Principles of Digital Communications
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Lecture - 47
Quadrature Phase Shift Keying - I

We know from our study of power spectral density of the binary modulated signal that in order to increase the bit rate, we require to increase the bandwidth of transmission. Now it is always desirable in digital communication to increase the bit rate without increasing the bandwidth. There are modulation schemes which achieve this goal and the state for extension of the binary modulation scheme which provide higher bit rate without increase in bandwidth are QPSK Quadrature Phase Shift Keying, offset QPSK; MSK which stands for Minimum Shift Keying.

We will start our study with QPSK that is Quadrature Phase Shift Key.

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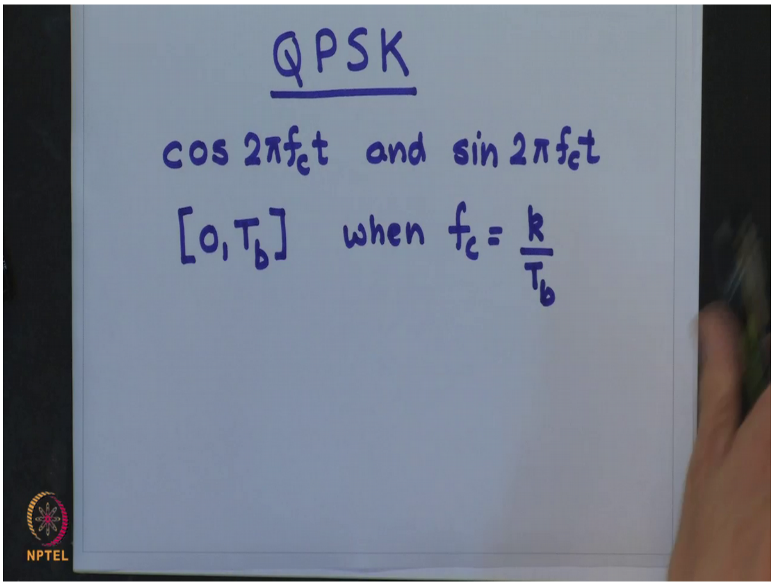

So, the bandwidth efficiency modulation schemes basically form a class of emilee modulation in general which we will study later on. So, the basic idea behind QPSK exploits the fact that the 2 carrier signal; cos 2 pi f c t and sin 2 pi f c t; this 2 signals are orthogonal over the interval 0 to T b, when your f c is chosen as k is an integer by T b.

So, just as in analog communication this can be used to transmit 2 different messages over the same frequency band. So, to accomplish this; the bit stream is taken 2 bits at a time and mapped into signals as shown in the following table.

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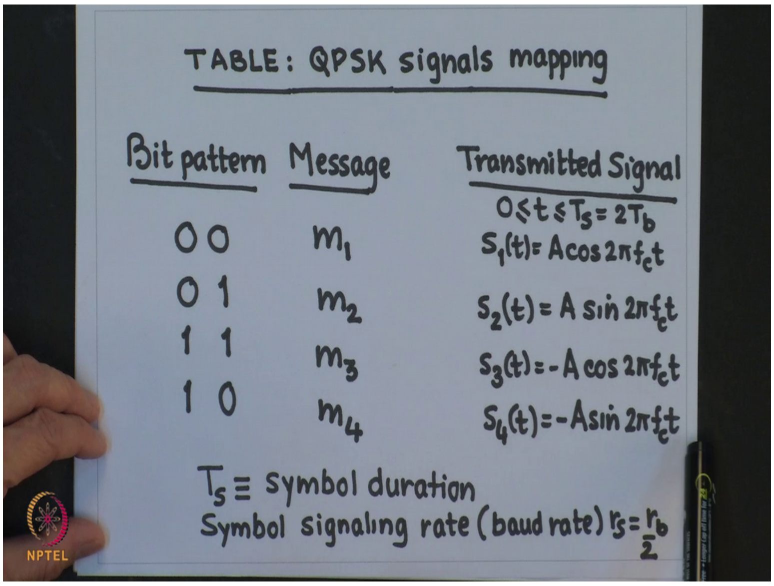

So, bit pattern 0 0 is mapped as m 1; 0 1 as m 2, 1 1 as m 3 and 1 0 m 4 and for this messages the choice of the signals are given here. So, m 1 and m 3 are polar signal with the carrier as cos 2 pi f c t and m 2 and m 4 is again polar signal but the carrier is now sin 2 pi f c t.

