Principles of Digital Communications

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Lecture - 52
Minimum Shift Keying - II

We were studying minimum shift keying and we will continue with a study. Quickly let us recollect where we had left last time.

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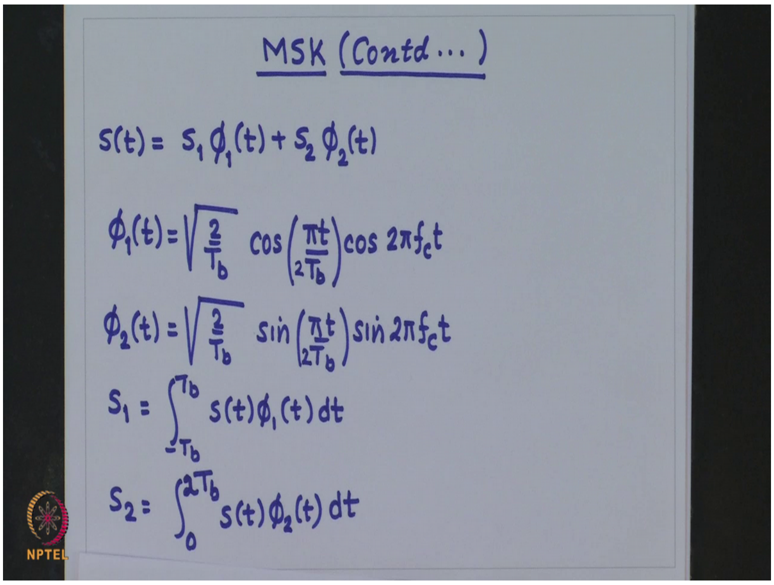

We said that we could express MSK signal in terms of orthonormal basis signal, phi 1 and phi 2 t where phi 1 is as given by this expression and phi 2 is as given by this expression. These are base band here is half cosine wave and here the base band is half sine wave. And this S 1 and S 2 are obtained by taking the projection of S t on phi 1 and S t on phi 2 respectively. And please note that for the projection, the interval duration is 2 T b, where T b is the bit duration and this two integrals are offset by interval T b.

Now we will try to find the modulator for the MSK; to do that, let us quickly write down the trigonometric identity cos 2 pi fct multiplied by cos pi t 2 T b is given by what is shown here on the right hand side.

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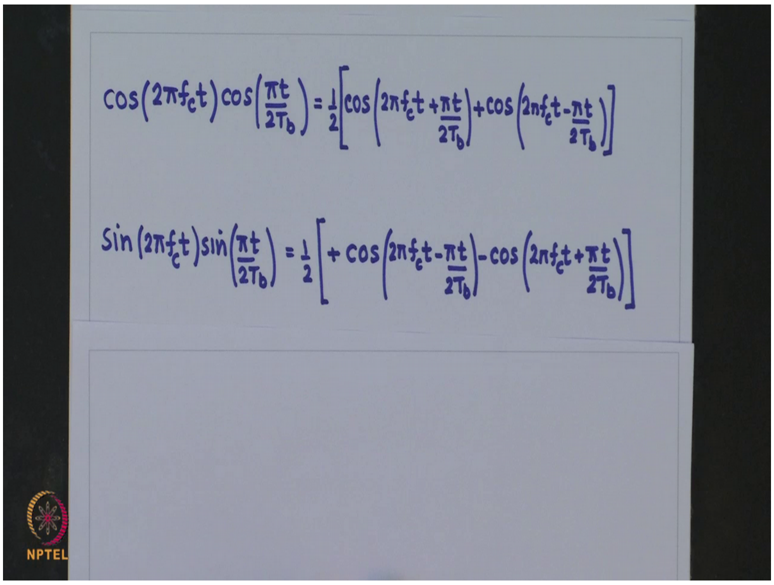

And similarly sine 2 pi fct sine pi t 2 T b is given here on the right hand side.

Now using these two identity we will generate the orthonormal basis signal first phi 1 t and phi 2 t and this will be done using the block diagram as shown in this slide.

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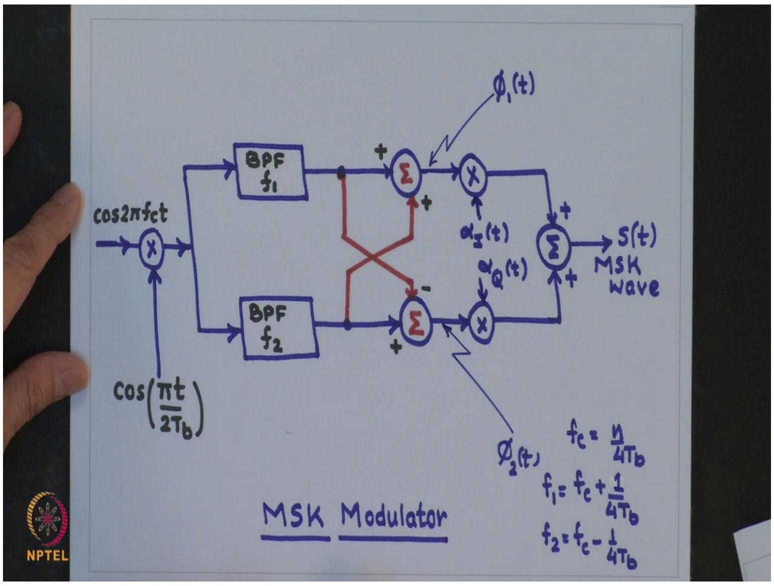

