Principles of Digital Communications
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Lecture - 45
Binary Frequency Shift Keying - I

Digital information can be encoded in any one of the parameter of the sinusoidal carrier signal. It could be amplitude which will give us amplitude shift keying, it could be phase which will give us phase shift keying and we have studied binary amplitude shift keying and binary phase shift keying.

Today we will study the variation of the frequency parameter of the sinusoidal carrier and we will examine binary frequency shift keying. In binary frequency shift keying, we have 2 signals of frequency f 1 and f 2 the sinusoid with the frequency f 1 would be used to transmit the symbol 1 and sinusoid with the frequency f 2 would be used for transmitting of the symbol 0.

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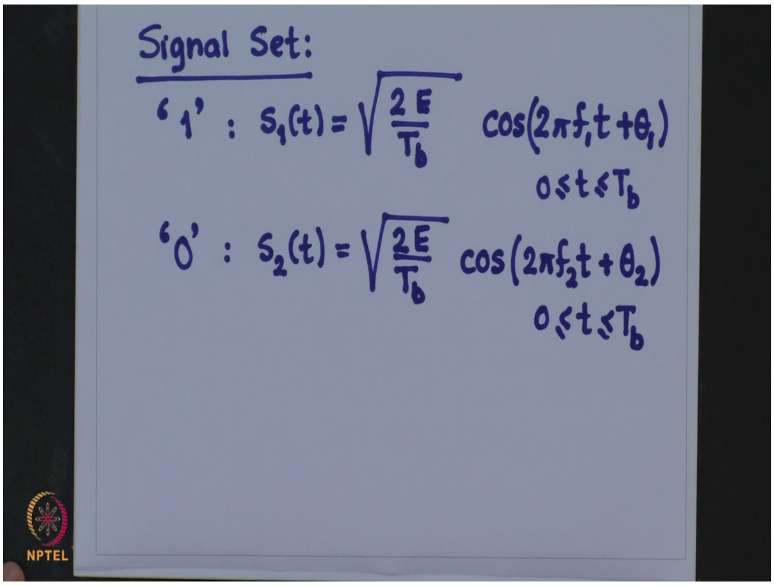

So, the signal set which we use for the binary FSK is as follows for symbol 1 we will use the signal S 1 t which is given as follows. We will also assume that it has some phase theta 1 to make the problem more generic and for symbol 0, we will transmit the signal S 2 t which is another cos function, but a frequency of f 2. Theta 1 and theta 2 need not be the same and we will assume that f 1 and f 2 are multiples of 1 by T b where T b is the rate at which we transmit each of this symbols.

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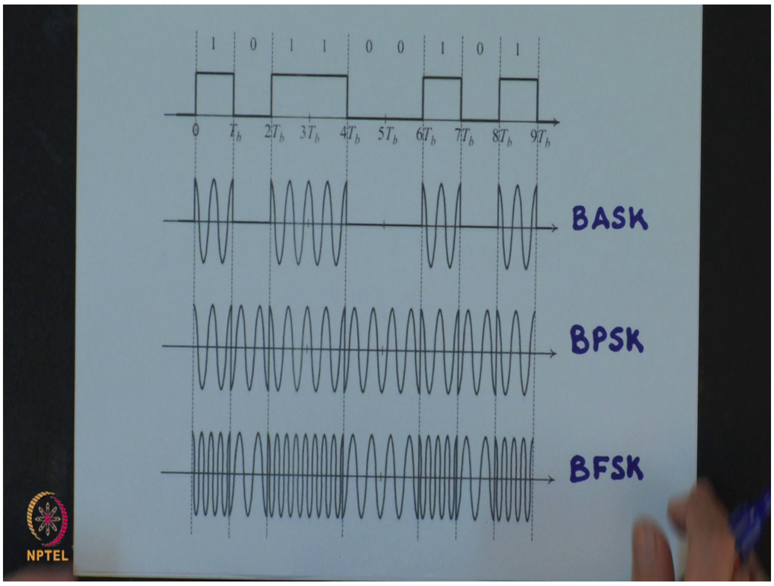

Our binary FSK modulated signal would look something like shown in this figure. So, this is your BASK and this is BPSK which we have studied earlier and this figure out here corresponds to binary frequency shift keying. So, we see that for symbol 1 we have one frequency and for symbol 0, we have another frequency

Now, before we go ahead it is important to know when with these 2 signals S 1 t and S 2 t would be orthogonal over the period from 0 to T b, let us try to find the answer to this question.

