Principles of Digital Communications
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Lecture – 59
Differential Phase Shift Keying

In the earlier class, we studied non coherent binary FSK and we also said that the results can be extended in principle to any non coherent binary orthogonal signaling scheme. Today we will study non coherent version of PSK which is popularly known as DPSK; that is Differential Phase Shift Keying.

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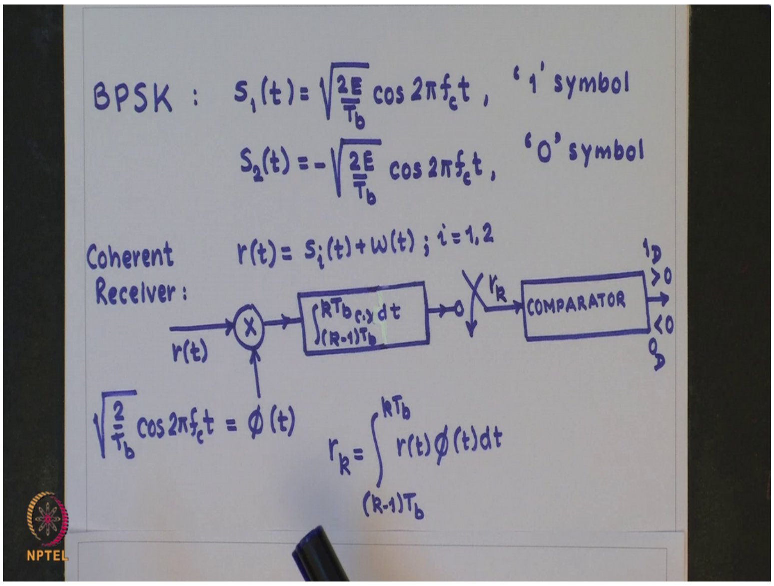

In the normal BPSK transmission we have 2 antipodal signal S 1 t and S 2 t which are being transmitted say for symbol 1 and 0 respectively. Now, we assume at the receiver that the there is a synchronization between the transmitter and the receiver; in which case the optimum receiver is a coherent receiver which is depicted in this figure, which we have studied earlier.

So, we have a reference signal out here which is in synchronization with the transmitted signal. Now, if we have a phase uncertainty in the received signal then we will get the following model for the received signal.

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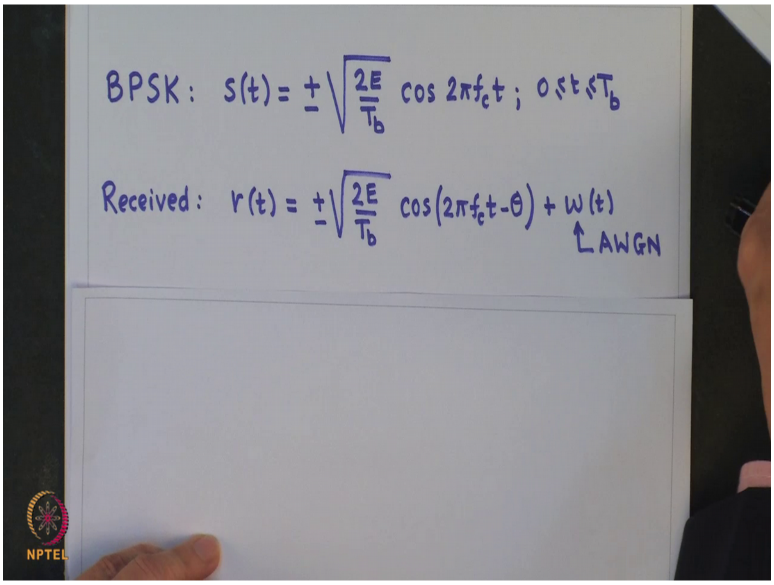

We have the received signal which is plus minus cosine with a phase uncertainty and additive white Gaussian noise as usual. Now, if this is the received signal and if that signal is demodulated using this coherent receiver, then we will get at the output this signal contribution from the received signal will get multiplied by cos theta term and depending on the value of theta the output will vary.