So, we take the input here cos 2 pi fct and another input cos pi t 2 db, which is fed to a product modulator and the output of this modulator are 2 phase coherent sinusoidal waves at frequency f 1 and f 2, which are related to the carrier frequency fc and the bit rate 1 by T b by this equations given here f 1 and f 2.

Now, these 2 sinusoidal waves are separated by 2 narrow band filters centered at f 1 and f 2. The resulting outputs are next linearly combined to produce the pair of the quadrature carriers or the orthonormal basis signal phi 1 t and phi 2 t. Finally, phi 1 t and phi 2 t are multiplied by 2 binary waves alpha I t and alpha Q t, both of which have a bit rate equal to 1 by 2 T b. These 2 binary waves are extracted from the incoming binary sequence in the manner studied earlier through an example and then it is combined here to produce S t which is the MSK wave.

Advantage of this method of generation of MSK signal is that, signal coherence and deviation ratio are largely unaffected by variation in the input data rate.

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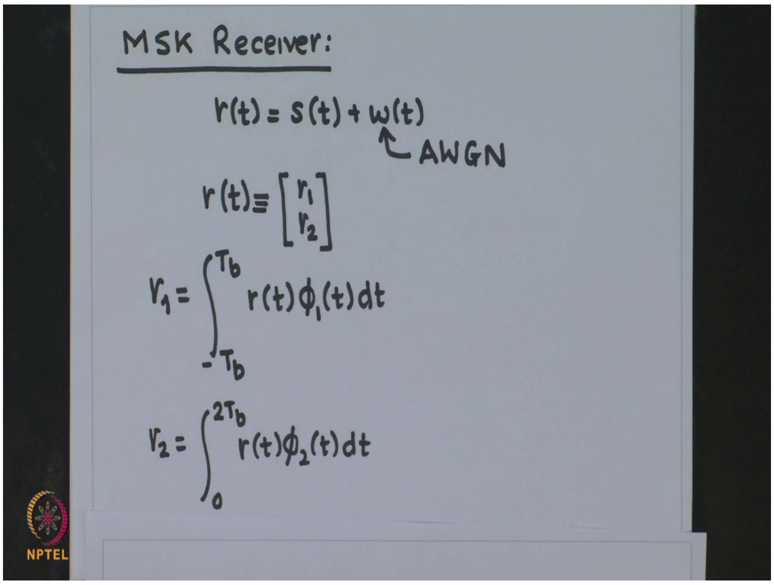

Now let us try to derive the MSK receiver our received signal r t is composed of the input signal S t, which is the MSK wave plus wt which is additive white Gaussian noise. So, the first thing we do is project r t onto the vector space using the basis signal phi 1 and phi 2, there are its a 2 dimensional signal space. So, we get 2 components for r t, r 1 and r 2 and r 1 and r 2 are obtained as given by these 2 expressions.

And given this now we can see that, we can get the optimum MSK receiver as follows; r t is project on phi 1 phi 2 t.

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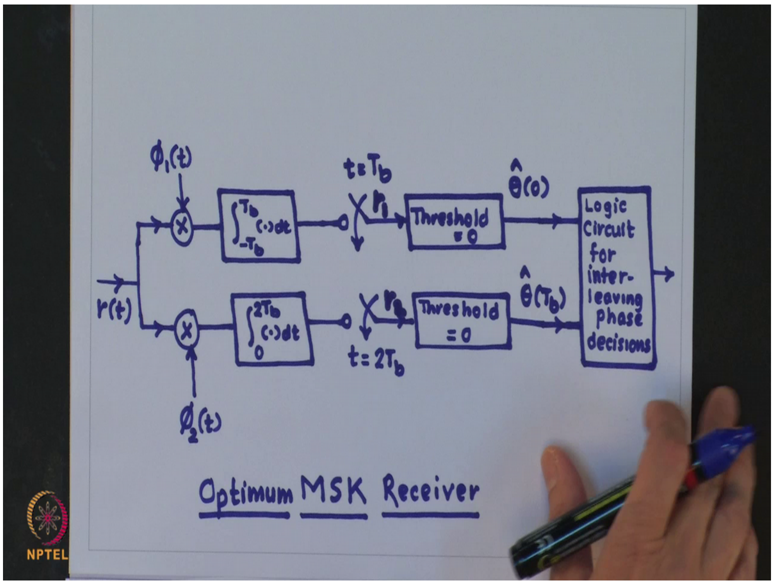

. So, here you get the outputs are this is r 1 and this is your r 2 and we will see now this r 1 is applied through a threshold, to decide what is the phase status at time t equal to 0, whether theta 0 is equal to 0 or theta 0 is equal to pi and similarly from this channel we will get the phase status for theta T b, whether theta T b could be pi by 2 or it could be minus pi by 2.

