raise to power minus t. So, you can verify this. And you can even try to plot this response. So, you can have the individual response, the natural response and the force response, you can plot them and see what you get.

I thank you very much for your attention. In the next session, we will see some other aspects, some other tools which will help us to analyze the power electronic circuits.