Principles of Digital Communications
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Lecture - 44
Binary ASK and PSK

In baseband transmission, there are transmitted signal power lies at low frequencies; typically around 0. It is desirable in many digital communication system for the same reason as in analog communication system for the transmitted signal to lie in a frequency band toward the high end of the spectrum. And this is achieved by encoding digital information has a variation of the parameters of a sinusoidal signal called the carrier signal.

Typically as for analog communication systems, the carrier frequency chosen is much higher than the highest frequency in the modulating signal. So, there are 3 basic forms of digital modulation and these are as follows.

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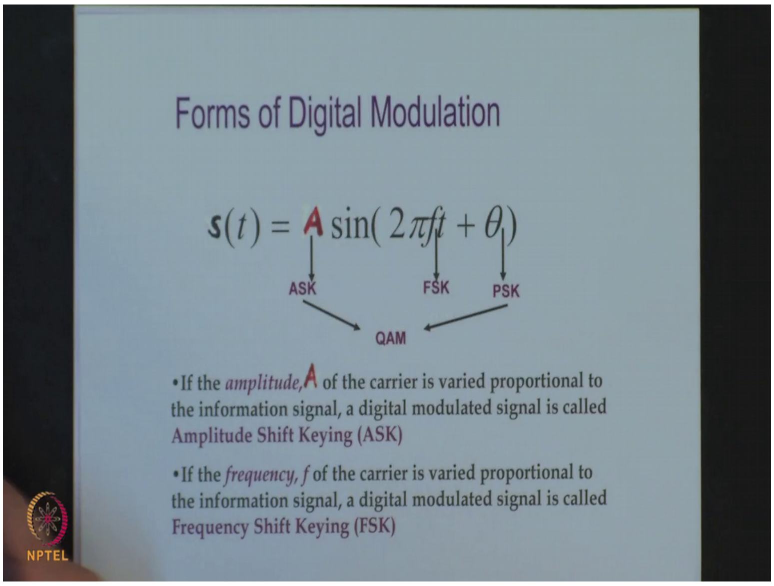

So, here this is the carrier with the amplitude A, frequency f and the phase theta; if the amplitude A of the carrier is varied in proportional to the information signal, then what we get? A digital modulates signal called as Amplitude Shift Keying, if the frequency of this carrier is varied then we get what is known as Frequency Shift Keying. And if the phase theta of this carrier is varied in proportional to the information signal, then a digital modulated signal we get is what is known as Phase Shift Keying.

Now, if both the amplitude and the phase theta of the carrier are varied proportional to the information or message signal, a digital modulated signal is generated called Quadrature Amplitude Modulation; QAM; Q A M; Quadrature Amplitude Modulation. The probably the first type of digital modulation to be practically applied was amplitude shift keying.

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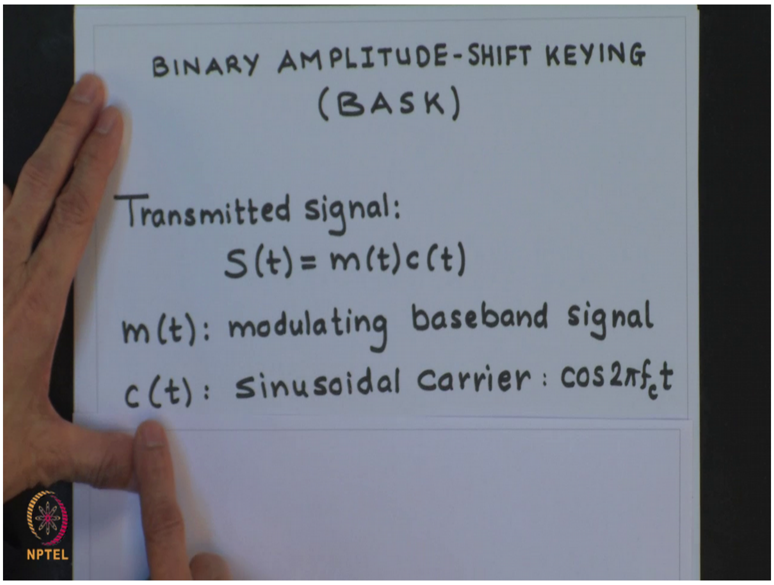

So, we will start with our study on binary amplitude shift keying; in short BSK. So, we will denote the transmitted signal S t as product of modulating signal m t and c t is the carrier; m t is the modulating baseband signal and c t is a sinusoidal carrier in the form of cos 2 pi f c t; f c is the carrier frequency.