So, there will be uncertainty about the contribution from the signal component itself in the received signal.

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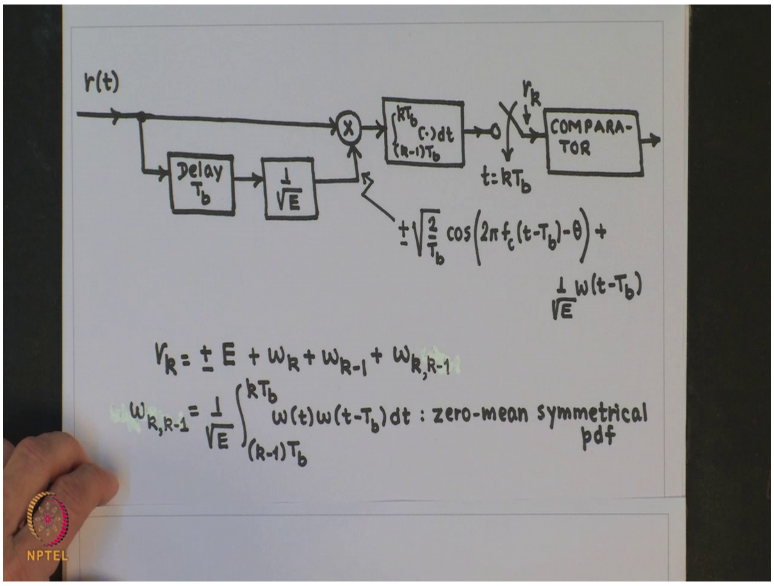

So, to take care of that we can provide the following solution provided we assume that the phase variation between the two consecutive signaling interval is slow enough. So, what we could do at the receiver as a local oscillator we can choose the delayed version of the received signal itself. In that case, our reference signal which will serve as a local oscillator would be given by this term where r t is this signal including the additive white Gaussian noise.

Now, the output here r k will consist of the following terms; you will have plus minus E which is a contribution from the signal part. Here again I repeat this is does not have any theta term because we are presuming that at this point and this point, the phase cancels out because of the slow variation and then these are the 3 noise terms.

So, W k and W k minus 1 are the usual noise terms W k is the projection of the input W t onto this signal W k minus 1, which is the projection of this noise onto the this portion of the received signal. And this term is equal to the cross product term between the 2 noises delayed by T b duration and this is zero mean symmetrical pdf.

So, for this configuration it is easy to observe that r k will be greater than 0, when present bit equals previous bit. And r k will be less than 0 when the present bit is complement of the previous bit; so, your decision would be as follows.

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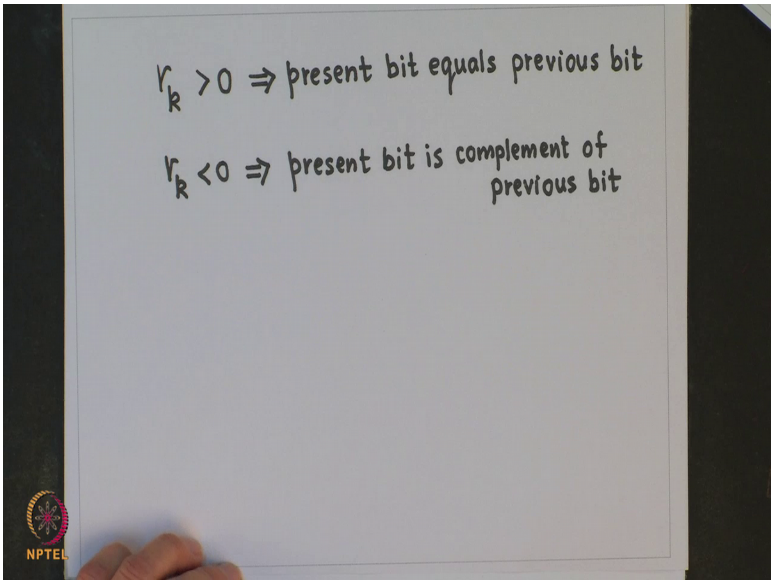

Now, the problem with this decision making process is that if one bit goes wrong then there is a cumulative effect because the decision is based on the previous bit. So, in order to take care of this we could do differential encoding at the transmitter end as follows.