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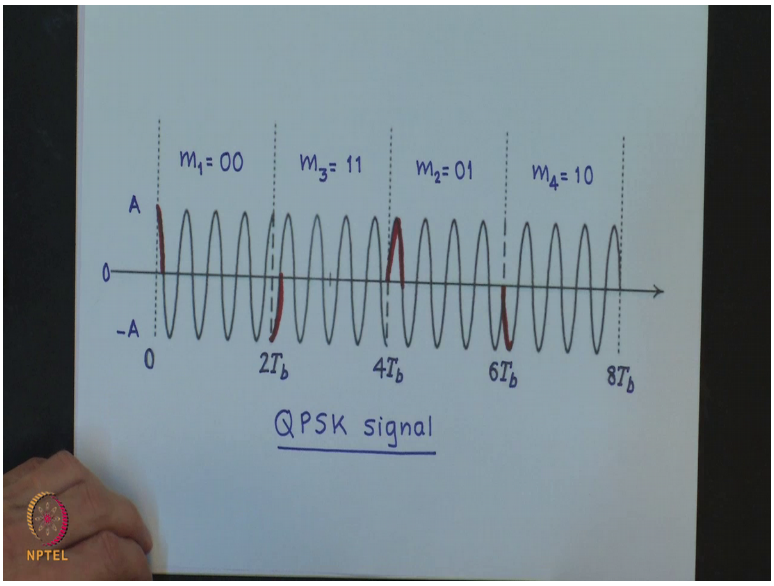

And corresponding with this the waveform would be as shown in this figure. So, this is cos 2 pi f c t corresponding to the bit pattern 0 0 or message m 1; this is for m 3, this is m 2 and this is m 4. So, since each bit in the original sequence occupies T b seconds; the signals corresponding to the symbols or messages last for a symbol duration which is T S equal to 2 T b seconds.

Now, we define what is known as baud rate that is message or symbol rate as r s and this is equal to r b by 2 because T S is equal to twice T b. So, since bandwidth is proportional to r s, it can be reduced by half for a given bit rate r b. Conversely for a fixed bandwidth the bit rate r b can be doubled ok; so now, though the bit rate has been increased without a corresponding increase in bandwidth; it is also necessary to look at what happens to the bit error probability and we will examine this issue.

So, to accomplish this signals S 1 t, S 2 t, S 3 t and S 4 t as represented by this message or signal set have to be represented as usual by an orthonormal basis set.

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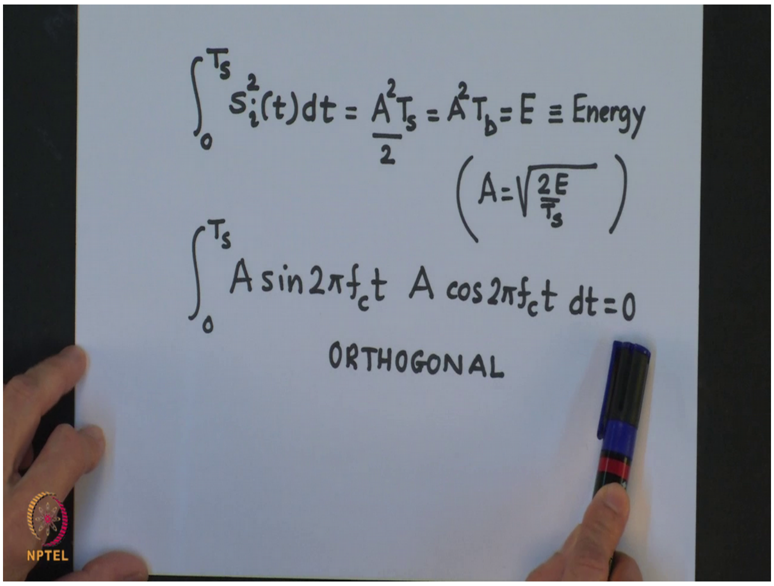

Now, note that the signals in the message signal set S i t; the energy in that signal will denoted by E and this 2 signals sin 2 pi f c t and cos 2 pi f c t are orthogonal. Given this conditions, the only 2 orthonormal basis signals needed to represent these 4 signals can be derived very easily and they are as shown on the slide.

