Principles of Digital Communications

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Lecture - 53
M-ary Coherent ASK (M-ASK)

We have learned that there are benefits to be gained when M-ary M equal to 4 signaling methods are used rather than straightforward binary signaling. In general M-ary communication is used when one needs to design a communication system that is bandwidth efficient. It is based on the observation that as the time duration of a signal T s that is the symbol duration increases, the bandwidth requirement decreases. Typically unlike QPSK and its variant and MSK the gain in bandwidth is accomplished at the expense of error performance.

Typical application of M-ary modulation is one where a binary source has its bit stream blocked into groups of n bits. The number of different bit patterns is 2 raised to n, which means the number of message is M is equal to 2 raised to n; where each bit pattern is mapped or modulated into a distinct signal, each block of n bits is a symbol and given that the source bit rate is R b which is equal to 1 by T b bits per second. The symbol transmission rate is R s is equal to 1 by T s is equal to 1 by n times T b or is equal to R b divided by n symbols per second.

A signal from the modulator occupies the symbol duration which is T s is equal to n times the bit duration that is T b seconds. And the implication is that the bandwidth requirement is on the order of 1 by symbol duration that is T s or there is a bandwidth saving of 1 by n compared to binary modulation.

We will study M-ary ASK, PSK and hybrid of this two modulation scheme which is known as quadrature amplitude modulation; popularly as form and M-ary FSK signaling methods. In particular the demodulator which minimizes the message error probability or symbol error probability will be derived and applied to the different signaling methods. The error performance of this modulation methods and bandwidth saving will be analyzed.

Now, it should be noted that though a message will usually represent a block of binary digits or bits. Minimizing message error probability is not the same as minimizing bit error probability and in general there is no simple relationship between message error probability and bit error probability. Relationships however, will be derived for the discussed modulation L bit only in terms of bounds for some of this modulation.

Now, we will begin our study with M-ary amplitude shift key that is M-ary ASK.

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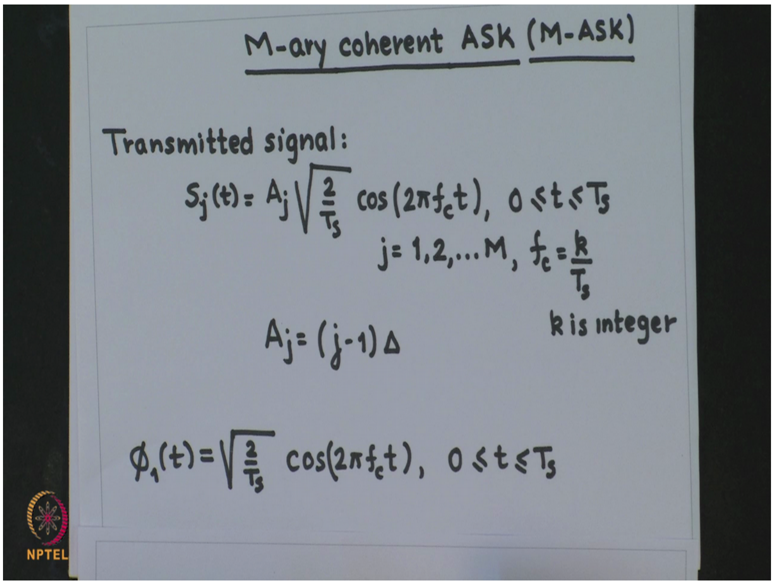

So, M-ary ASK consists of transmitting M signals and the M signals are given by this expression S j t is equal to this term multiplied by the carrier term cos 2 pi f c t, and the signal duration is T s, and the choice for f c is k by T s where k is some positive integer and the amplitude of this carrier wave is given by this expression.

So, looking at this transmitted signal set its easy to see that this can be represented in signals phase diagram by using just one basis signal and this is given as phi 1 t is equal to root of 2 by T s cos 2 pi f c t and the duration is from 0 to T s. Now, given this let us calculate the optimum receiver for this.