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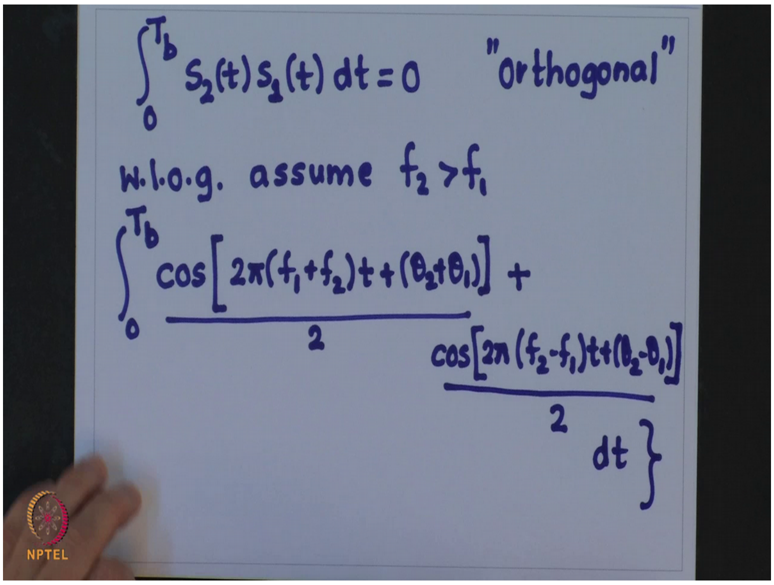

So, the question is that when would the 2 signal S 2 t and S 1 t be orthogonal that means, it satisfies this condition. So, this has to be equal to 0 without loss of generality. let us assume that f 2 is greater than f 1. Then if this has to be satisfied then what it means that this also has to be satisfied.

So, we have cos functions; cos 2 pi f 1 t plus theta 1 multiplied by cos 2 pi f 2 t plus theta 2 we have to integrate that product. So, we will use the trigonometric relationship and immediately we can write this first plus there will be another term. So, this term I am writing below and this whole thing has to be integrated correct; so, sum of these 2 terms this is integrated ok.

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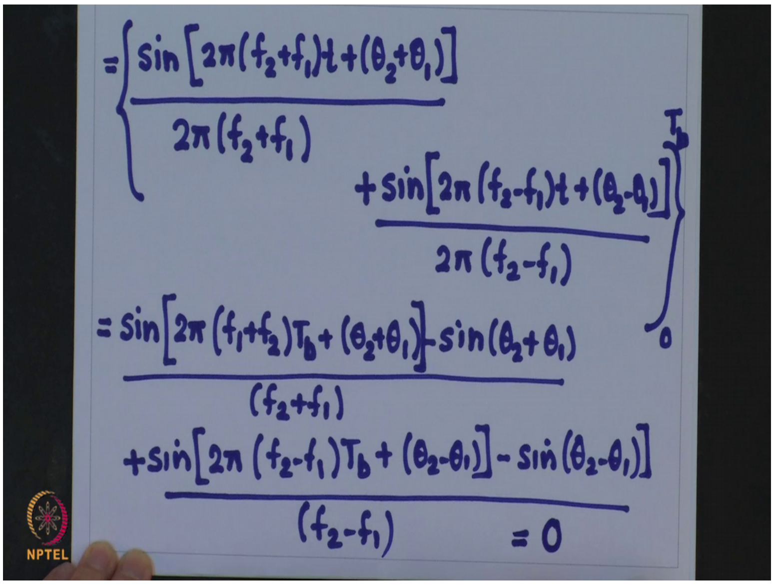

So, if we integrate this we get the following expression; this is the expression which we get after integrating the first term. And the second term would be as follows, now the both these terms addition have to be evaluated for 0 and T b because the integration limits was 0 to T b. So, if we do this; we get, so this would be the evaluation of this term from 0 to for 0 and T b and the other term would be equal to.

Now, there are 2 cases let us consider one where the 2 phases theta 1 and theta 2 are same and the other is when theta 1 is not equal to theta 2. So, let us take the first case when theta 1 is equal to theta 2 and now we want that for the orthogonality condition to be satisfied this summation should be equal to 0. So, if that is the condition to be satisfied, then we see from here it is very clear that. So, when theta 1 is equal to theta 2 then if you want this expression to be totally 0 then we will get this condition.