We have seen the signal constellation earlier and based on that signal constellation, we choose this both this input binary data is equiprobable and they are independent from time instants to time instant or time interval to time interval. So, we choose the, for both the channels the threshold to be 0 and then there is a logic circuit for interleaving phase decision.

this I have shown the circuit for the first bit interval that is from t equal to 0 to t equal to T b, and you can easily extend this to any time interval as follows.

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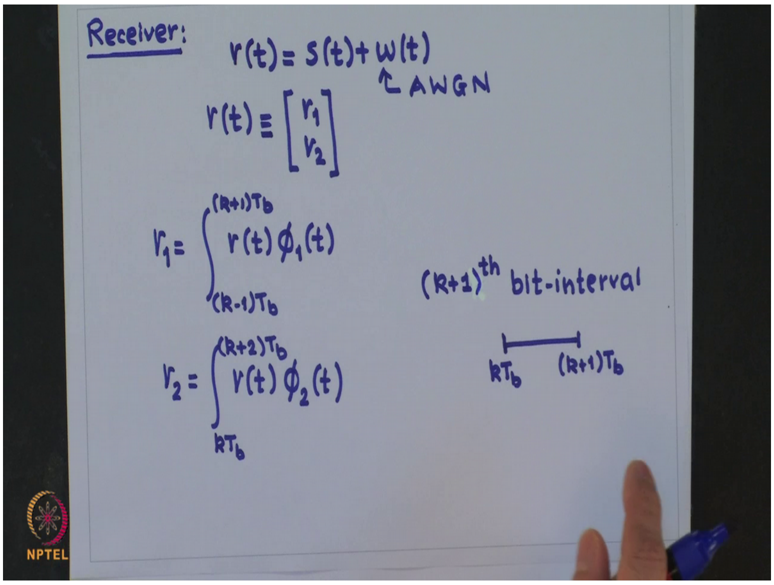

So, now, your projection r 1 and r 2 will be given by these 2 expressions right this is for k plus 1 th bit interval. So, for this analysis at this interval this becomes a zero for you and this becomes T b ok.

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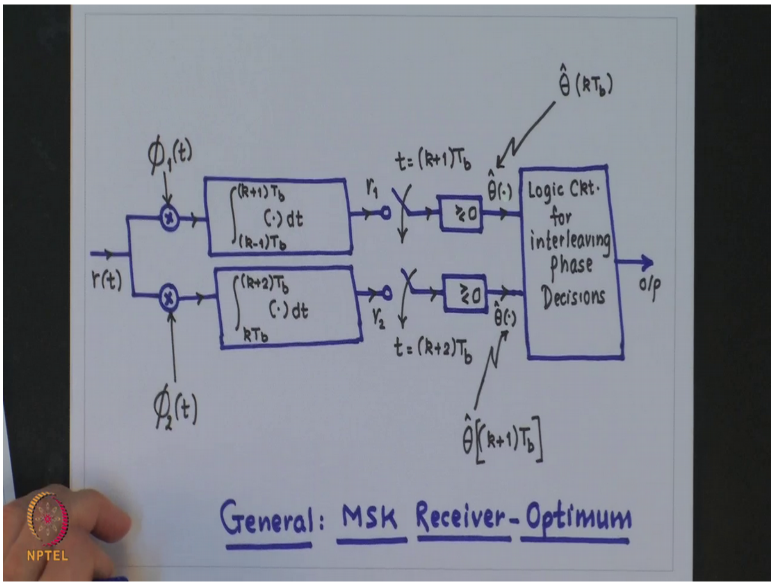

And based on this we can get the optimum MSK receiver as shown in this figure, it is exactly the same except that this is for any general interval ok. So, this r 1 and r 2 should be the sampled version.

Now, let us calculate the probability of error for this optimum receiver.

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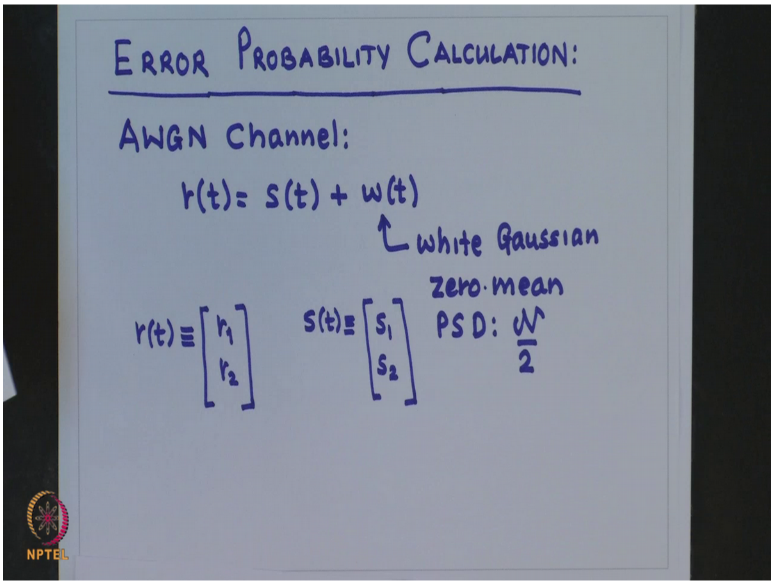

