Principles of Digital Communications
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Lecture – 57
Comparison of M-ary Schemes

Important parameters of any modulation or demodulation technique include transmission bit rate, bandwidth requirement, error performance in terms of bit or symbol error probability and transmitted power, which is usually quantified by the signal to noise ratio per bit, to achieve a certain error performance.

Now, to have a meaningful comparison of different modulation techniques, which we have studied so far, this important parameters need to be taken into account. In fact, we have done this kind of comparisons between M-ary PSK and binary PSK and between M-ary QAM and M-ary PSK.

A more compact and meaningful comparison of different modulation techniques is the one based on the bit rate to bandwidth ratio, which is bits per second per hertz of bandwidth versus the signal to noise ratio per bit, required to achieve a given error probability. This ratio is commonly called the normalized bit rate, which measures the bandwidth efficiency of a signaling scheme. So, let us determine this ratio for different modulation of signaling techniques which we have studied so far.

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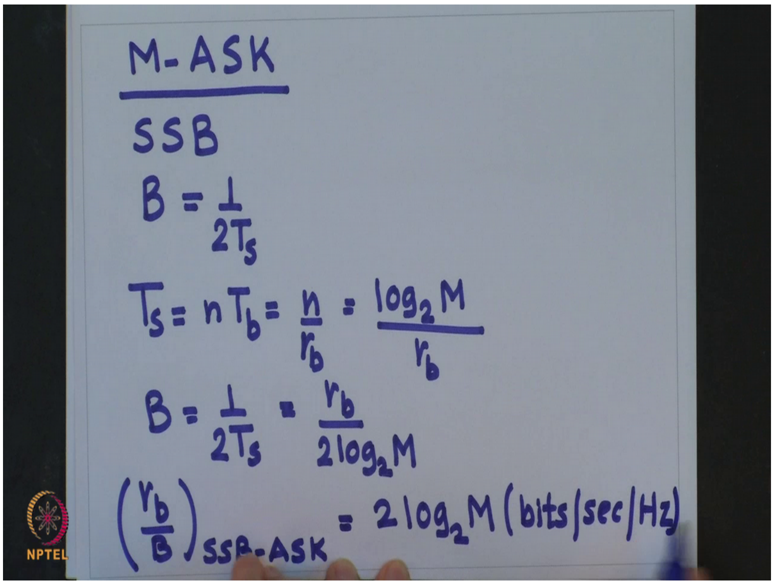

So, first we will start with M-ary ASK; we will assume that ideal Nyquist filtering has been done, in and since M-ary ASK is a form of amplitude modulation the bandwidth efficient transmission would be to use single side band transmission. So, for this transmission, the bandwidth requirement would be equal to 1 by 2 T S, T S is the symbol duration.

Now, we know that T S is equal to remember this is this bandwidth equal to 1 by 2 T S is because of single sideband, if it was a double sideband, then it would be 1 by T S. T S is equal to n T b which is equal to n by r b, where r b is 1 by T b. So, this is same as writing log to the base 2 of M by r b. Now, bandwidth is equal to 1 by 2 T S. So, we can rewrite this as r b by 2 log M and therefore, this ratio r b by B, this is the bit rate by the bandwidth required for single sideband M-ary ASK would be equal to twice log to the base 2 M bits per second per hertz.

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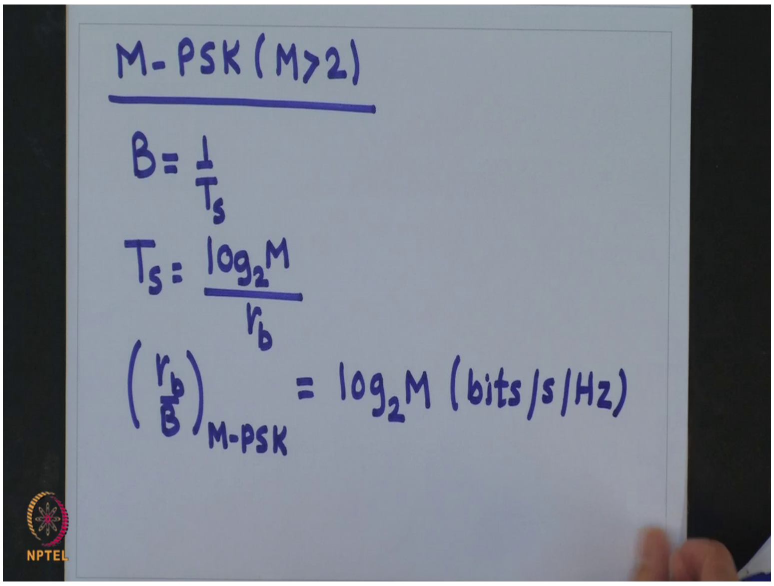