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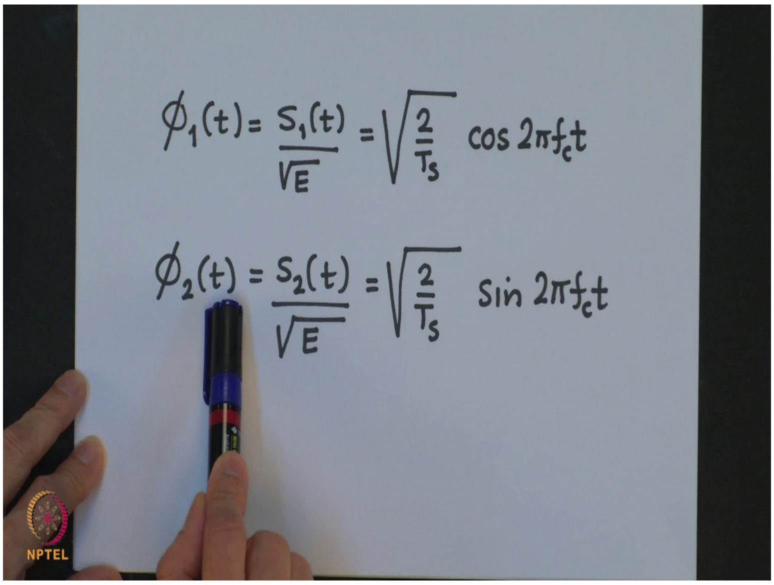

Phi 1 t which is the normalized version of the signal S 1 t and phi 2 t, which is the normalized version of the signal S 2 t; both of these signals are orthonormal. For this orthonormal basis and for the given message signal set the signal consolation will be as show in this figure.

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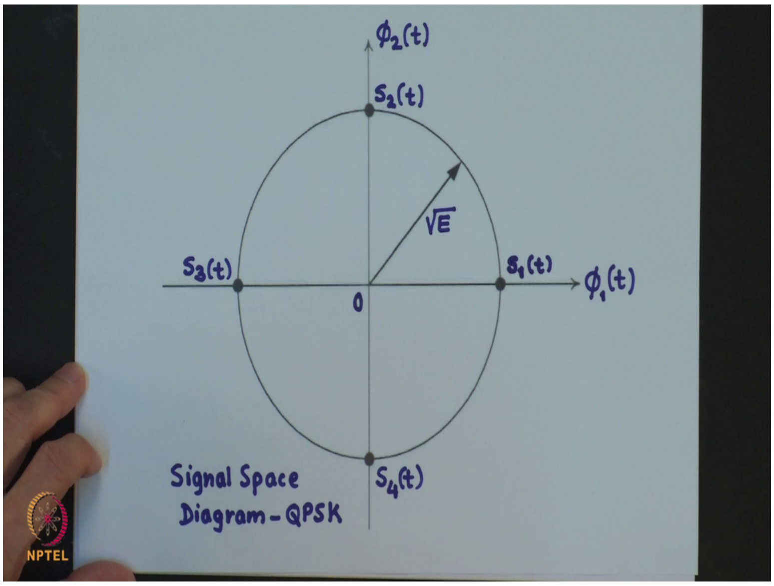

We have a phi 1 t axis, phi 2 t axis and using this as your basis signal S 1 t, S 2 t, S 3 t, S 4 t can be represented as vectors in the signal space diagram. So, this is the signal constellation which we get for the QPSK signal; now let us determine the optimum receiver for this. So, we assume that all this 4 signals are equiprobable and we have additive white Gaussian noise channel a zero-mean noise process with the variance as italic N by 2 as usual.

So, for this we know from our earlier study that we can use the maximum likelihood detection rule and if we deploy that rule let us see how do we generate our optimum receiver?