So, we have r t is equal to s t plus w t; r is going to be projected first on phi 1 and phi 2 I will get 2 components r 1, r 2 and similarly we have the for S t its projection on phi 1 and phi 2 in terms of S 1 and S 2.

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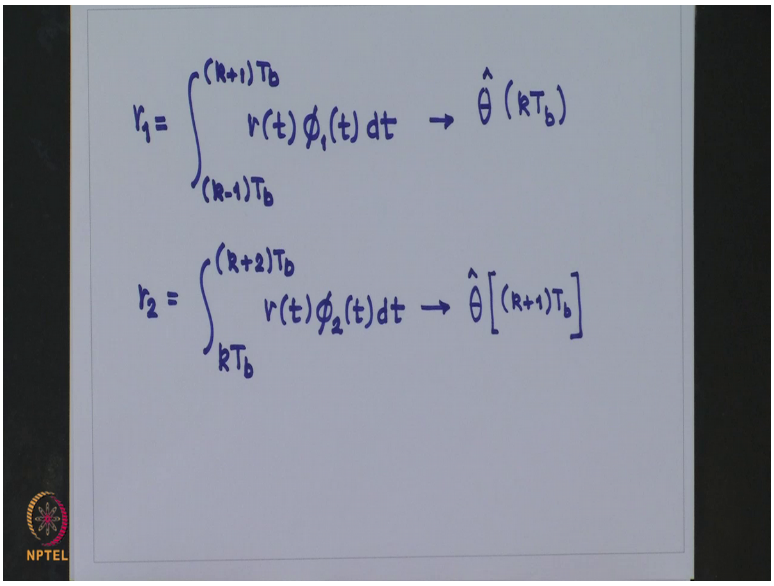

And we have seen this projection r 1 and r 2 are obtained by taking the projection of r t on phi 1 t and r t on phi 2 t correct ok. So, this is for any general k plus 1 th interval correct ok.

So, without loss of generality, let us restrict our discussion to the first bit interval that is from t equal to 0 to t equal to T b, and the same argument can be extended to any other interval ok.

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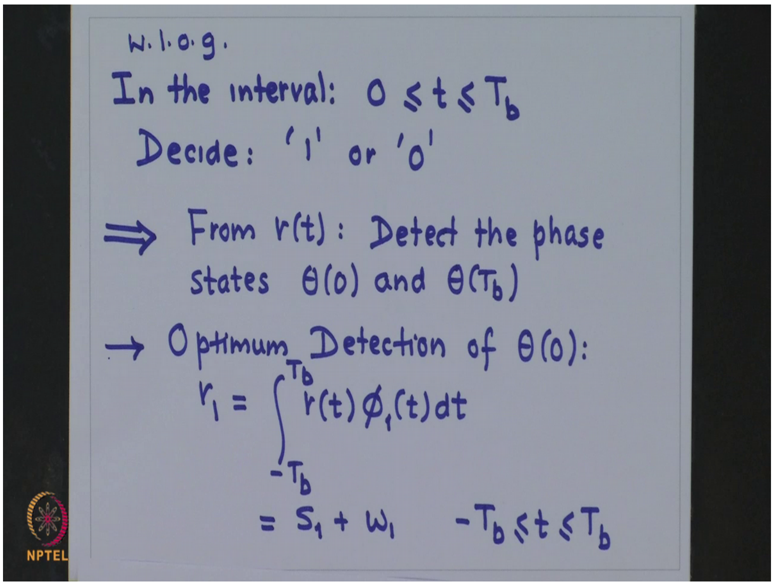

So, in the interval between 0 and T b, we have to decide which symbol was transmitted 1 or 0. Now to do that basically what we will do is that will take r t and project it on phi 1 and phi 2, and then detect the phase states theta 0 and theta T b.

Knowing these phase states, we will be able to find out what was transmitted during this interval between 0 to T b. So, optimum detection of theta 0 first let us see, we have seen from the signals phase diagram that the channel corresponding to the phi 1 axis and the onecorresponding to phi 2 axis both are independent. So, r 1 we will get it by this expression.

So, you will get S 1 plus W 1 and for this the signal constellation would be given as shown here phi 1 t.