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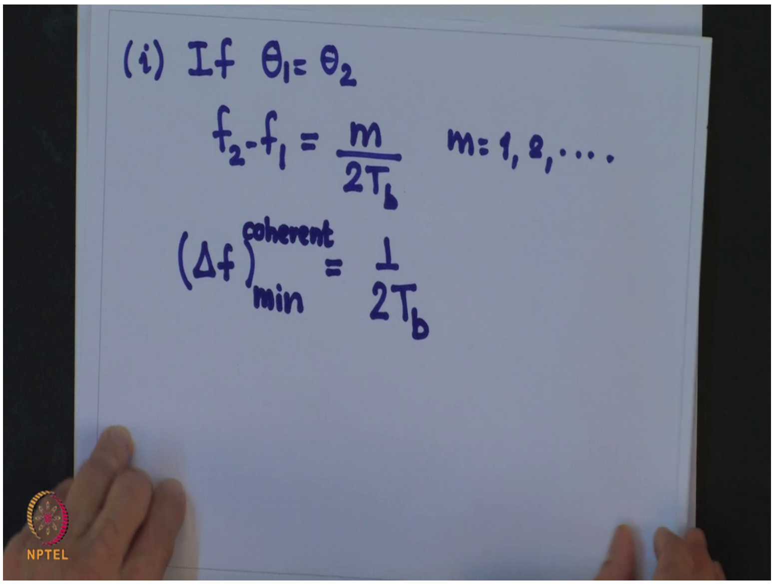

. So, first is if theta 1 is equal to theta 2, we get the condition that f 2 minus f 1 should be equal to m by 2 T b where m is equal to 1, 2 and so on. So, the minimum frequency separation f 2 minus f 1 for orthogonality will occur when m is equal to 1 and then in that case this separation is indicated as follows minimum is equal to 1 by 2 b.

Now, in this case this separation is also known as coherent because the 2 sinusoidal carriers are set to be coherently orthogonal coherent because the 2 phases are the same correct ok.

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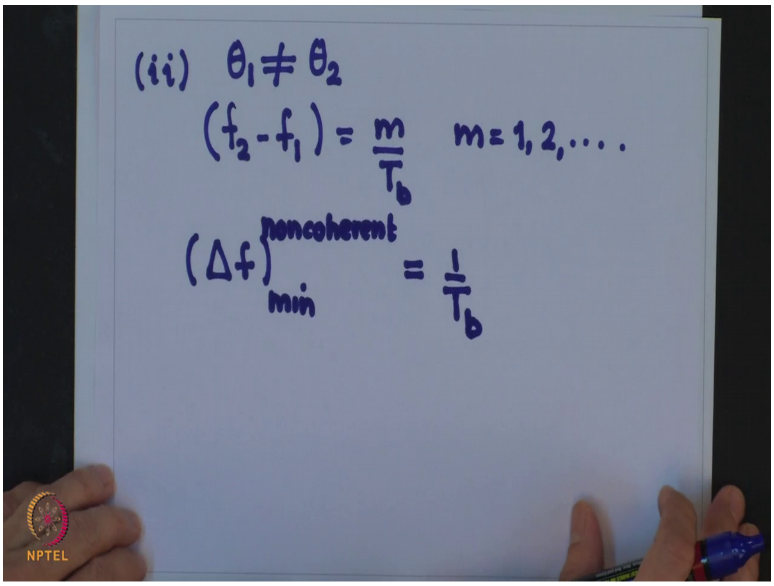

Let us take another case when the 2 phases are different; that means, theta 1 is not equal to theta 2. In that case, again from here it is easy to see that the difference f 2 minus f 1 should be m by T b correct because you will have to take care of this terms, theta 2 is not equal to theta 1. So, this sin terms have to get cancelled and that is why we get that relationship is simple to see that m is equal to 1 2 and so, on.

So, the minimum frequency separation for this case which we call as non coherent orthogonality, non coherent because there is no relationship between the 2 phases; theta 1 and theta 2 and this minimum separation is indicated as follows. Minimum and we will write it this is for non coherent case; this is equal to 1 by T b. So, above shows that relaxing phase synchronization of the 2 carriers requires doubling of the minimum spacing in order to maintain the orthogonality of the 2 carriers.