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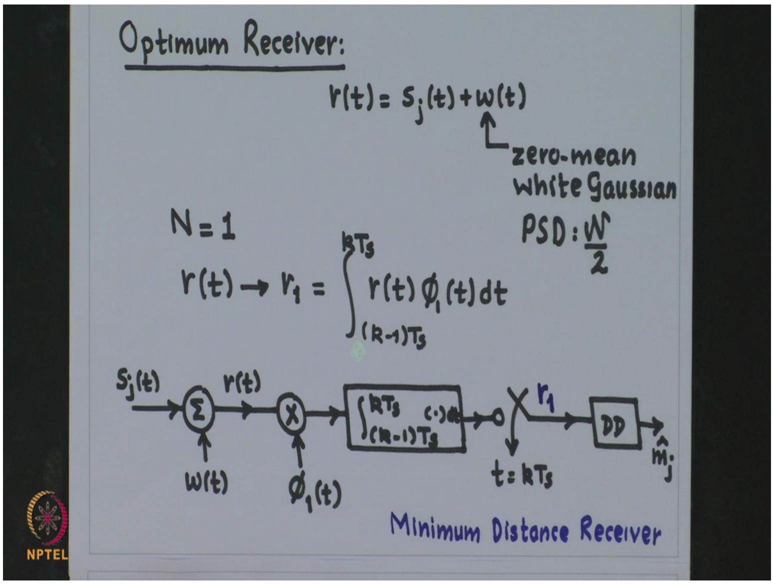

We will again assume that our signal is being transmitted over additive white Gaussian noise channel; this white Gaussian noise channel is a zero-mean and it is has a power spectral density of italic N by 2. In this case our signal space diagram is going to be one-dimensional. So, N is equal to 1; we will take the receive signal r t and project it on the basis signal phi 1 t. This is the expression for the general symbol duration.

So, the block diagram for the optimum receiver would look as shown here. This is the channel model white Gaussian noise is being added to the transmitted signal. The received signal is projected on the basis signal using the correlation receiver we sample it, we get the output r 1 and we have a decision device here. Let us look at the signal constellation for this, the signal constellation will be as shown in this figure it is a 1 dimensional.

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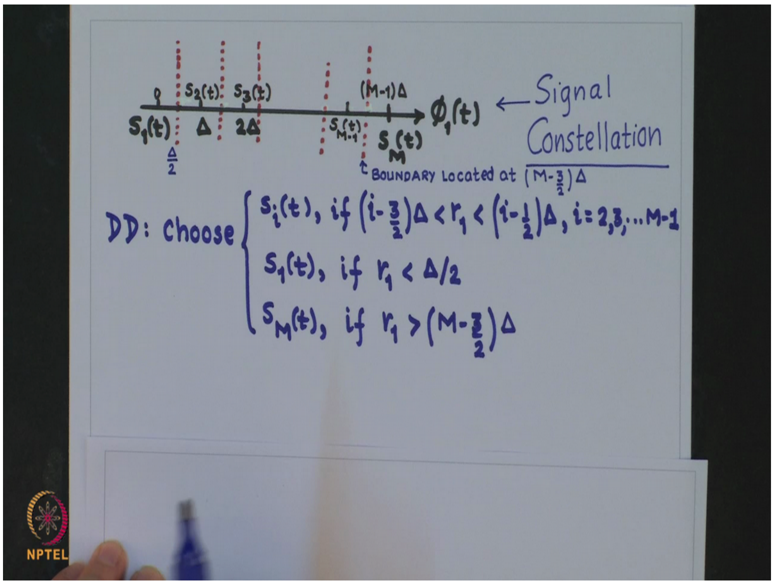

So, the signals S 1 t up to S M t are going to be projected on this axis. The distance between the points is going to be delta, all this message symbols are equiprobable. So, the decision boundaries are going to be the perpendicular bisectors of the line joining the two message signal points; like here I have shown for S 1 and S 2. The is S 1 and S 2 this is the perpendicular bisector between S 2 and S 3, this is the perpendicular bisector.

Now, the decision device we will choose S i t if this condition is satisfied and this will be valid for all i from 2 to M minus 1. So, from this point for this message signal, this message signal up to S M minus 1 t this condition will be valid whenever, this condition is satisfied the appropriate S i t would be chosen. For S 1 t we require r 1 to be less than delta by 2 and for S M t we require r 1 to be larger than this point here, the boundary is located at M minus 3 by 2 multiplied by delta.