Now, let us take M-ary PSK M is larger than 2, again we assume ideal Nyquist filtering and this is a double side band transmission. So, the bandwidth required would be 1 by T S. T S is equal to log to the base 2 M by r b. So, r b by B for M-ary PSK would be equal to log to the base 2 M bits per second per hertz.

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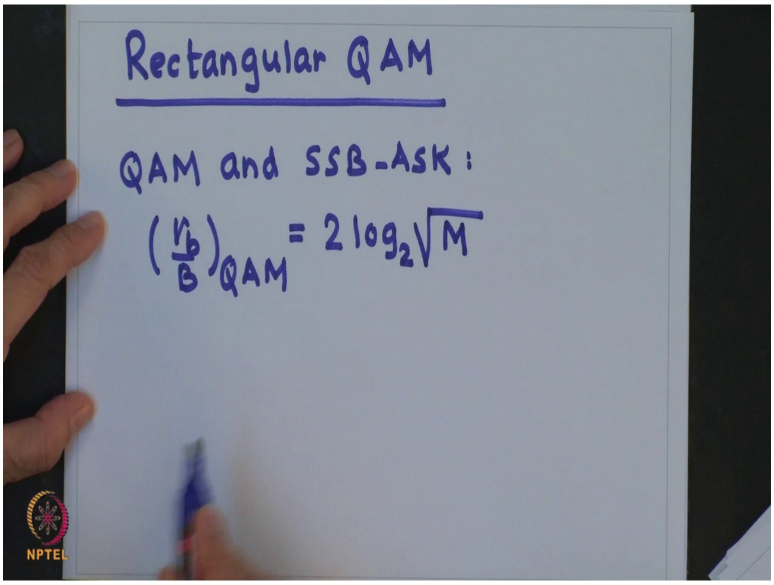

Now, for the rectangular QAM, we have seen it is 2 independent ASK signal transmission on orthogonal quadrature carrier. So, the transmission rate is twice that of ASK. And transmission is via double side bands. So, QAM and single sideband ASK, will have the same bandwidth efficiency. And it will be equal to 2 log to the base 2 and here we will assume that this QAM is a square QAM. So, in that case I can write this as square root of M, this ratio r b by B is also known as spectral efficiency in the literature. And now finally, let us look at the M-ary FSK.

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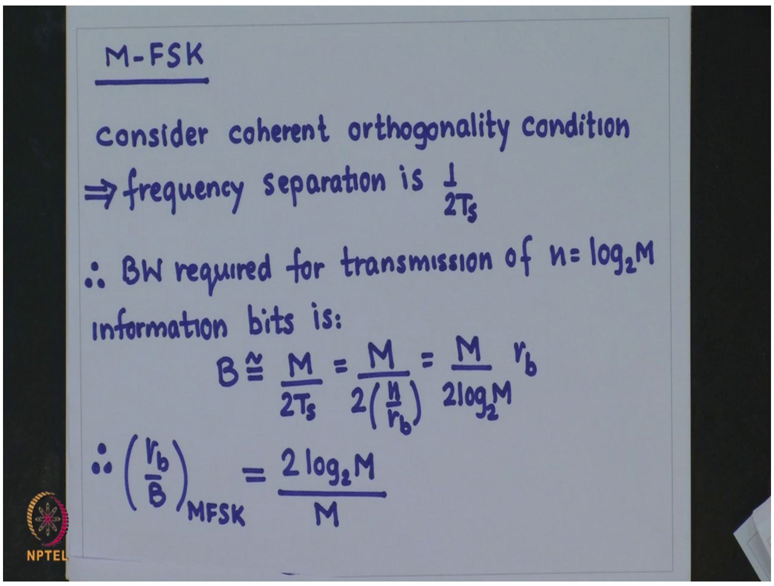

We will assume that the message signals, in this message set for M-ary FSK, have coherent orthogonal condition is satisfied. So, frequency separation is going to be 1 by 2 T S and, in this case the bandwidth required for transmission of M messages would be equal to approximately M times 1 by 2 T S. And this I can again rewrite as M by 2 log to the base 2 M r b fine. And now for this the bandwidth efficiency or spectral efficiency would turn out to be this quantity.