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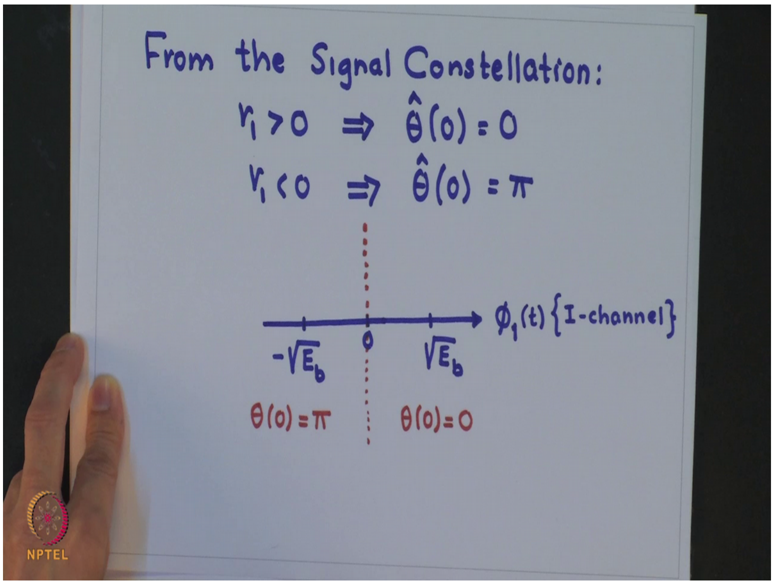

So, we will have root E b remember and this would be minus root E b. So, if lies this division will be the perpendicular bisector of this the distance between this point and this point, these are all equiprobable. So, this site of the region is theta naught theta 0 is equal to 0 and this is theta 0 is equal to pi. So, your decision will be if r 1 which is the projection on this phi 1 t axis is greater than 0, then we say that theta naught is equal to 0 and if when r 1 is less than 0 we say theta naught is equal to pi.

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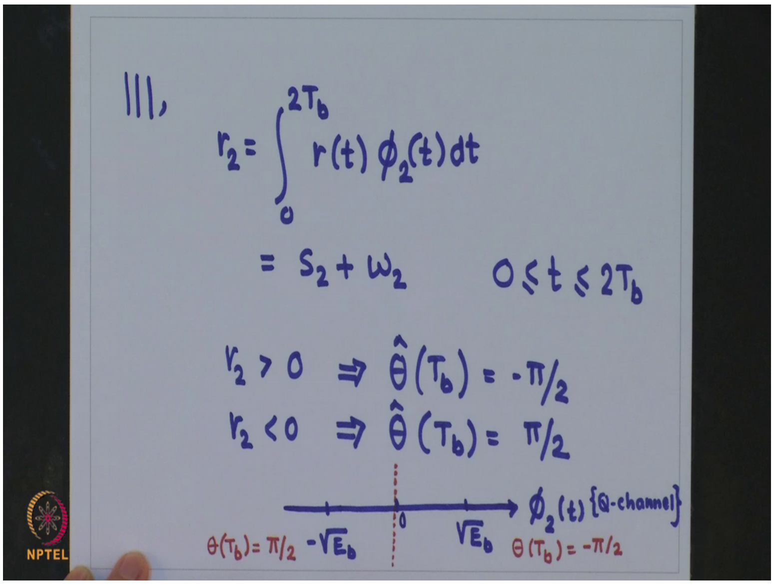

And similarly the signal constellation for the phi 2 axis would be given by this diagram out here, signal phase diagram. Now here when your r 2 is greater than 0 we say theta T b is equal to minus pi by 2 and when it is less than 0 we say theta T b is equal to pi by 2. And once we know what is the theta 0 and theta T b, then we can find out what transmission has taken place.

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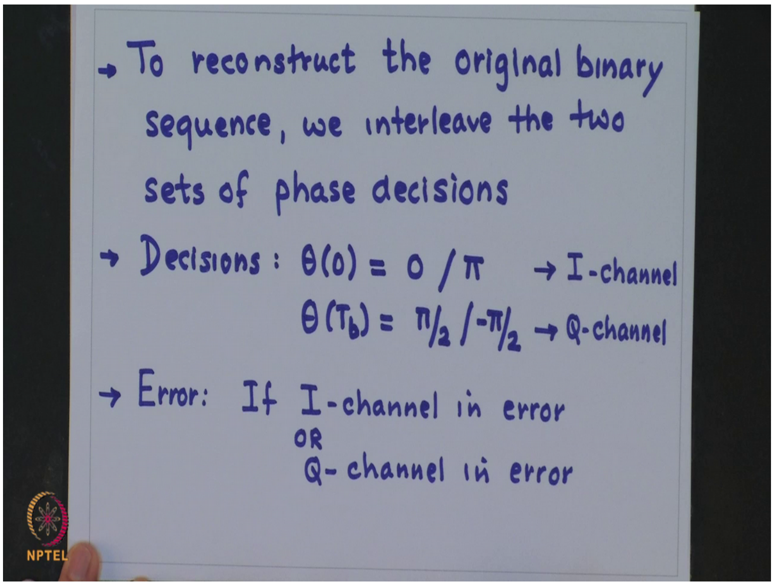

To reconstruct the original binary sequence, we interleave there are 2 sets of phase decisions correct. So, theta 0 will be 0 or pi, that will be an I channel that is Q 1 orthonormal basis signal and theta T b is either pi by 2 or minus pi by 2 this is the Q channel which is decided by the orthonormal basis signal phi 2 t.