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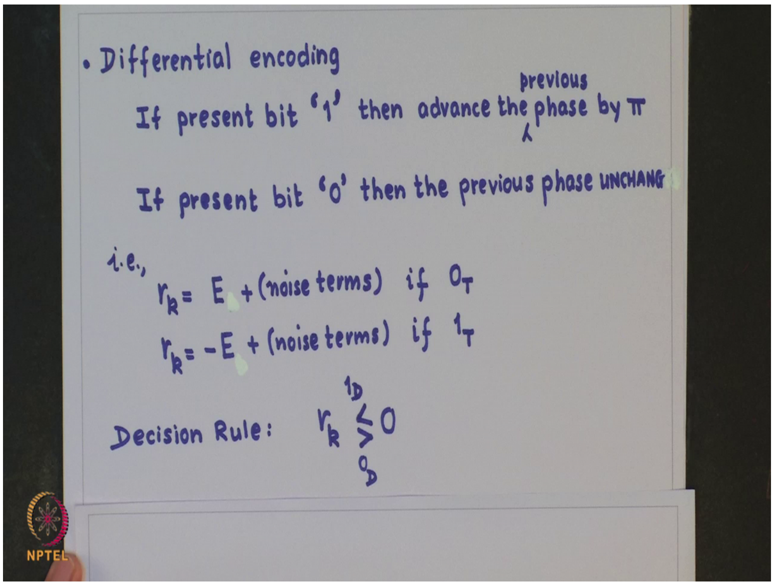

If present bit is 1, then while transmitting I advance the previous phase by pi radians; so 180 degrees. And if the present bit to be transmitted is 0 then the previous phase remains unchanged. If we use this strategy of differential encoding and then use phase shift keying for transmission, then it is easy to see that this game will work with the following decision process.

Now, r k will be equal to E plus some other noise terms will be there and this will happen if 0 is transmitted. And r k will be less than 0 that is minus E this is signal contribution if 1 is being transmitted because we have used the following encoding procedure. So, decision rule will be r k greater than 0 implies that I decide in the favor of 0 symbol and when it is less than 0, I say that 1 has been transmitted.

Now, the disadvantage of this procedure is that we have additional noise terms. So, the signal to noise ratio has deteriorated. So, the question is it possible to find an optimum receiver for DPSK? Now, we can observe that in DPSK, the transmission in the current interval is based on what has happened in the previous interval. In the sense that if I want to transmit symbol 1 in the current interval, then I will have to change the phase of the previous transmitted signal. And then I am transmitting the symbol 0 in the current bit interval; then my earlier phase remains unchanged. So, now, if you look DPSK signal over duration to T b we will get some additional information.