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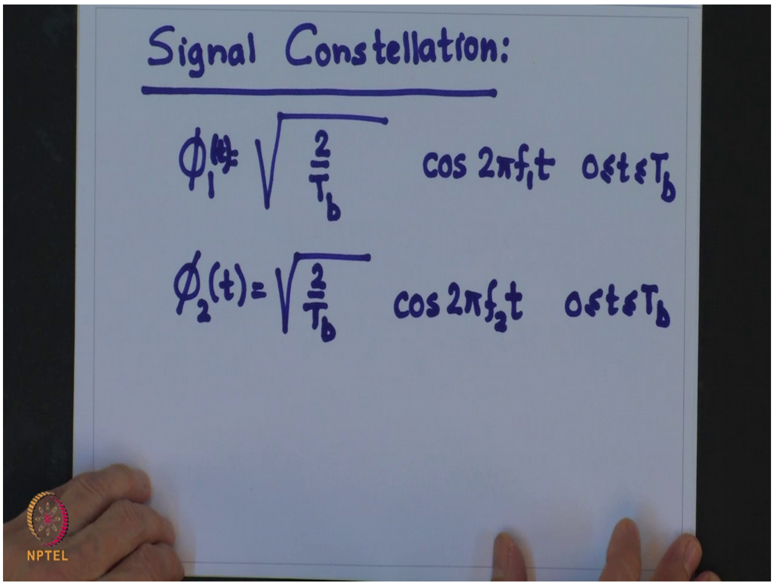

Now, given this let us look at the signal concentration now. So, we choose your S 1 t and S 2 t with the frequencies f 1 and f 2 respectively in such a way that it satisfies the orthogonality condition. So, let us take a very generic case for the discussion where theta 1 and theta 2 need not be the same correct.

So, in that case the minimum separation has to be 1 by T b, but we will not consider that in writing our basis signal. We know now for this will require 2 basis signals and those 2 basis signals are as follows function of time. So, given this basis signals we can write the transmitter block as follows.

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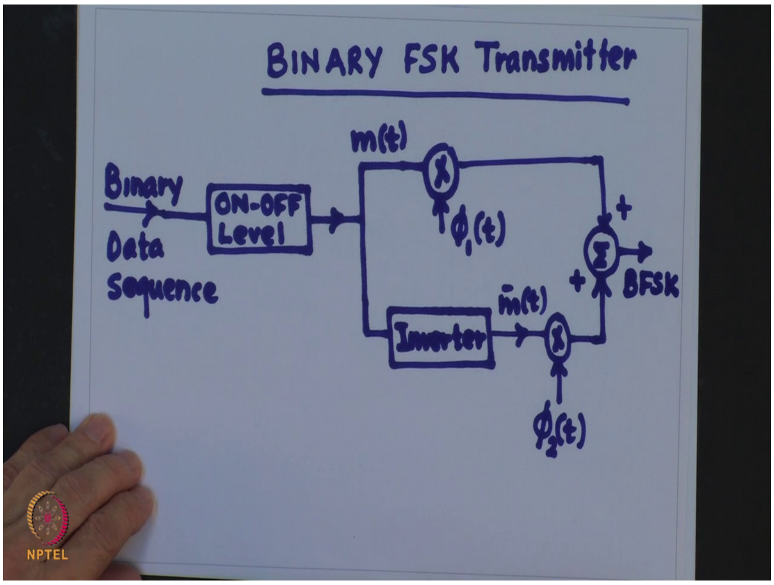

So, binary FSK transmitter would be something as shown in this figure. We have binary data sequence 0s and 1; we pass it through on off level encoder. So, the output of that this becomes my message signal, we multiply this by phi 1 t, we take this pass it through a inverter.

So, 0 will become 1 and 1 will become 0 this we give it to the product modulator with the other input to be phi 2 t. And the output of this summer would be your binary FSK signal correct this is how we can generate our binary FSK signal. Now, let us look at the signal space diagram for this. So, this will help us to design the optimum receiver or detector for this.