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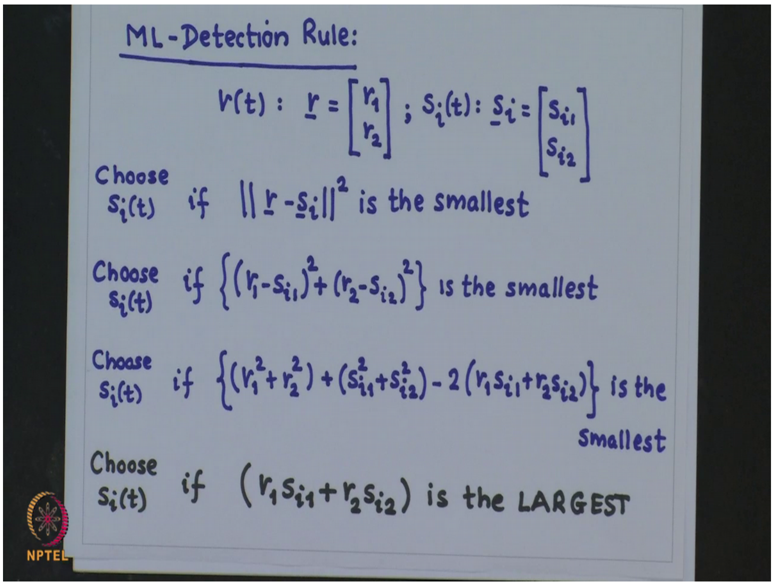

So, the first thing we do is basically take your receive signal r t and since our basis signals are 2 in number, our signal constellation is 2 dimensional. So, we project r t onto the basis signal phi 1 t and phi 2 t to generate 2 components of this vector r as r 1 and r 2. And similarly for each of this signal S i t; we can generate the message or signal vector with the 2 components. This is the component which is projected on phi 1 t and this is the component which is projected on phi 2 t.

Given this now we know that the optimum receiver which minimizes the probability of error is ML detector. And it says that choose that signal S i t for which this condition is valid that is basically we are trying to calculating the equilibrium distance between the received or observed vector and the message vectors. So, this is basically minimum distance receiver; now we can expand this quantity and if you do that, we will get this expression out here, so choose S i t if this is the smallest.

Now, further expanding this expression and collecting the terms together, we get this 6 terms. Note that this is the energy in the received or observe vector which is constant for any transmitted signal. And in our case this is the energy in the signal which is equal for all i equal to 1, 2, 3, 4. So, when we try to compare this expression for different i's; this 2 quantity remains the same. So, this rule can be modified now as choose S i t if this is the largest correct?

So, if we do this basically then we get the following receiver.

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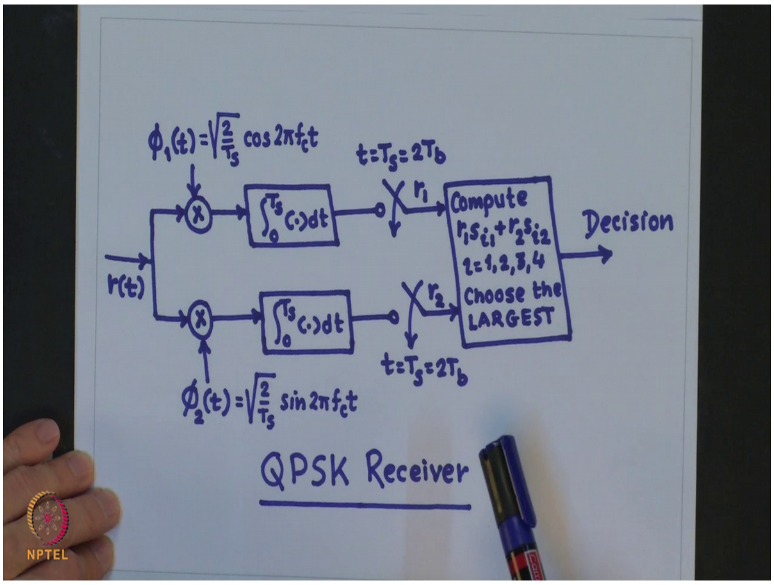

So, you have a r t, we have 2 basis signals; we project that using the correlation receiver, we sample it at appropriate time to get the 2 components of the observed vector r 1 and r 2. And we have the templates for the message vector, we take the dot product of the received vector with the message vector and then choose the largest; this is how we generate the optimum QPSK detector, receiver or demodulator.