So, remember this r 1 is Gaussian and it is a sufficient statistic; it has a variance which is italic N by 2 and a mean value will be determined by the transmitted signal. So, the conditional pdf of r 1 would be the Gaussian with mean given by the choice of the transmitted signal and the variance equal to italic N by 2.

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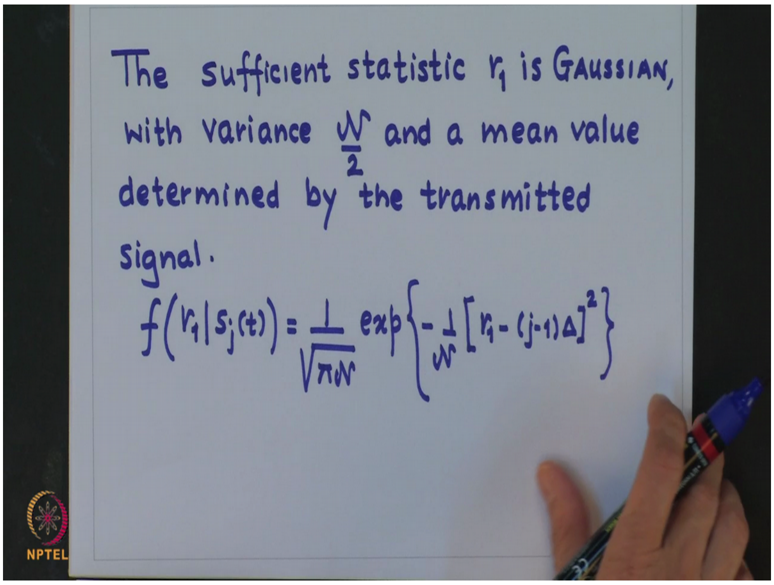

So, the decision regions would look something as shown in this figure.

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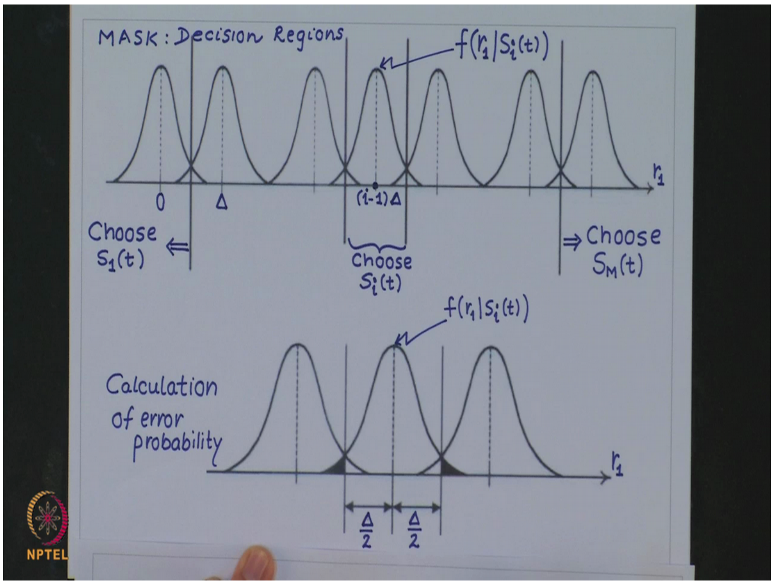

So, this is for S 1, this is for S 2 and so, on. This is for S i t and this point out here is for S M t each of this is a Gaussian distribution with the variance equal to italic N by 2 and the means decided by the transmitted signal. So, if we take specifically any S i t signal for that S i t signal; the decision boundaries will be given as shown here. So, your if your r 1 lies in this region; we will decide that S i t has been transmitted, but if it lies outside this region or outside here then it is decided other than S i t.

Now, we have to calculate the probability of errors for this. The probability of error would be given a summation of probability of transmitting a particular signal say S j t and what is the probability of error given that signal has been transmitted.