Let me just show you the waveform for such a modulation scheme.

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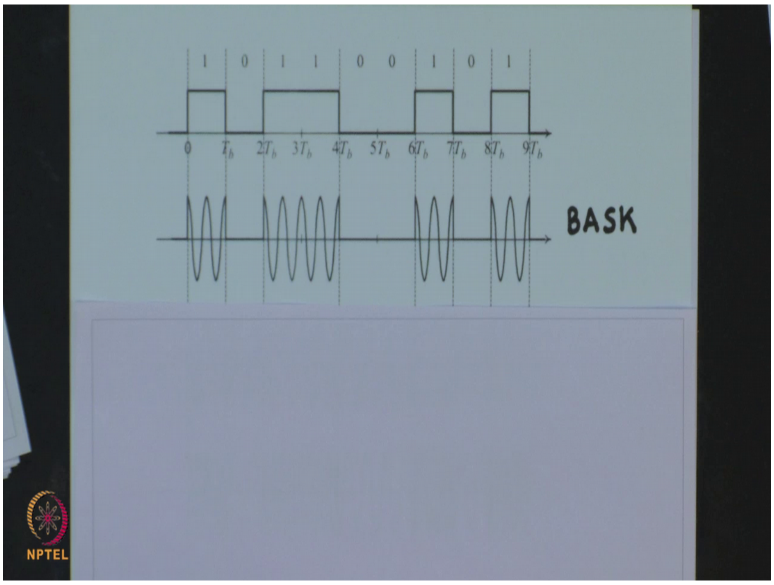

Let us assume that we have binary data in the form of 1, 0, 1, 1, 0, 0, 1, 0, 1 and it has been converted to a line code using a unipal polar scheme. So for 1, we have a pulse and for 0 we do not have a pulse and this becomes your modulating signal m t and it modulates the carrier cos 2 pi f c t; so, the output which you will get is shown here. So, whenever there is a 1, you transmit the carrier, whenever it is 0; you put it off.

So, in binary amplitude shift keying a sinusoidal carrier that is cos 2 pi f c t is gated on and off by the binary digit or bit sequence to be transmitted. So, this is your binary ASK modulation and in its simplest form, it has been used for radio telegraphy transmission in morse code.

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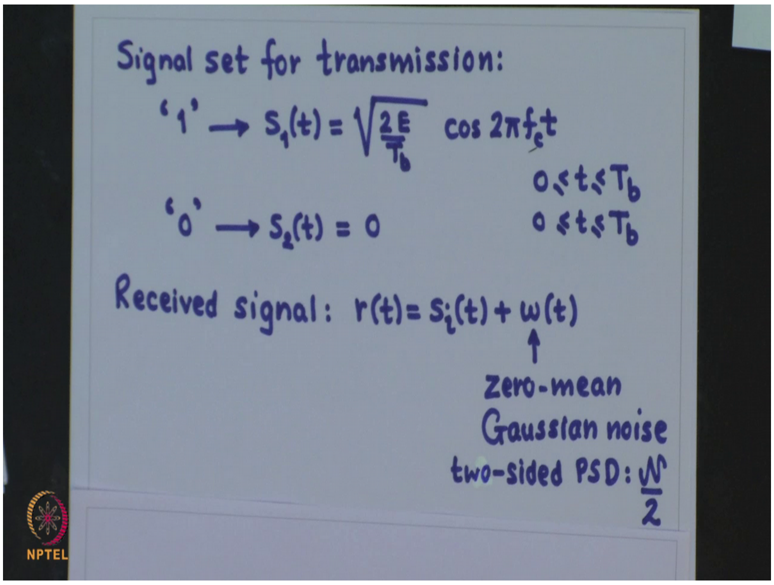

So, let us look at the signal set for this transmission; we have 2 signals for symbol 1, signal would be S 1 t which is given here with the amplitude given this by root of 2 E T b. Now the reason for choosing this kind of an amplitude is that if you find out the energy in this signal S 1 t, then it will turn out to be E. And that is the reason for writing the amplitude in this form. And the other signal S 2 t is equal to 0 for the symbol corresponding to 0 binary digit; both these signals are over the big duration which we denote by T b.