Now, having done this let us look at the calculation of the probability of errors for this; now to do that we need to first look at the decision regions in the signal constellation.

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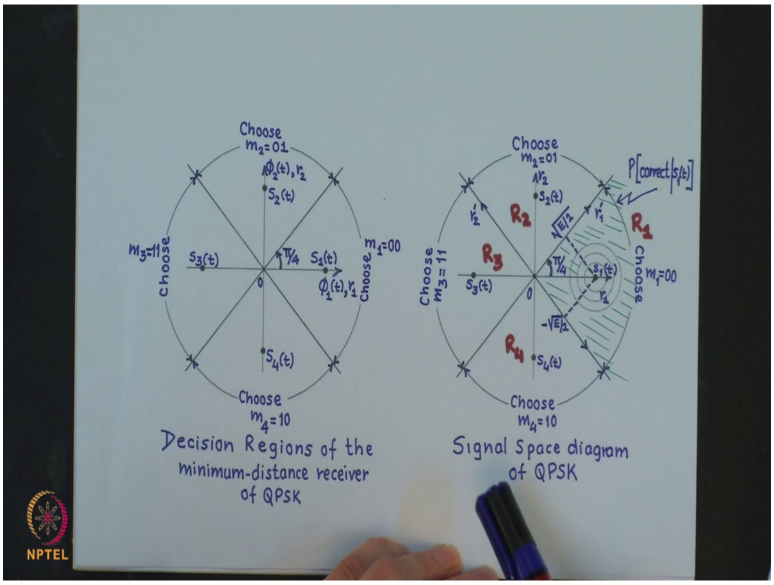

So, this is the signal constellation; these are 4 points here now you have to partition this space into 4 regions corresponding to the 4 message vectors. Now since all of these are equiprobable; the region between S 2 and S 1 will be the perpendicular bisector joining the line S 2, S 1.

Similarly, the region between S 4 and S 1 will be partitioned by the line which is perpendicular to the line joining S 1 and S 4. So, this is the sector that is the region 1, this is region 2, region 3, region 4. So, whenever your receive vector lies in this region; this would be r 1, it would be decided in favor of signal S 1 and if it lies here in this sector, it will be decided in favor of S 3 correct? Now to calculate this probability of error; what we will do is basically, we will use the principle of rotational invariance which we have studied earlier.

So, the philosophy for doing that is as follows; remember all this are equiprobable S 1, S 2, S 3, S 4. So, all this regions basically have equal probabilities what I mean by that is that what is the probability that the observe vector falls in this region? That probability will be same as the probability of that vector falling in this region or this region or this region. So, because of the symmetry and equal a priory probabilities of 4 signals; probability of error for this receiver would be the same as the conditional probability of error when you transmit any one particular signal, so this is what I mean.

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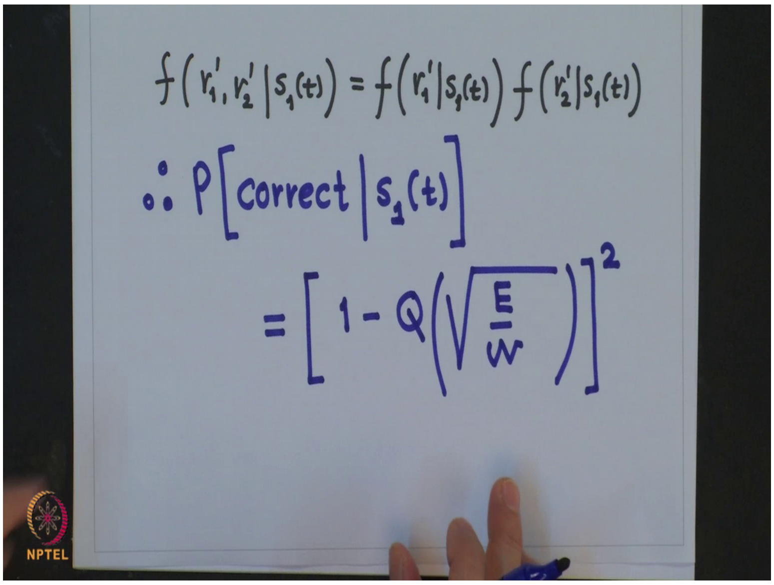