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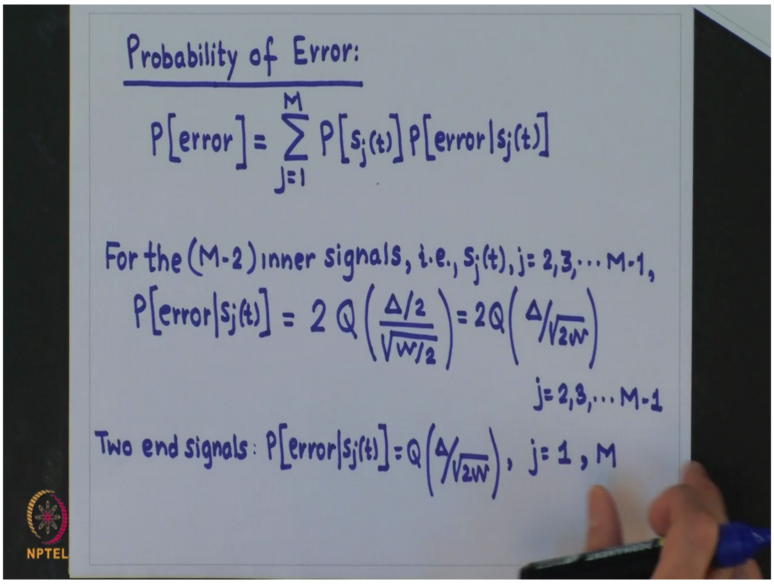

So, based on this principle we can calculate the probability of error for this signal and to calculate this remember that for the M minus 2 inner signals that is S j t, j equal to 2, 3 up to M minus 1. These are the inner signals up to this point, for this tool for all these signals the probability of error given S j t would be equal to twice of the area under this curve.

And the area under this curve would be given by Q times this value remember, its we have seen this earlier that the distance between the two point is delta; any two point is delta. So, delta by 2 and divided by italic N by 2 which is the power spectral density of the noise projected on to phi 1 t. So, you will have this twice because you have to consider both the areas and you will get this value out here this is for the inner M minus 2 signals.

And for the two end signals which are S 1 t and S M t will have Q given by this expression here. So, the only difference is that the area is considered only once and assuming that we have equiprobable symbols and independent transmission from one interval to another interval.

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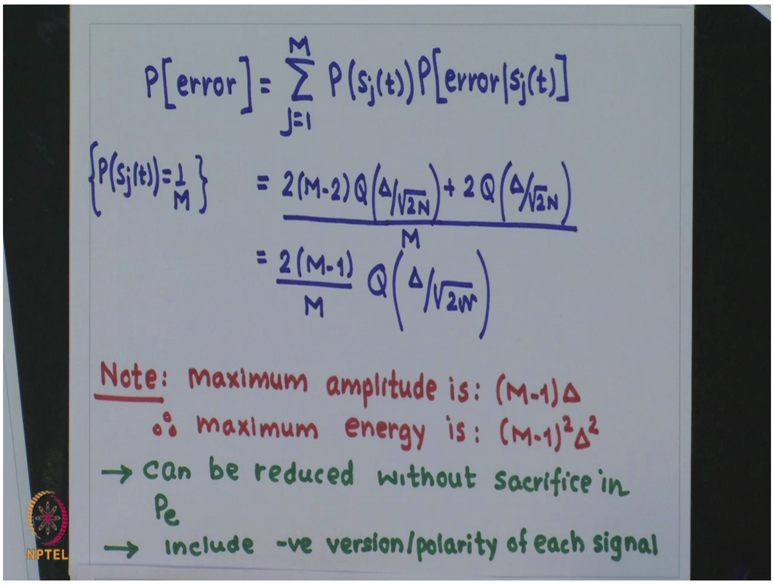

The probability of error which is given by this expression can be rewritten as this is the probability of error summation from M minus 2 symbols plus from 2 N symbols and this will get this and each of this basically gets multiplied by 1 by M. So, get this whole expression gets divided by M because equiprobable and on simplification we get this.

Now, note that when we use this kind of signal levels; maximum amplitude of the signal being transmitted is equal to this quantity M minus 1 delta. So, the maximum energy is equal to square of this, now this can be reduced without sacrifice in the probability of error which we have calculated and this can be easily done if we include the negative version or polarity for each transmitted signal.