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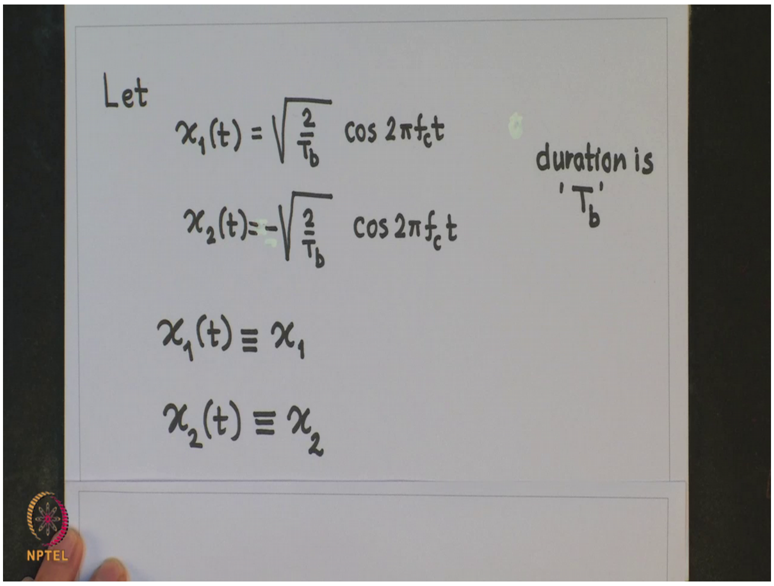

So, before we do that let me define 2 signals x 1 t which is a cosine of with the frequency f c and x 2 t which is antipodal of x 1 t; both these signals are of duration T b and for sake of brevity I denote x 1 t by just x 1 and x 2 t by x 2.

Now, if I see the DPSK signal then let me define signal S 1 t as the transmitted DPSK signal which is for the duration 2 T b. And this S 1 t is for the case for transmission of binary symbol 1 at the transmitter input for the second part of this interval that is between T b and 2 T b. So, the transmission of symbol 1 advances the carrier phase by pi radian.

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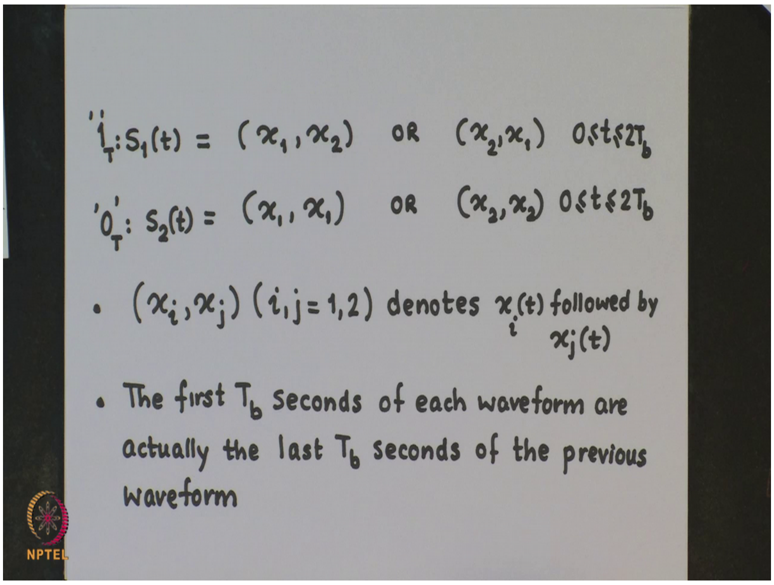

So, what it means that my S 1 t signal over a duration of 2 T b for the transmission of 1 will consist of x 1, followed by x 2 or it could be x 2 followed by x 1. So, the symbol is being transmitted during the second part of this interval 0 to 2 T b. So, it will change the phase; so, if it was x 1, it will become x 2 and if it was x 2 and x 1.

So, similarly let S 2 t denote the transmitted DPSK signal again for the duration 0 to 2 T b for the case when we have a binary symbol 0 at the transmitter input for transmission during the interval T b to 2 T b. So, the transmission of 0 will leave the carrier phase unchanged over the interval 0 to 2 T b and so, we define S to t as x 1 t followed by x 1 t or it could be x 2 t followed by x 2 t correct.

Now, x i and x j are taking values i j from 1 or 2 and it means that x i t is followed by x j t. So, the first T b second of each waveform are actually the last T b second of the previous waveform. Now, having defined this S 1 t and S 2 t over the duration to dB, we see that S 1 t and S 2 t can each have either of 2 possible forms.