So, this is the conditional probability of error given that I have transmitted S i t. So, in this case S i t being equiprobable, the probability of overall error is the same as this. And to calculate this is same as 1 minus the probability of correct detection given S i t correct. So, the probability of the vector lying in this region is the probability of correct decision given that I have transmitted S 1 t fine. So, what we will do? That we will try to calculate the probability of correct decision given S 1 t.

Now, in order to do this we require to find the joint pdf of the 2 components of the vectors r 1 and r 2; given S 1 t and that has to be integrated in this region r 1. So, to do that what we will do is we will change this coordinate axis and rotate it. So, if we take this axis and rotate it by 45 degrees, if I rotate by 45 degrees, I get new coordinate axis like this. So, this is your new coordinate axis; so, in this new coordinate axis your received vector with the components r 1 and r 2 can be projected as r 1 dash and r 2 dash, so, if we do this what we are doing is as follows.

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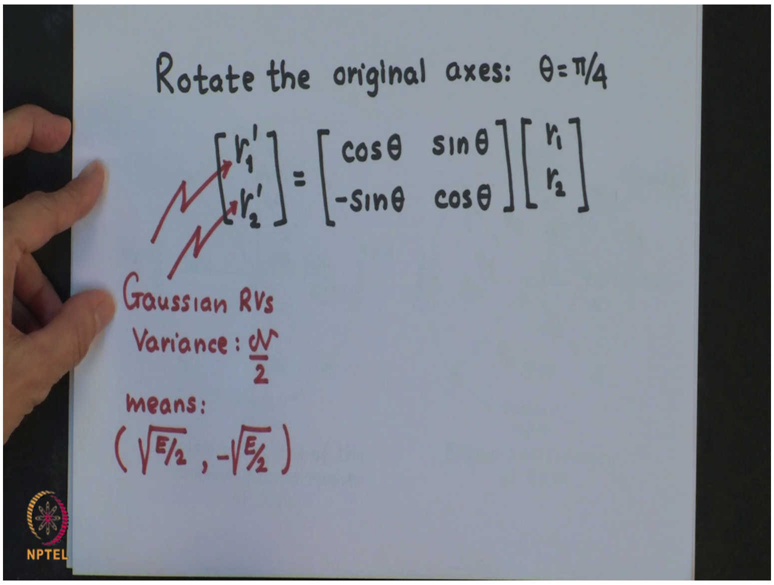

You have this new coordinate points for the receive vector in terms of the old coordinate points of the receive vector through this transformation matrix, this transformation matrix remember is orthonormal you can easily show this. So, based on our principle of rotational invariance, the probability of error which we calculate now should be the same as we get in the earlier case. And we are also seen that the components of the observed vector in the rotated coordinate axis will have this property now, it will still remain Gaussian random variable because this is Gaussian random variable, this is a linear transformation.

The variance will still be italic N by 2 because the variance of each of this is italic N by 2. And it is being rotated through an orthonormal matrix and the means for this will be as given here root of E by 2 and minus root E by 2. This is very clear from the figure out here, remember look here at this point if; if I transmit S 1 t then your receive vector will be somewhere around this circles. So, I have drawn this circles dotted circles depending on the strength of noise, this receive vector will be on this circumference of any of this circle.

So, now if you take this point; this point gets projected in the new coordinate axis as minus root of E by 2 on this axis. And on this axis it gets this point gets projected as root of E by 2 correct. So, if you look at this point as far as your noise distribution is concerned; the Gaussian distribution. Now your Gaussian distribution is going to be of something like this form correct and the mean for this is going to be minus root of E by 2. As for this point is concerned the Gaussian distribution is going to be something of this form and the mean will be located at root of E by 2 correct.

And so, given this and remember this; the noise components along both these axes are independent.