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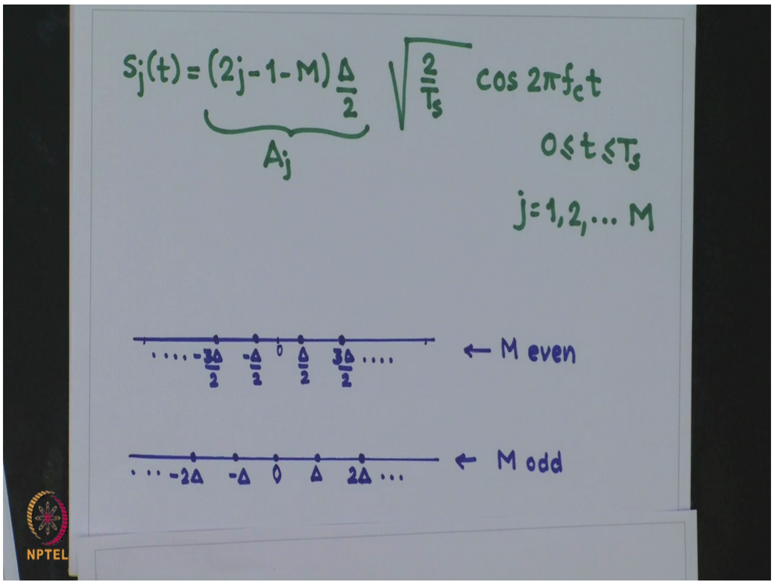

So, you will have the signal transmission of this form where A js take both positive values and negative values. So, that this is the only difference of this scheme compared to the earlier scheme where A j where all positive from 0 to j minus 1 multiplied by delta. Now, since the distance between the message points still remains delta; the probability of error will not get affected and the signal constellation for the this choice of signal set would be given as shown here, depending whether M is even or M is odd.

So, if M is even 0 will not be one of the message point. If M is odd then we have 0 as one of the message signal point in the signal space diagram or signal constellation. Now, the maximum energy is obviously, has also reduced.

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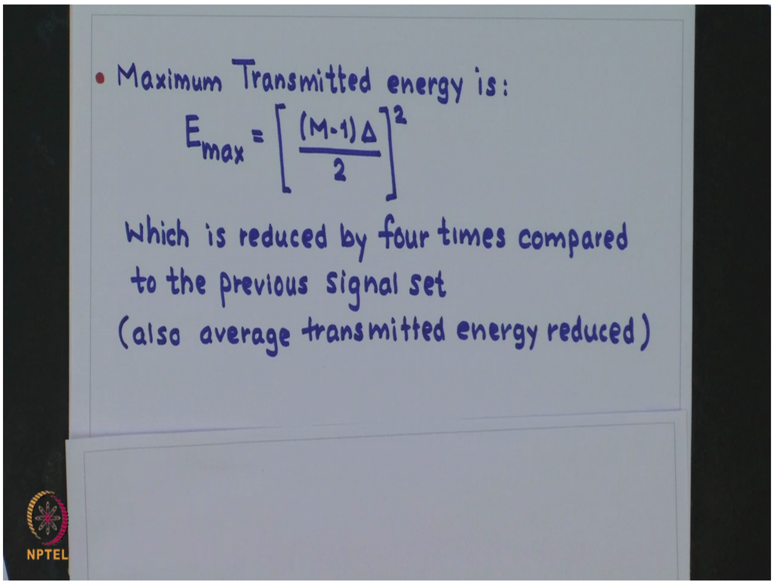

This is the maximum value which you will get for the amplitude. So, square of that and what this shows that it is reduced by four times compared to the previous signal set and this also implies that the average transmitted energy is also reduced.

Now, we can calculate the probability of error for this modulation, but remember that for fair comparison we would like this calculation of probability of error in terms of average bit rate. So, we can compare the different modulation schemes for same value of M or for different values of M. So, with that motivation let us try to calculate first the average bit energy.

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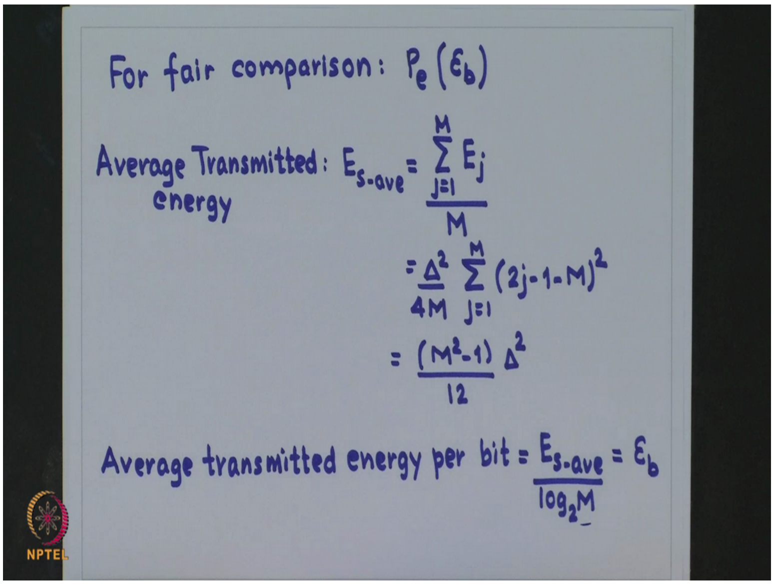