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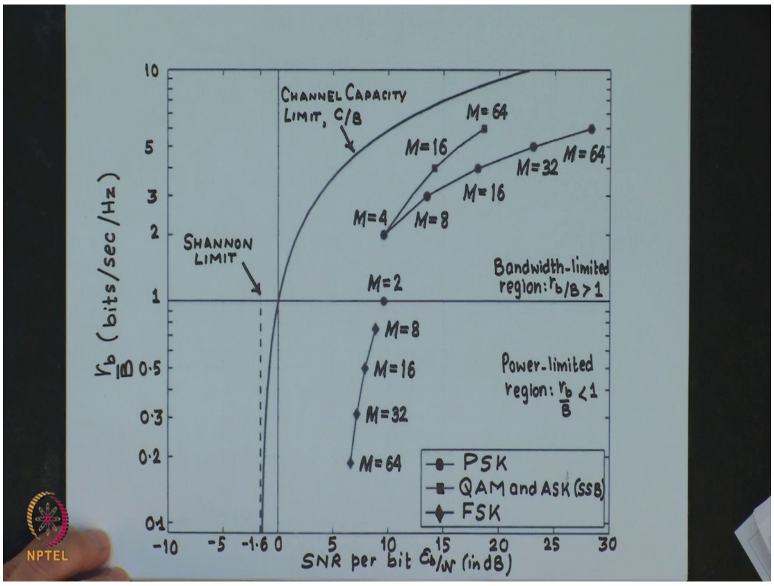

The figure out here shows the plot of the bandwidth efficiency, or the spectral efficiency that is r b by B versus signal to noise ratio per bit. For single sideband ASK, PSK and QAM and FSK, when symbol error is 10 raised to minus 5. Now, observe that in the case of ASK, PSK and QAM increasing M results in a higher bandwidth efficiency; however, the cost of achieving the higher data rate per unit of bandwidth is an increase in signal to noise ratio per bit.

So, as I increase this for PSK, I have go from M equal to 4 to 8, 16, the bandwidth efficiency improves, but SNR per bit also increases. So, consequently this modulation techniques are appropriate for communication channels that are bandwidth limited. So, you see there is a line here, dividing line and where above this, this bandwidth efficiency is greater than or equal to 1. So, this modulation techniques are appropriate for communication channels there are that are bandwidth limited, where it is desired to have a bit rate to bandwidth ratio greater than 1. And when there is sufficiently high signal to noise ratio per bit to support this increase in M.

So, telephone channels and digital microwave channels are example of such bandwidth limited channel. In contrast M-ary FSK modulation provides a bit rate to bandwidth ratio which is less than 1. So, it lies below this line. So, as M increases r b by B that is bandwidth ratio decreases due to the larger increase in required channel bandwidth; however, the signal to noise ratio per bit, required to achieve a given error probability, in this case is 10 raised to minus 5 decreases as M increases. Therefore, the M-ary FSK signaling technique is appropriate for power limited channels that have sufficiently large bandwidth to accommodate a large number of signals, but cannot afford a large signal to noise ratio per bit.

So, for the case of M-ary, FSK as M tends to infinity. The error probability can be made as small as possible as desired, provided this signal to noise ratio per bit is larger than 1.4 to decibel, which we have studied earlier. So, note in this figure there is a graph for the normalized channel capacity of the band limited additive white Gaussian noise channel, which is due to Shannon and which we have studied earlier.

So, this ratio C by B, where C is the capacity in bits per second represents the highest achievable bit rate to bandwidth ratio on this channel that is an additive white Gaussian noise channel. So, this serves as an upper bound on the bandwidth efficiency of any modulation technique. And this bound we have studied earlier in the course.