Now, both the bits input bit stream is independent correct symbols from 1 time instant to other time symbol are independent and they are equiprobable. So, what is implies that, the error will occur if I channel is an error or Q channel is in error correct. And if we look at the signal constellation for any one of them this is the for Q channel, we can find out what is the probability of error here that will be given by the Q function and the argument of that would be the distance between this 2 divided by 2 and the whole expression is divided by root of italic N by 2.

So, in this case the distance is 2 root E b. So, if we take that we immediately get this probability of error calculation.

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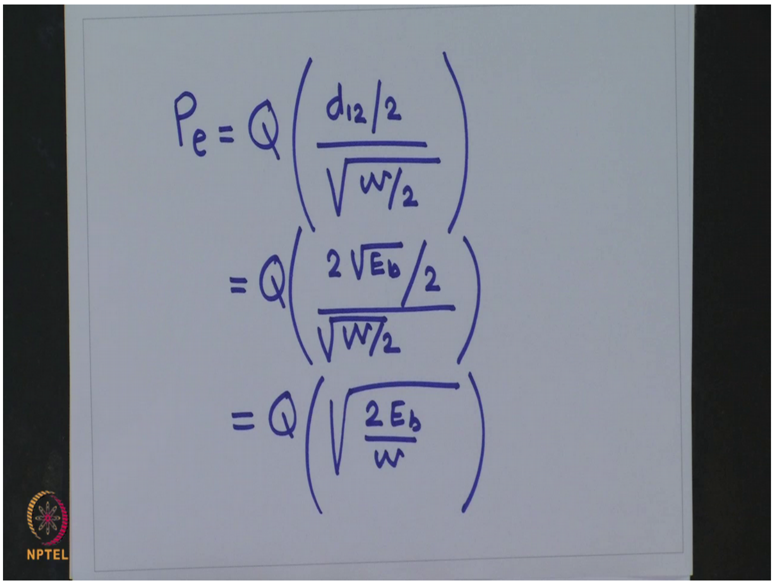

d 1 2 by 2 root italic N by 2, this is d 1 2 the distance is this and then we get this. So, what it shows that, the probability of error for the MSK case is exactly the same as you get for the BPSK case or for the QPSK and its variance correct. But the bit rate f or MSK case is higher than what we get for BPSK case, but with no deterioration in the error performance.

And let us look at the bandwidth requirement for the MSK signal and that would be obtained by finding out the power spectral density of the MSK signal.

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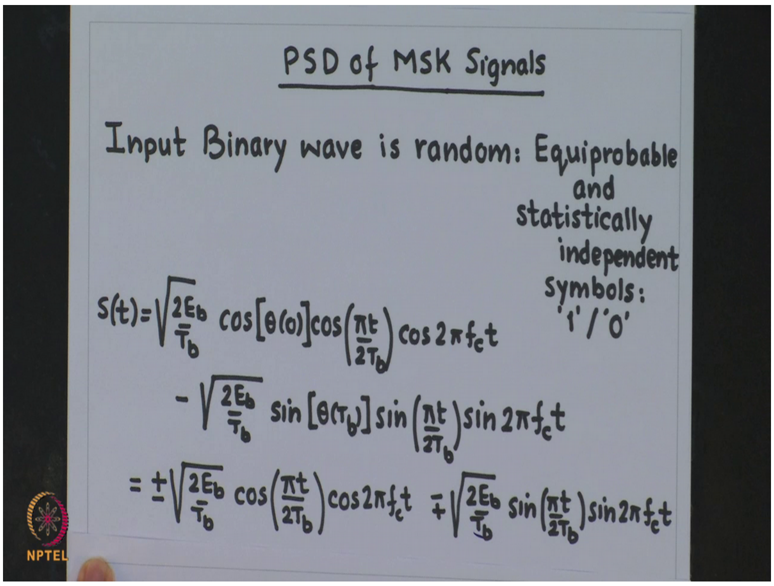

Now, remember that input binary wave is random, what we mean that equi probable and statistically independent symbols 1 and 0. Our MSK wave is given by this expression, this is the in phase and this is the quadrature phase and this we can write it as this expression the sign plus minus will be decided by the phase state of theta 0 and similarly here we can write this expression as given here, where minus plus the sign will be decided by the phase state of theta T b.

. So, we will try to evaluate first, the power spectral density at the baseband. Once we know the power spectral density at the baseband we know how to translate it to the carrier frequency f c. So, there are 2 components for calculating the baseband power spectral density, this is the in phase component and this is the quadrature component.