The average transmitted energy is the energy each of this signal in the message signal set. Since, we are assuming equiprobable this is divided by M and if we solve for this we will get this expression.

Now, once we have this average transmitted energy; we can find out the average transmitted energy per bit that is take this average transmitted energy divided by the number of bits, remember M is equal to 2 raised to n. So, n the number of bits is equal to log to the base 2 of M, where M is the number of messages and we will denote this average transmitted energy per bit is equal to E b and now we know that probability of error is equal to this.

So, what we will do we will use this expression and try to eliminate delta and obtain it in terms of average bit energy. So, we will get finally, the probability of symbol error in terms of bit energy that is our goal to be achieved. So, using this expression, using this relationship and this relationship we can write down the probability of error as given by this expression.

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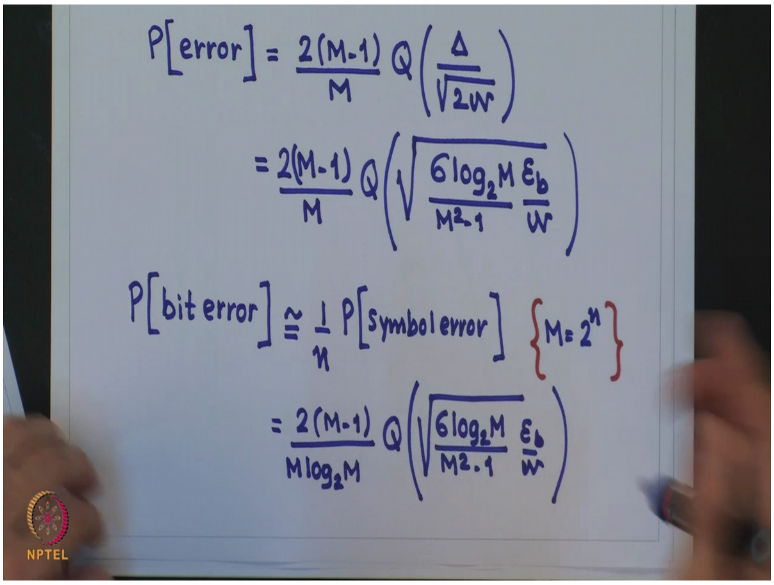

Now, it is important to note that we had derived the same expression earlier when we was studying M-ary pulse amplitude modulation scheme or pulse or M-ary PCM modulation scheme.

Now, the symbol error probability of MSK is plotted as shown in this figure. So, this figure clearly illustrates how one can trade power for bandwidth by using a higher order modulation.

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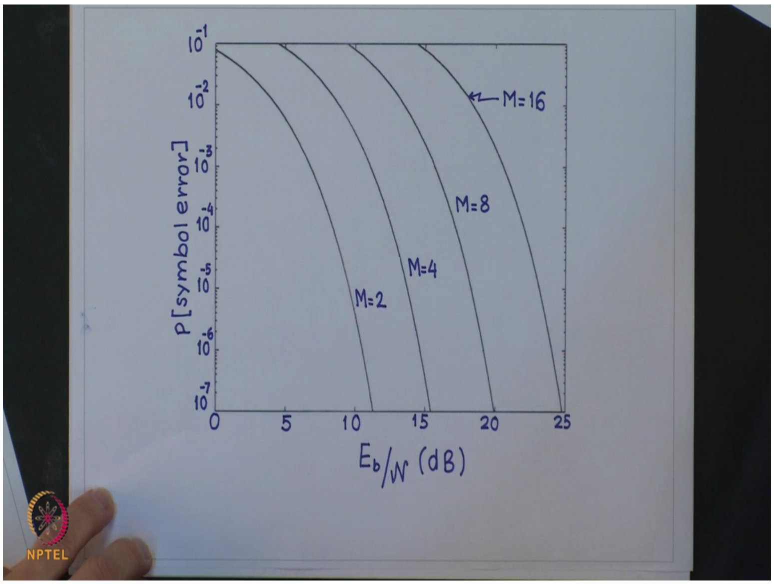