So, in our study we have assumed that the transmitted signal is received at the receiver without any phase uncertainty, but in practical applications phase uncertainty can arise due to different aspects. One possibility is slow drift in the receivers local oscillator that is used to demodulate the incoming signal to base point. Another common effect is changes in the propagation time of the signal between the transmitter and the receiver.

So, significant phase uncertainty can therefore, be introduce quite readily into the received signal. What this means that using BPSK as a modulation technique, in the phase of this phase uncertainty is an impossible proposition, but we will see a way out of this. We will also study the effect of this phase uncertainty on binary FSK. Now, in order to do this kind of study we need little some background and, we will prepare that background as follows.

So we will see, what is the effect of the phase uncertainty on the transmitted signal, when it is received at the receiver and projected on some basis signals, which we have used to represent that message signal set?

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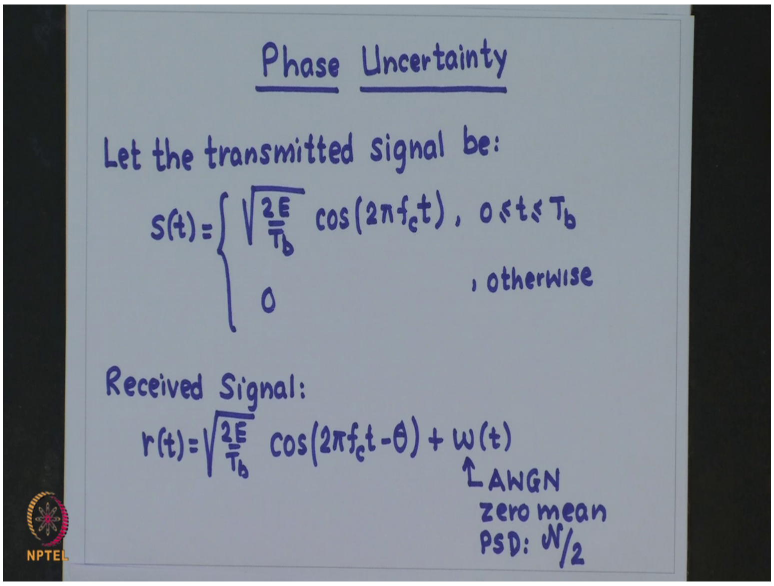

So, let us take a transmitted signal to be a sinusoid of the form given here, and that sinusoidal signal is of a duration T b. Now, at the receiver we will assume that this transmitted signal has some kind of phase uncertainty in the form of this theta.

So, far we had neglected this theta, we had assumed it to be 0 without loss of generality. And we had assumed that this received signal is only corrupted by additive white Gaussian noise with 0 mean and power spectral density of italic N by 2. Now, let us assume that we have 2 basis signal and this basis signals are phi 1 t and phi 2 t which are nothing, but 2 quadrature carriers.

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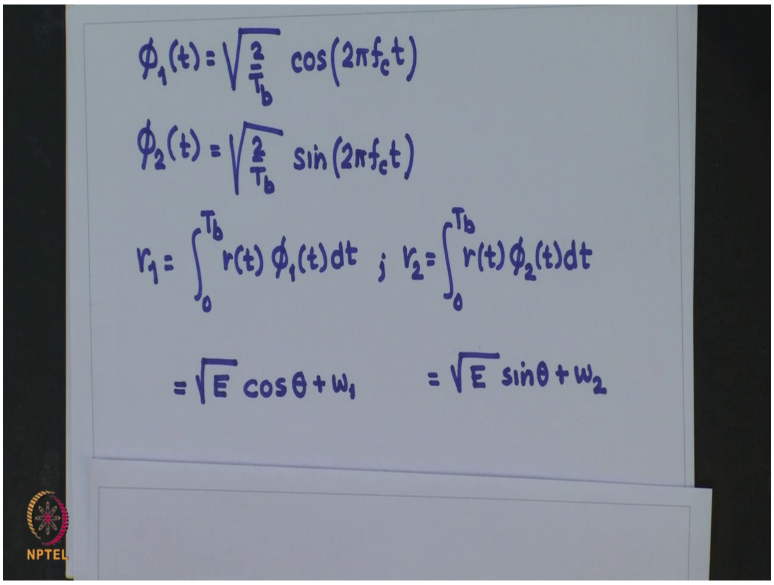