So, received signal would be equal to S i t plus W t where we assume that W t is additive white Gaussian noise of zero mean and the 2 sided power spectral density as usual is given by italic N by 2 Watts per hertz. So, depending on this symbol we have transmitted this S i t; we will take one of this form

Now, the first thing when we will try to analyze all this modulation scheme is that we have to decide how to represent this signal set in terms of signal vectors. So, what we are looking is basically to get the signal constellation or the signal space diagram for the given signal set. So, in this case it is very easy to see for the this 2 signals which we have, we have only one orthonormal basis signal and that basis signal we can choose to be as phi 1 t is equal to S 1 t normalized by the energy E.

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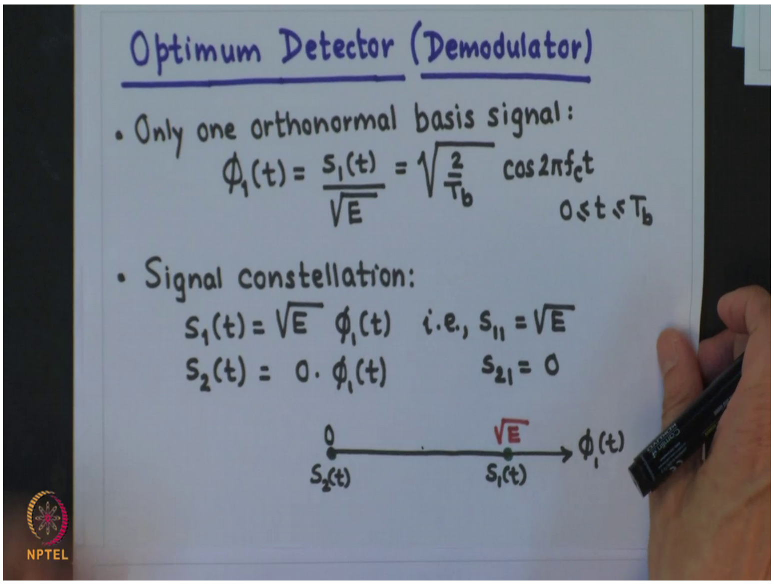

So, this will become of energy 1 over the duration 0 to T b; one more thing to remember that if we choose f c to be some integer multiplication of 1 by T b, then the energy in this will always turn out to be 1, but if it is not indeed your multiple of f c; then approximately it will be equal to 1 because when we integrate this over the duration 0 to T b for large value of fc, we can show that this will turn out to be equal to 1.

Now, so signal constellation for this would be given as follows. So, first is the S 1 t signal; I take the projection of that signal onto phi 1 t; I will get it root E; that means, the S 1 1 denotes the projection of the signal S 1 t on to phi 1 that; so, S 1 1 is equal to root E. And the projection of the signal S 2 t on to phi 1 t is equal to 0. so, S 2 1; that means, this one denotes the projection on the phi 1 basis signal and 2 here denotes the signal. So, S 2 t is being projected on phi 1 t and that value is equal to 0. So, the signal space diagram or signal constellation will look as shown here; so, this is 0.

Now, given this once we have the signal constellation, once we know the orthonormal basis signal it is very easy to find out the optimum detector for this, we have done this based on our results which we have studied earlier immediately we can write the optimum detector for this case binary amplitude shift keying which is as shown in this figure.