So, let us look at the first the in phase component. So, if we look at the in phase component, the symbol shaping function pit becomes this expression out here and plus minus is decided by the input bit stream. And similarly for the quadrature component we will see that the symbol shaping function becomes this half sine wave and the sine is decided by the input binary stream.

. So, let us take first the in phase component.

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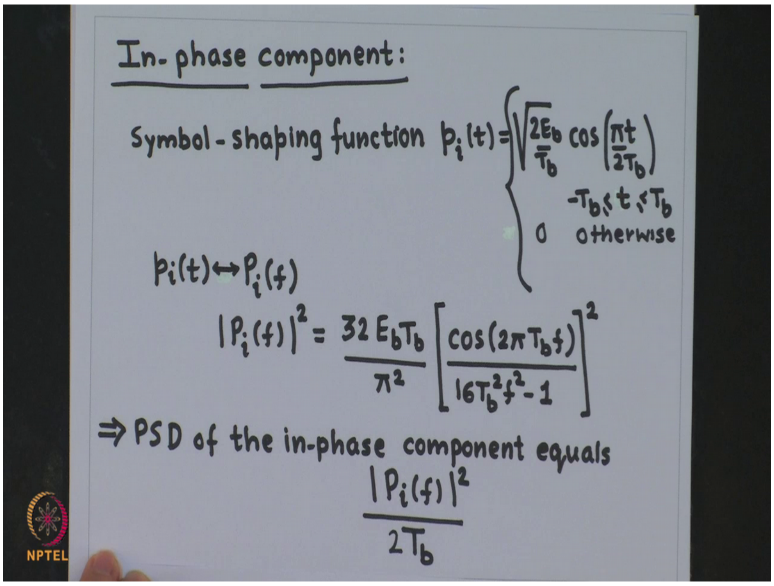

So, once I have the symbol shaping function p i t, now we can find out its Fourier transform. Once we get the Fourier transform of this we can find out the energy density of that by taking mod and square of that and we can show that it turns out to be the expression given here.

Once I have this energy spectral density, I can find out its power spectral density now is given by this expression. Remember these are nothing, but polar wave, we can consider this as a polar baseband wave and we have seen basically how to evaluate the power spectral density for such polar wave. So, we get the power spectral density of the in phase component.

Now, let us calculate the power spectral density for the quadrature component.

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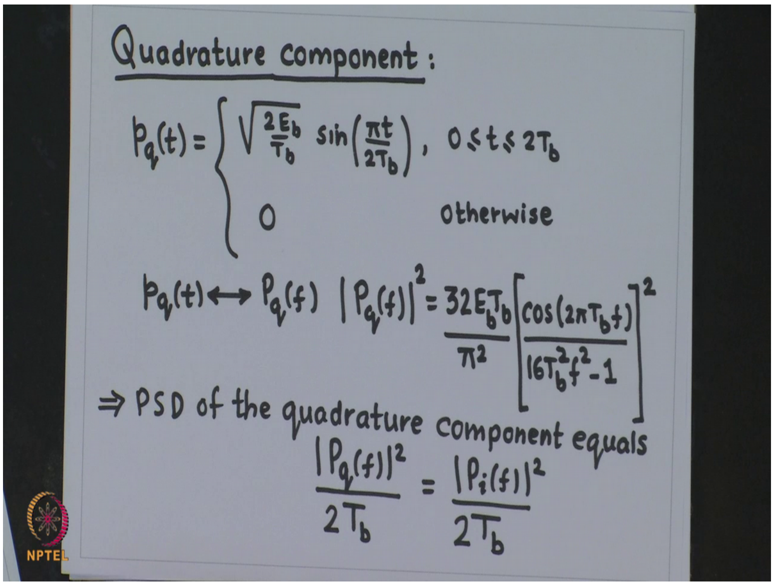

For the quadrature component, the symbol shaping function would be given by this expression again we calculate this we know that this is just the phase shift compared to the in phase component. So, this will not affect the power spectral density. So, immediately we can find out its energy spectral density, which will be the same as the 1 which we had for the in phase component and once we have the energy spectral density, the power spectral density is obtained as given here.

Remember this 2 T b comes because our polar wave, bit duration is now 2 T b correct. So, once and since this both this in phase and quadrature components are independent statistically, we can get the baseband power spectral density of the MSK signal as the summation of the 2 correct.