So, it could be either this pair or it would be this pair correct; so, any pair you take from these 2 pairs.

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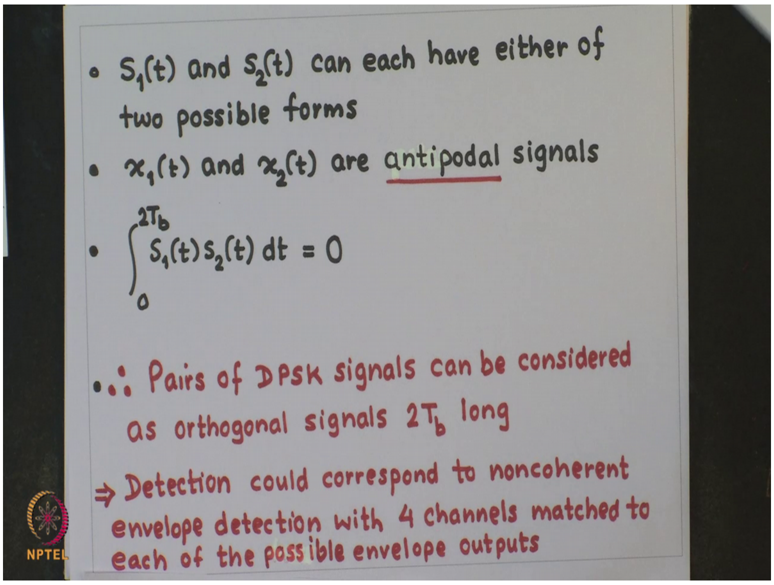

It is not difficult to see that because x 1 t and x 2 t are antipodal signal; S 1 t and S 2 t are orthogonal over the duration 2 T b. So, what this implies that pairs of DPSK signals can be considered as orthogonal signals, when we consider 2 T b long duration.

So, now we can extend what we have studied earlier for non coherent binary FSK case. We said that it is extendable to non coherent binary orthogonal signaling scheme also. So, detection now could correspond to non coherent envelope detection with 4 channels match to each of the possible envelope outputs; we have 4 cases here correct.

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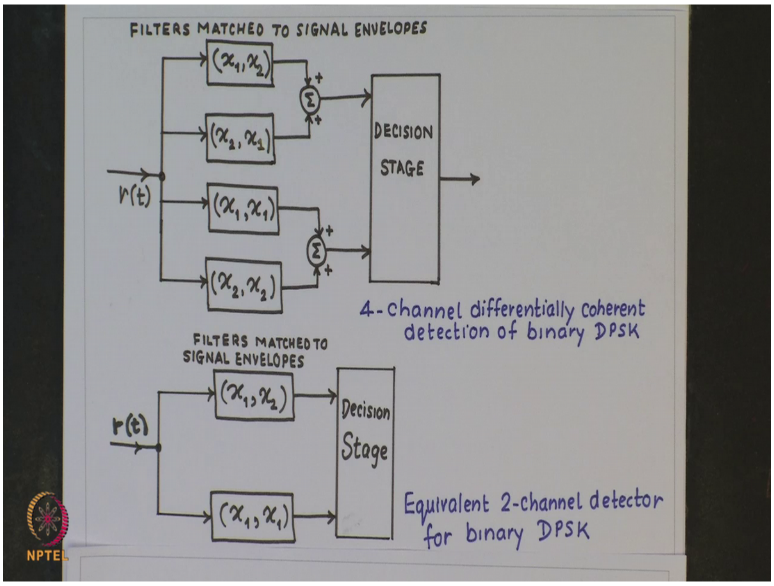

So, based on this reasoning; we can have the optimum receiver for the DPSK signal with the phase uncertainty as shown in this figure correct. So, this corresponds to the transmission of symbol 1 during the second part of the duration of 0 to 2 T b; that is between T b and 2 T b and similarly this corresponds to a transmission of symbol 0.