For example, if we take the probability of symbol error to be less a 10 raised to minus 6 then to achieve that with M equal to 2 we require certain value of E b by italic N ratio, but for higher modulation we require higher value of this. But in that case the bandwidth required by this modulation scheme M equal to 8 would be one-third of what is required for the M equal to 2 or we could fix up this E b by italic N ratio and in that case we see that as we keep on increasing the level of modulation the simple error probability will also increase.

So, there is a tradeoff between the bandwidth and the power and for large M it is clear that the additional power per bit needed when M increases by a factor of 2 will approach approximately 6 dB. This we had seen earlier also that we were studying M-ary PCM case. So, now one more thing is to be remembered this probability of error is a symbol error probability.

For M-ary modulation the relationship between symbol error probability and the bit error probability is often tedious and this is due to its dependence on the mapping from the n bit patterns into the signal points. If gray mapping is used then two adjacent symbols differ in only a single bit.

And since the most probable errors due to noise result in the arenas selection of a signal adjacent to the true signal, more symbol errors contain only a single bit error. So, this equivalent bit error probability for MSK modulation can be approximated by this relationship here. So, this is just divided by the number of bits which is equal to log to the base 2 of M.

Based on our early study of M-ary PCM we can easily calculate the baseband power requirement for this and it would be given by this expression; where your R 0 is the average signal energy, where A j takes this value because we are using the polar form of this M-ary ASK and p t would be the rectangular function for which the Fourier transform is the sinc function in the frequency domain.

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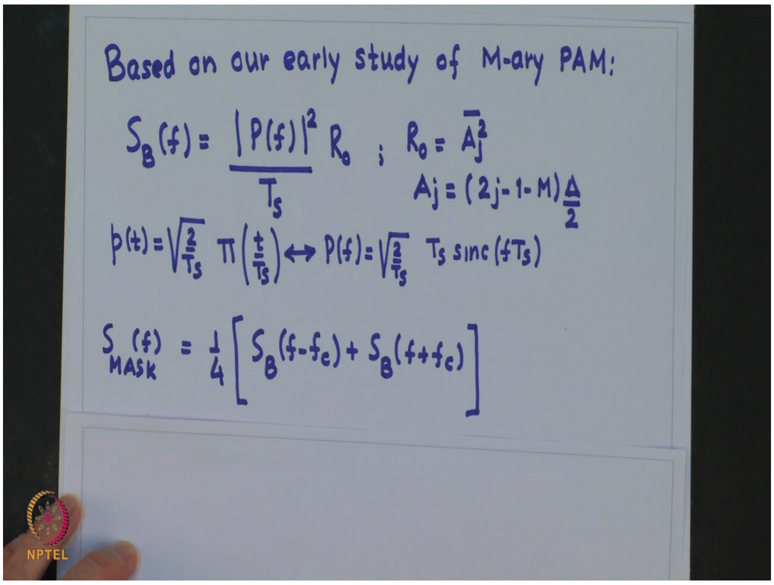

So, once we have the baseband power spectral density we can obtain the power spectral density for the M-ary ASK signal using this relationship. So, we see that for the M-ary ASK system the power spectral density is the inverse function of frequency raised to the power of 2.

So, though M-ary ASK gives us a bandwidth saving; it comes at the expense of either error performance or an increase transmission power if the same error performance is required. This problem arises from the fact that the energy grows linearly with n, while the number of signals one needs to place on the signal axis phi 1 t grows exponentially as 2 raised to n.

So, perhaps if we go to two-dimensions this effect if not overcome completely could at least be M-ary related. So, M-ary phase shift keying which uses two orthonormal basis functions may be a possible solution and it was certainly successful for the M equal to 4 that is QPSK situation.

Now, next time we will study general M-ary PSK modulation scheme.

Thank you.