And we are interested in studying the behavior of the received signal r t onto this orthonormal basis signal. So, the projection of the received signal onto this phi 1 and phi 2 t would be denoted as r 1 and r 2. For the model, which we have used for the received signal on this case the projection of r t on phi 1 and phi 2 t would be equal to as given by these expressions for r 1and r 2. Now, our task is basically is to find out what is the joint pdf for r 1 and r 2 given this uncertainty about the theta. So, we will determine the joint pdf of r 1 and r 2 as follows.

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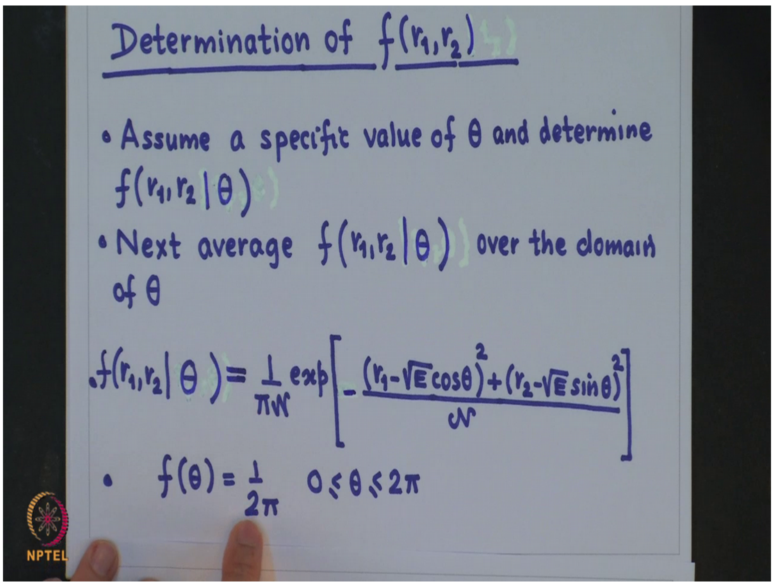

So, the procedure is assume a specific value of theta and determine the joint conditional pdf for the specific value of theta. And, then we average this conditional pdf over the domain of theta, which is a random variable. First write this conditional pdf joint pdf for the particular value of theta and, that is very simple to write because, we know that r 1 and r 2 are going to be Gaussian and, with the power spectral density of italic N by 2 and the mean determined by root E cos theta and root E sin theta. So, this is the joint pdf we write it, for the particular value of theta.

Now, we have to average this over the domain of theta. So, for this we have to assume some kind of probability distribution function for the random variable theta, we will assume the pdf for the theta to be uniform between 0 and 2 pi. So, f theta is equal to 1 by 2 pi, using this pdf for theta we can find out the joint pdf for r 1 and r 2 as follows.

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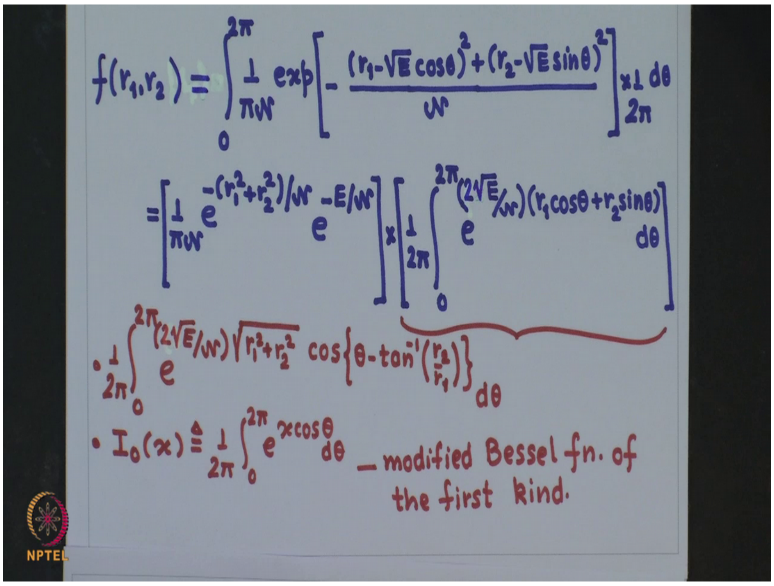

So, this is the conditional pdf, for r 1 and r 2 given theta and, then we multiplied by the pdf of theta and integrate this over the domain of theta that is 0 to 2 pi.