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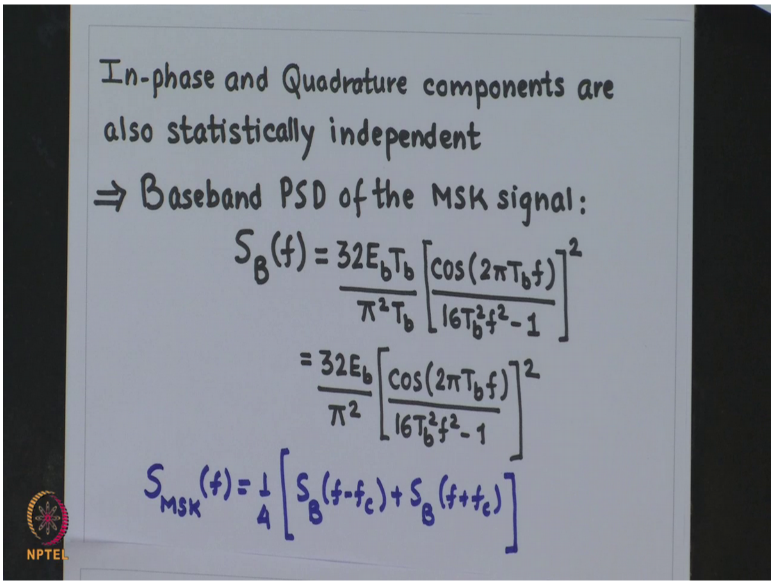

And if we do that so, it will become it becomes twice of this quantity. So, if we do that, we get this expression out here. And once we have this we know how to translate it to the carrier wave by using this expression.

And here is the plot of this power spectral density, which is being shown here.

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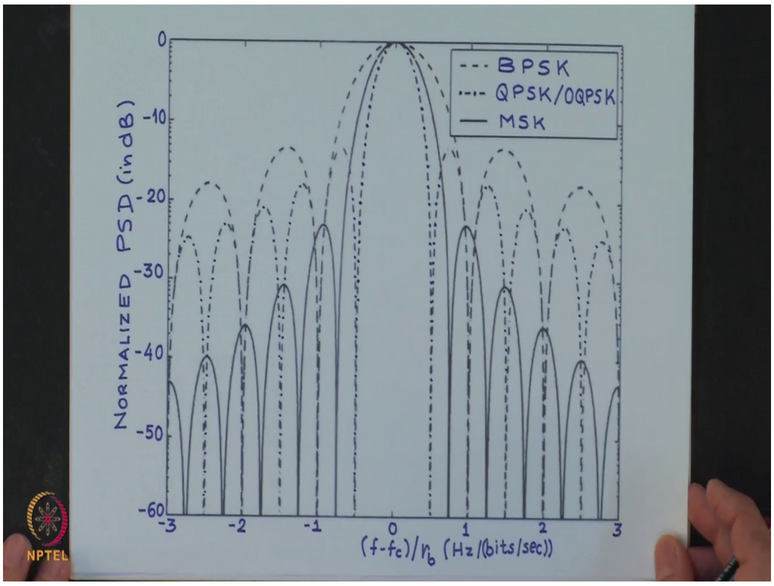

So, in this plot the solid line refers to the MSK power spectral density with a dot in between refers to QPSK or its variant and the outside 1 here refers to the BPSK power spectral density.

So, from this equation it is clear that, the power spectral density for the MSK decays as fourth power of the frequency which is considerably faster than the second power of the frequency decay behavior of the power spectral densities of the BPSK and QPSK or offset QPSK correct. So, that is a difference between MSK and the BPSK, QPSK and its variant power spectral density.

So, this decay reflects the fact that there are no discontinuity in the transmitted signal. And from this normalized power spectral densities for this three modulation scheme, it is cleared that MSK has an advantage over other 2 schemes as far as the side lobe levels are concerned. So, remember that BPSK requires more bandwidth than the others for any reasonable definitions of bandwidth such as, the null to null of fractional out of band power definition whatever definition for bandwidth we take, the BPSK requires more bandwidth. And the theoretical null to null bandwidth efficiency of BPSK is half that of QPSK we have seen this earlier.

So, as predicted by the decay rate, it is observed from this figure that MSK has lower side lobes than QPSK or BPSK. And this is a consequence of multiplying the binary bit stream with a sinusoid yielding more gradual phase transitions. So, the more gradual the transition the faster the spectral tails drop to 0. But it is important to observe that MSK spectrum which is the spectrum out here solid line has a wider main lobe compared to the QPSK or OQSK, which means that when compared in terms of null to null bandwidth MSK is less spectrally efficient than QPSK or offset QPSK. But MSK due to its constant envelope continuous phase and lower side lobe levels and the fact that it can be demodulated as easy as FSK, MSK has become a very popular modulation technique in mobile communication.

So, with this we come to the end of our study on MSK modulation. So, far we have studied basic binary passband modulation schemes for digital communications like, binary ask binary BPSK and binary FSK. The focus of our study was on the power needed to achieve a certain performance measured as the bit error probability and on the bandwidth requirements of the specific modulations.

So, besides the basic binary baseband modulations, we also studied higher level modulations namely quadrature phase shift key that is QPSK and its variant and the minimum shift key. This by introduced as methods to conserve bandwidth without sacrificing error performance. In these modulations the number of messages that is m was 4. Next time we will study higher level modulations known as M-ary modulations where this value of m would be larger than 4.

Thank you.