Now, note that the 2 envelope deflectors representing each signals here and here are negative of each other. So, the envelop sample of each will be the same; here the envelope sample here it will be same and here and here will be the same. Therefore, we can implement the detector as a single channel for S 1 t match to either x 1, x 2 or x 2, x 1 and a single channel for S 2 t match to either x 1, x 1 or x 2, x 2. So, DPSK detector is therefore, reduced to a standard 2 channel non coherent detector has shown in this figure.

Now, in practice the filter can be matched to the different signal so, that only one channel is necessary.

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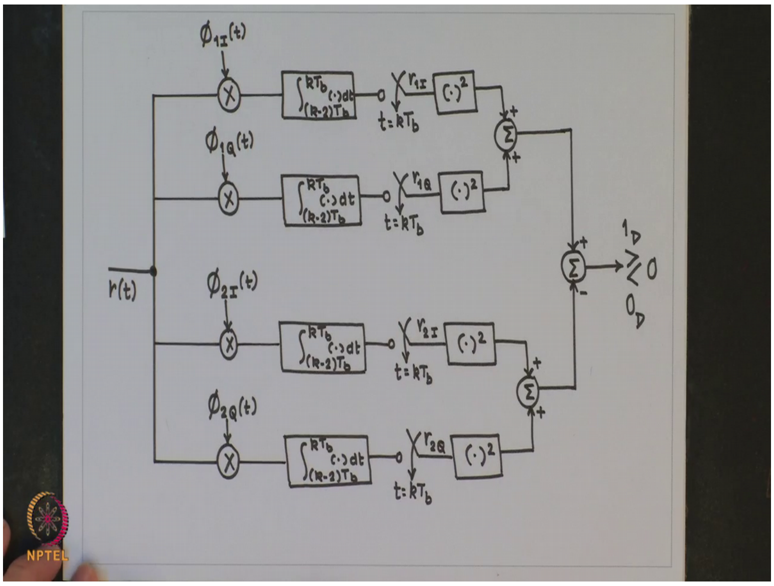

This figure can be expanded as shown here based on what we have studied earlier. We have phi 1 I t phi 1 Q t and similarly phi 2 I t phi 2 Q t to obtain this r 2 I and r 2 Q. And this signals phi 1 I phi 1 Q phi 2 I phi 2 Q is given by the expression shown here correct.

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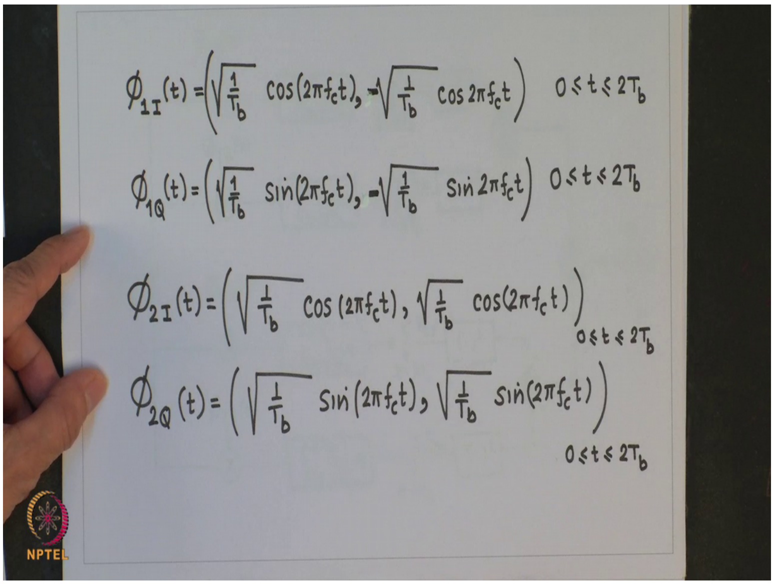

Remember these are orthogonal to each this is orthogonal to this similarly any of this signal is orthogonal to any other signal out here.