So, since we have assume the pdf to be uniform it becomes 1 by 2 pi. And, now we see that this can be rewritten as shown by this expression, this is simple to get after expansion of these 2 terms. So, this term will come once we expand these 2 terms and, what will be remaining here as a function of theta is shown by this expression. Now, this expression here can be rewritten in a different form, I can write this r 1 cos theta plus r 2 sin theta in a polar form, if I write this in polar form I get this expression.

Now, if you observe this integral in the polar form, it is equivalent to integral, which is known as modified Bessel function of the first kind and which is defined as given by the expression here. So, if you relate this expression and this expression, it is easy to see that I can write my joint pdf in terms of modified Bessel function of the first kind as follows.

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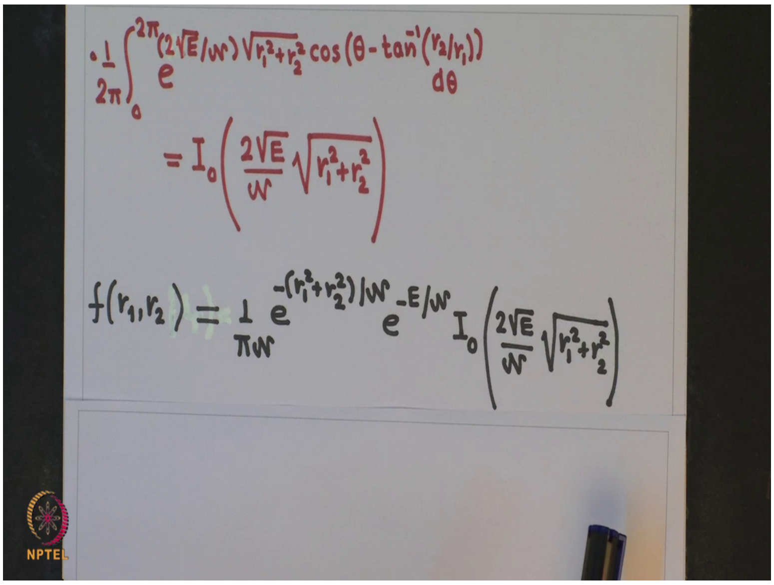

So, we get the result in terms of the modified Bessel functions of the first kind. And finally, the joint pdf for the r 1 and r 2 would be given by this expression on the right hand side of this equation. Now, we will study a different approach to compute this quantity, which is square root of r 1 squared plus r 2 squared and, we will do it using what is known has band pass filtering approach.

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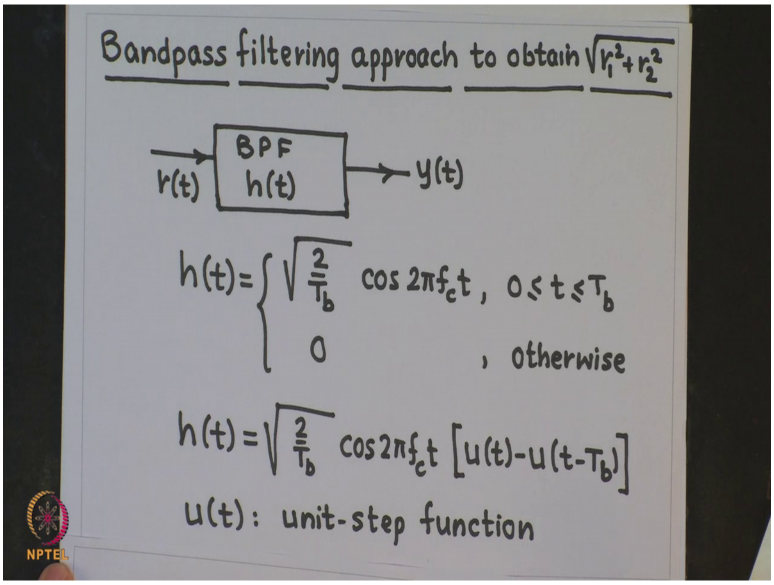

So, let us say we have a input r t passing through a linear time invariant system, which is in the form of bandpass filter with an impulse response h t and it is output is y t. The impulse response h t is specified here this is a sinusoid of frequency f c over a duration t b.