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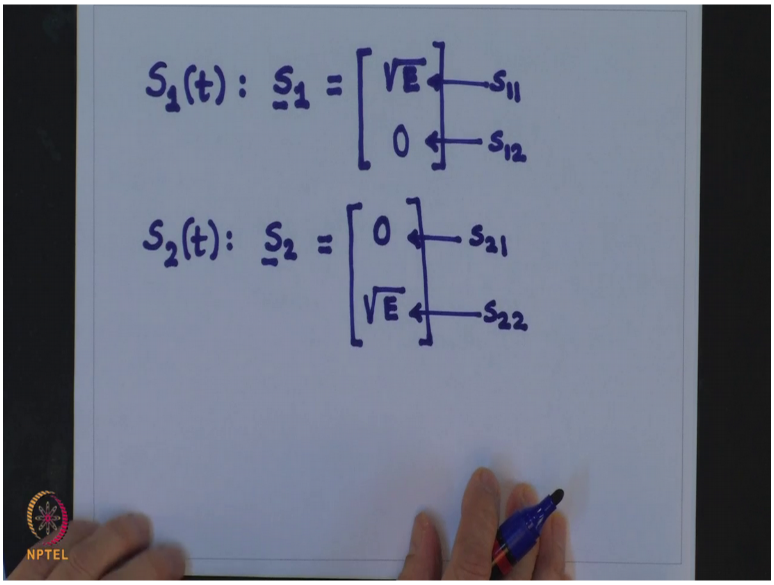

. So, S 1 t would get mapped to S 1 vector and it will have 2 components; the first component in this vector would be root E that is the projection of S 1 t on phi 1 t and the projection of S 1 t on phi 2 t would be equal to 0. So, this is your S 1 1 and this is your S 1 2.

Similarly, your signal S 2 t will get mapped to the vector S 2 which will be equal to the projection of S 2 t on phi 1 t is 0, projection of S 2 t on phi 2 t is root E. So, this is your S 2 1 and this is your S 2 2.

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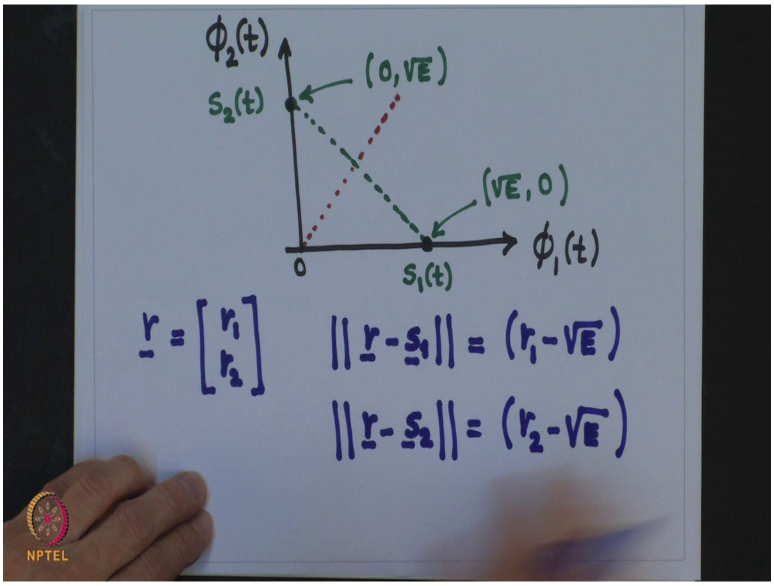

So, if we draw the signal space diagram for this or signal constellation it will looks something like this; this is my phi 1 t axis this is my phi 2 t axis your signal S 1 t will be let us say this is the root E and your signal S 2 t would be the same as this distance; so, it would be located here.

So, these are the 2 points we have this corresponds to S 1 t, this corresponds to S 2 t this is equal to 0 root E coordinates. And the coordinates for this is root E 0 the decision region for this can be obtained by joining this line and then taking the perpendicular bisector of this line; if you take the perpendicular bisector of this line I get this. We are assuming that both the signal S 1 t and S 2 t are equiprobable; having done this your receive vector can be anywhere in this space 2 dimensional space.

So, what we will have to do is that let us indicate the receive vector r; it will have 2 components r 1 and r 2; r 1 is obtained by taking the projection of r t on phi 1 t, r 2 is obtained by taking the projection of r t on phi 2 t. Now, let us calculate the distances of S 1 t and S 2 t from this receive vector r. So, the Euclidean distance r minus S 1 vector this would be equal to r 1 minus root E. And the Euclidean distance between the vector r and the message vector S 2 would be equal to r 2 minus root E; our maximum likelihood detection rule says that to whichever message vector r is closest to that we should assign the detected symbol.