So, now the probability of error for such a optimum receiver could be obtained very easily based on what we have studied earlier for the non coherent BPSK. The only difference is that T b will become 2 T b and your energy will become twice the energy because we are talking about the duration which is 2 T b.

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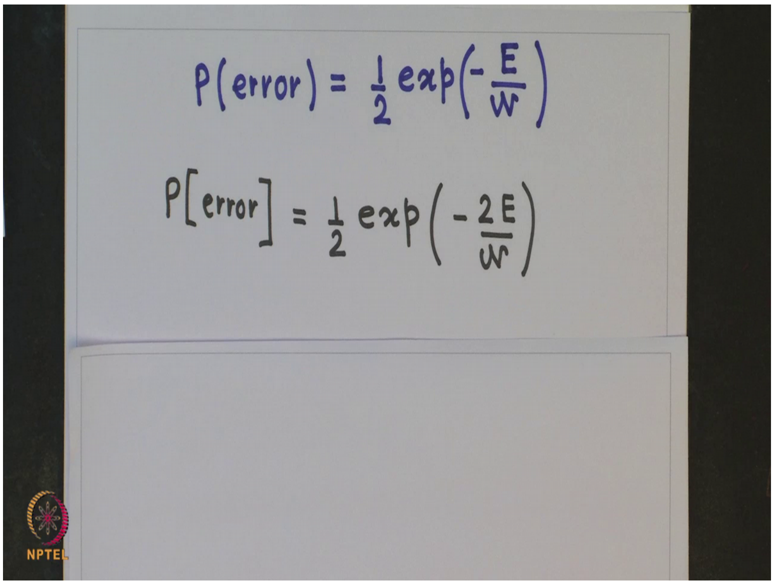

So, we know that probability of error for binary BPSK case was given by half exponential minus E by italic N. So, now in this case our E will become 2 E right because we are talking about duration 2 T b. And we get this is the relationship for the probability of symbol error for a non coherent PSK or DPSK. So, we see that DPSK requires 3 dB less signal to noise ratio per bit for the same performance as compared to the non coherent binary FSK.

Now, with this we come to the end of our study on digital modulation. So, let me summarize our study of digital modulation; we began our study with coherent binary ASK, binary FSK and binary PSK modulation schemes. We showed that binary PSK is efficient in terms of signal power, it requires 3 dB less signal to noise ratio per bit compared to the binary ASK or binary FSK case.

Binary FSK requires larger bandwidth compared to the binary ASK and binary PSK. Then we extended our study to bandwidth conservative modulation schemes and we studied QPSK and its variant and MSK. We saw that performance of all this modulation schemes is comparable to the binary PSK

MSK has constant amplitude, continuous phase and lower side lobe levels and easy to demodulate mechanism like the FSK modulation scheme and that is why it is quite popular in mobile communication applications. And then we moved over to the study of M-ary modulation schemes, we saw that M-ary ASK, M-ary PSK and the hybrid versions of this 2 amplitude and phase modulation is QAM; all these 3 modulation schemes are applicable in applications which are bandwidth limited. And in contrast we studied M-ary FSK modulation which can be applied in applications which are power limited.

We studied that in M-ary FSK as the size of the constellation increases for the same error performance signal to noise ratio per bit required decreases, but at the expense of bandwidth. And then lastly we studied non coherent modulations and we concentrated only on two of this and these are binary FSK and binary PSK; that is non coherent binary PSK is also known as DPSK Differential Phase Shift Keying.

And we saw that differential PSK requires 3 dB less SNR per bit for the same error performance compared to the non coherent binary FSK case. And the both this non coherent modulation schemes are special cases of a general non coherent orthogonal signaling scheme.

So, with this we conclude our study on digital modulations.

Thank you.