So, this impulse response can be rewritten, in this form where u t is a unit step function which says that for t less than 0, it is 0 and t greater than or equal to 0 the value is 1. Now, using this impulse response let us calculate the output of the band pass filter.

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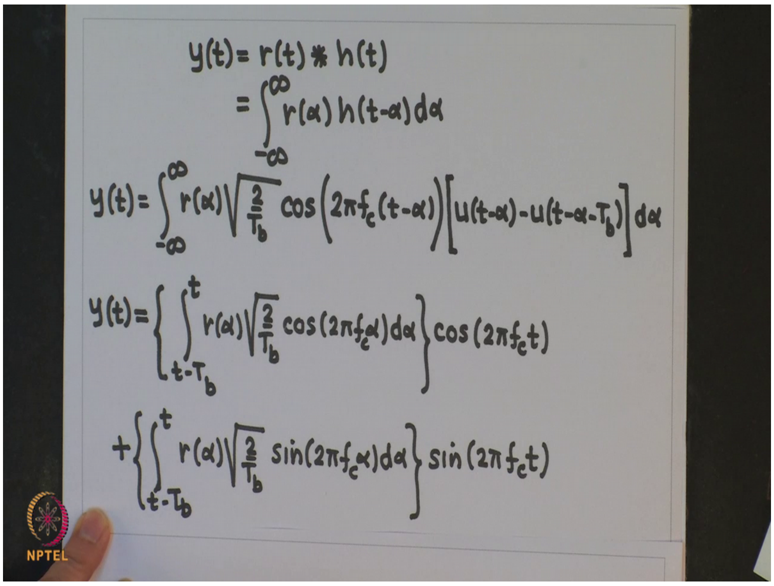

So, the output is given by the convolution of the input r t and the impulse response h t, which can be written as shown here. Now, we will substitute the value of h t minus alpha and if we do that we get this expression. And, now we will expand this term using the trigonometric relationship and, if we do that we will get 2 terms here, this is modulating cos 2 pi f c t and this is modulating sin 2 pi f c t.

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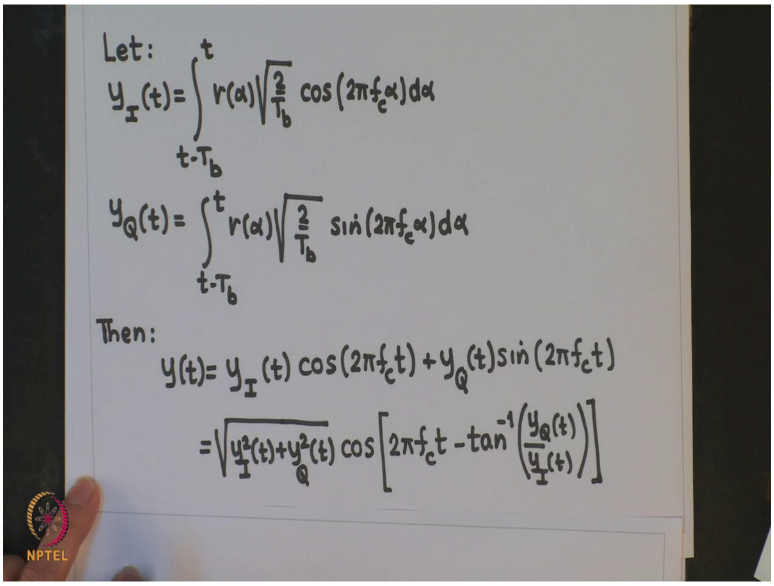

Now, if we define these 2 integrals as y I t is equal to this quantity and y Q t is equal to this quantity, then we can rewrite y t in terms of y I t and y Q t as follows. And this can be rewritten in the polar format as given by this expression. Now, this term out here is the envelope of the signal given by this expression.