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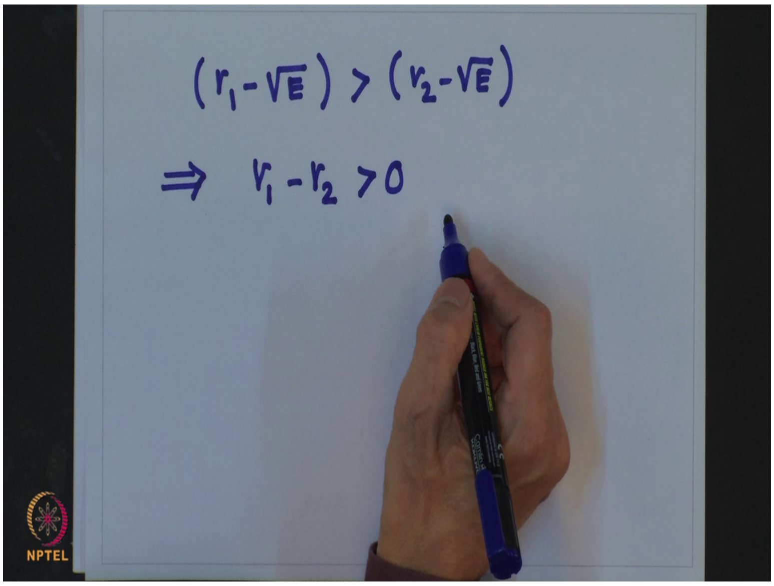

So, in this case now we will our decision would be as follows r 1 minus root E if it is greater than r 2 minus root E, my decision will be that I have transmitted 1. And if it is less then it means that I have transmitted 0; so, this implies that r 1 minus r 2 is greater than 0.

So, if it is greater than 0 I will decide in favor of 1 and if it is less than 0; I will decide in favor of 0.

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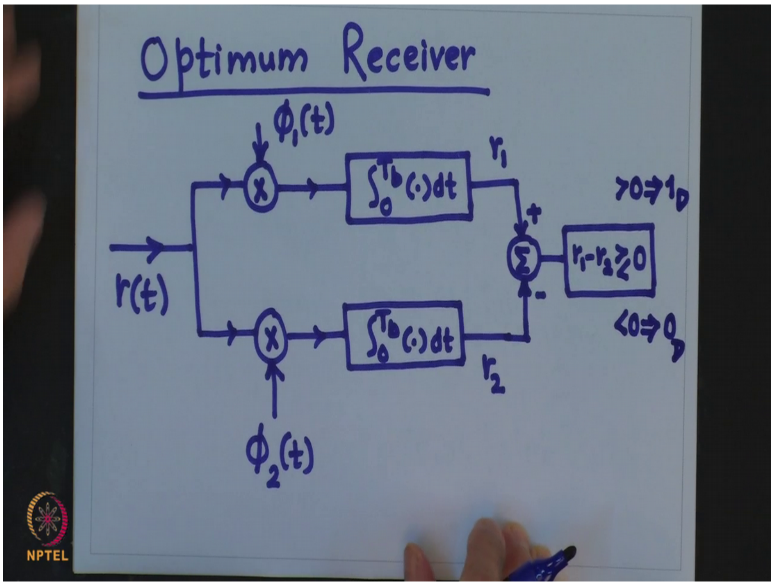

So, given this now we can get our optimum detector or the receiver or demodulator for the binary FSK as follows. I have my input signal r t; I take the product with phi 1 t, I integrate this over the interval 0 to T b to get the projection.

Similarly, r t we take its projection on phi 2 t this is a correlation receiver. So, I will get the output to be here r 1, here I will get the output to be r 2 I take the difference of the 2. So, I have my decision device which is r 1 minus r 2 greater than or less than 0. So, if it is greater than 0, it implies my decision is 1; so, 1 is detected and if it is less than 0 it implies that my decision is 0; so, 0 is detected.

Now, given the optimum receiver next thing we have to do is calculate its probability of error and power spectral density for the binary FSK. And this, we will do it in the next class.

Thank you.