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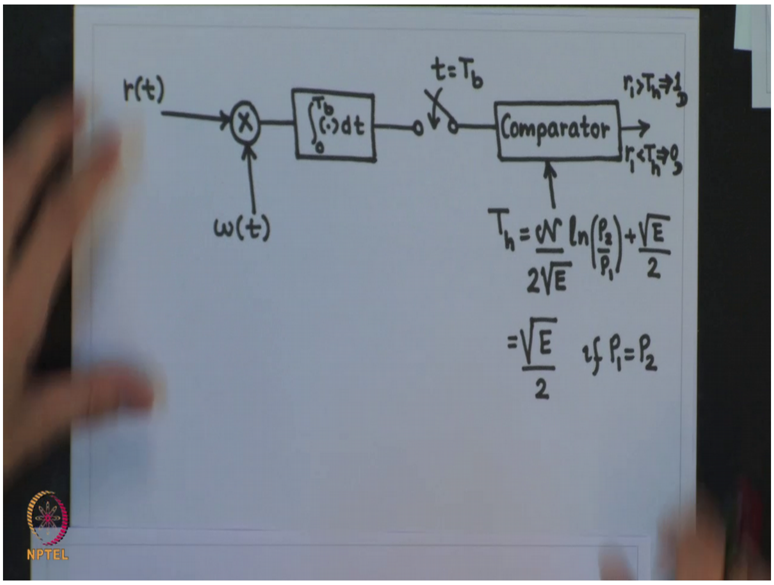

You have in the form of correlation receiver we have implemented and the comparator here. In general this would be the threshold depending on the probability for the transmission of the symbol 1 and symbol 0, but for our study unless stated otherwise; we will assume that probability P 1 is equal to P 2. In that case the threshold for this will turn out to be root E by 2 correct, half of this perpendicular bisector exactly we get this these are thing.

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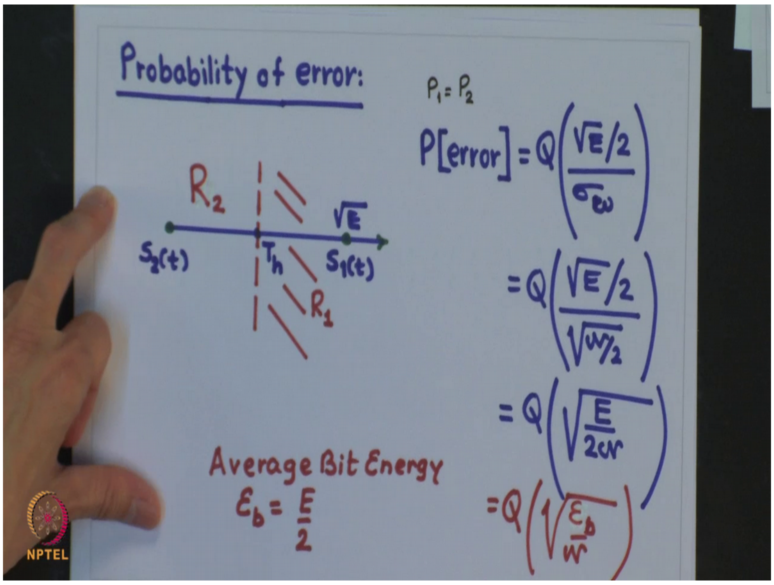

And then calculation of the probability error is again straightforward we have this signal consolation, decision boundary is midpoint; this side is R 2 region, this side is R 1 region and we have seen how to calculate the probability of error for any 2 equiprobable signals. And immediately we can write here the probability of error is Q times, the distance divided by 2 and the standard deviation of the noise; we are projecting the noise onto the phi 1 axis which is orthonormal axis therefore, the variance remains as italic N by 2.

So, standard deviation is square root of that and we get this quantity; now it is always more logical to express this ratios in terms of average bit energy; specifically when we move toward the higher constellations where more than 2 signals are involved that time you would be interested in finding out this expression in terms of bit energy.

So, if we do this in our case here the average bit energy will be equal to E by 2 because probability of 0 is half probability of standing 1 is half S 2 has 0 energy S 1 has E energy. So, the average bit energy turns out to be E by 2; so, if I plug in this equation I get this fine. And the last thing you are supposed to do whenever we will be evaluating the different modulation scheme is to evaluate the power spectral density of a particular modulation scheme.