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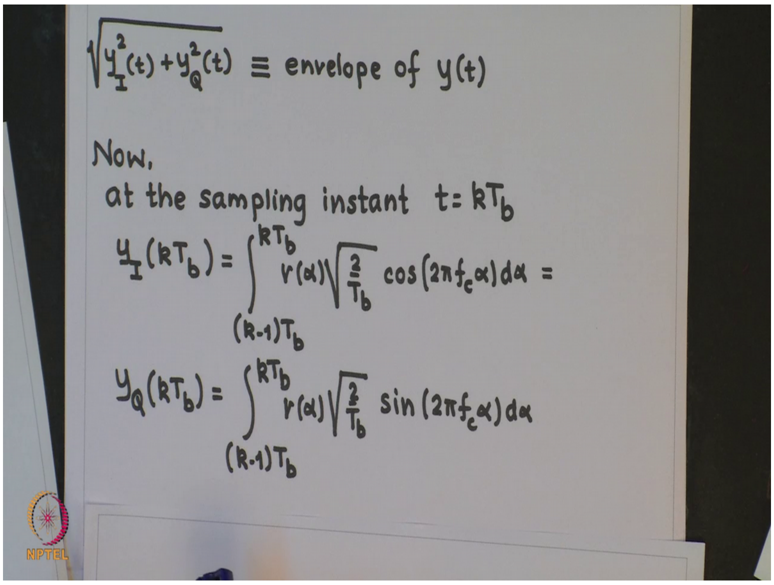

We get the envelope of y t to be this quantity. Now, if we evaluate this and envelope at the sampling instant t equal to k T b what we get are the following expressions for y I k T b and y Q k T b. Now, if we take the projections of r t on to cos 2 pi f c t and sin 2 pi f c t during the k-th bit interval, we will get this expressions.

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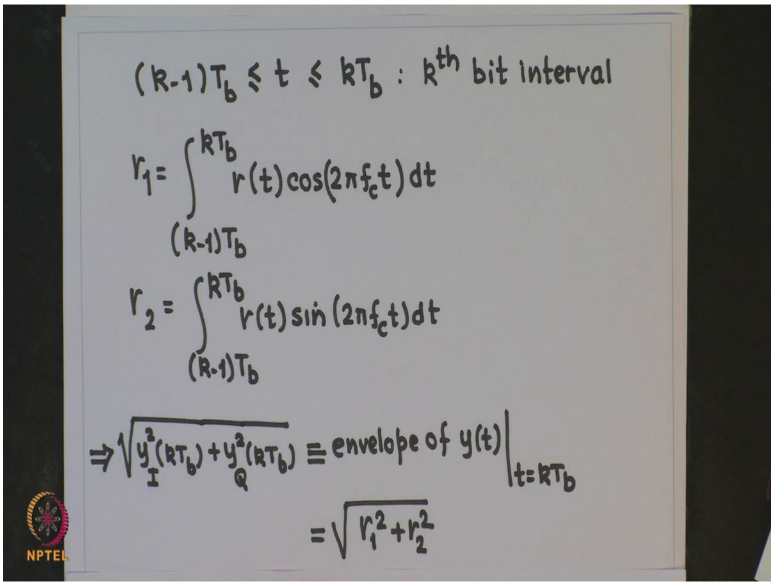

So, during the k-th bit interval r t projected on cos 2 pi f c t with give us this expression, an r t projected on sin 2 pi f c t during the k-th bit interval would give us the value for r t equal to this. So, from this relationship and from this relationship, we see that the envelope of y t sampled at t equal to k T b is exactly equal to the square root of r 1 squared plus r 2 squared.

So, this result can be interpreted in a different manner saying that, this bandpass filter with the impulse response h t is match to the input, which is a sinusoid of a duration T b. So, this is match filtering to the envelope of the input sinusoid of the finite duration. And in that case the phase does not play an important role. Now, we will see the application of this joint pdf of the sufficient statistic r 1 and r 2 and, the relationship to the envelope of the output of a bandpass filter, when we study the non-coherent binary FSK modulation scheme. And this we will do next time.

Thank you.