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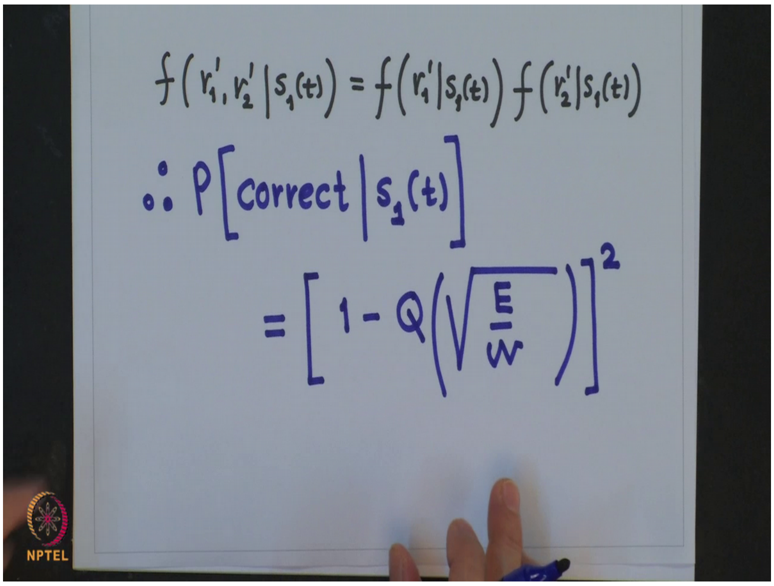

So, what it implies that the joint pdf of the observed components in the new coordinate axis given S 1 t will be the product of the marginal pdf's ok. So, given this now we can easily calculate the probability of correct detection given that I have transmitted S 1 t, this would be equal to look at this figure. So, what I want that if I place my Gaussian pdf with the mean at this point; I want that the thing should be lying here.

So, it means it is going to be 1 minus Q times this quantity divided by root of m correct. So, this we have done earlier correct? So, it is very simple if you look at the way this Gaussian distribution for the observed vector will look into this new coordinate axis, we can immediately write the probability of correct detection as follows.

So, this is the symbol or message error probability and not the bit error probability; remember this please. So, please note that this is the symbol or message error probability and not the bit error probably. So, even though a message error has been made; it does not mean that a specific bit is in error.

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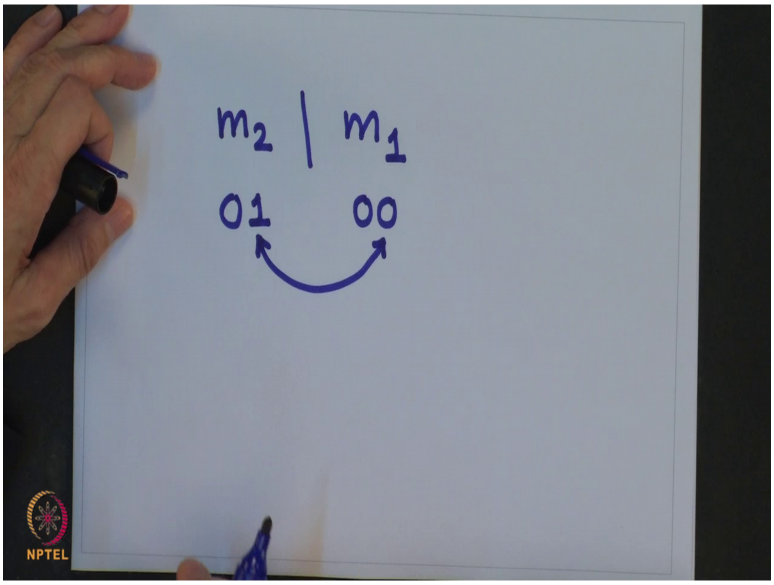

So, for example, if the receiver decides on message m 2 given that we have transmitted m 1; then in this case this message was 0 0 and this is 0 1. So, what it means? That only the second bit has gone in error. So, to determine the bit error probability, it is necessary to distinguish between different message errors.

And this we will do it in the next class, we will continue our discussion on QPSK error calculation and evaluate the probability of the bit error.

Thank you.