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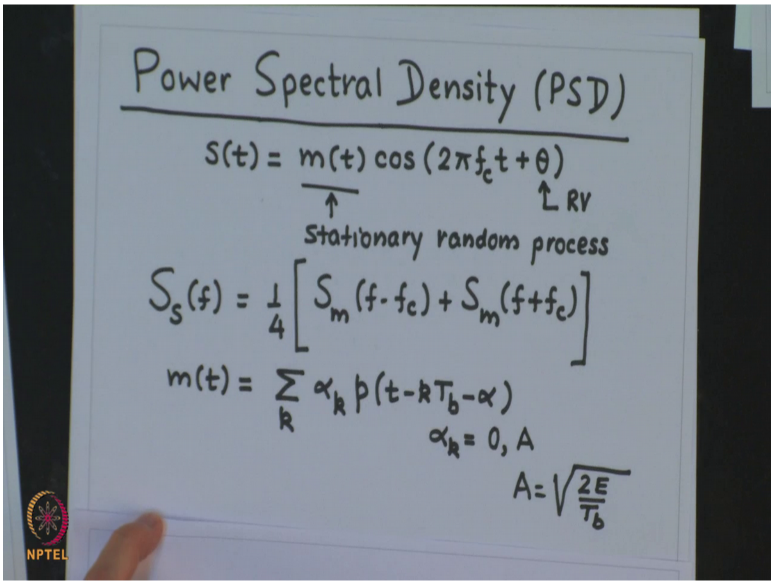

So, in general this idea concept will be useful for evaluating the power spectral density. If I have a signal S t which is a product of a stationary random process and a cos function like this, which I have shown here that theta is a random variable. So, from our study in random processes; we know that the power spectral density for this signal S t would be given in terms of the power spectral density of m t which is stationary random process and the carrier frequency f c correct.

Now, in our case for binary ASK, your m t is of this form p can be anything, but in the diagram which I had displayed your pulse basic pulse was chosen to be rectangle, but it could be anything; it could be raised cosine type also. So, for, but for our study just now we will restrict ourselves to the rectangle pulses without loss of generality. So, in our case now alpha k can take 2 values 0 and A; this A is nothing, but root of 2 E by T b it is easy for me to write the things in terms of A. So, we will write in terms of A, but remember A is equal to this quantity.

So, using this relationship we know how to calculate the power spectral density for this modulating signal m t correct; this we have studied earlier. So, we will use the results from there this is nothing, but a unipolar line code and for the unipolar line code we have the result here.

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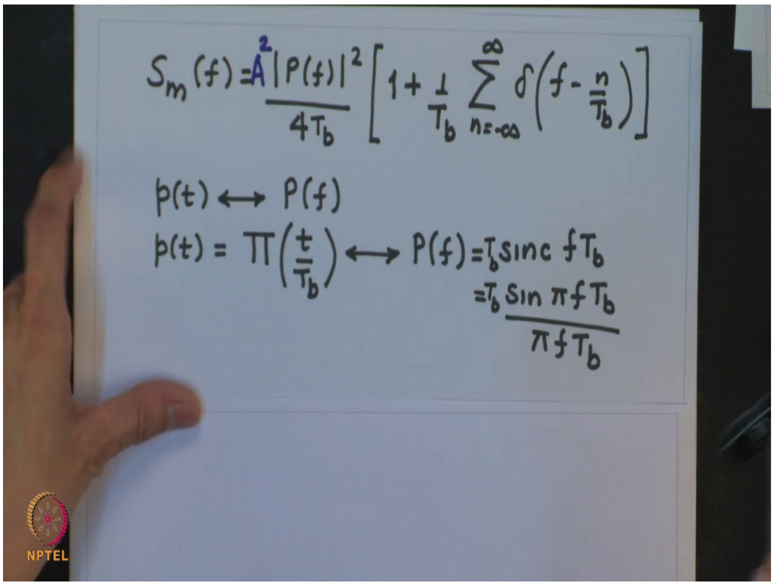

We have derived this is the power spectral density for the unipolar line code and using that the Fourier transform of pt is P f; if we use a rectangular type of a shape for the p t, then the P f is given by this expression and using this expressions we can plug into this expression for the power spectral density of the modulated signal it is a straightforward thing.

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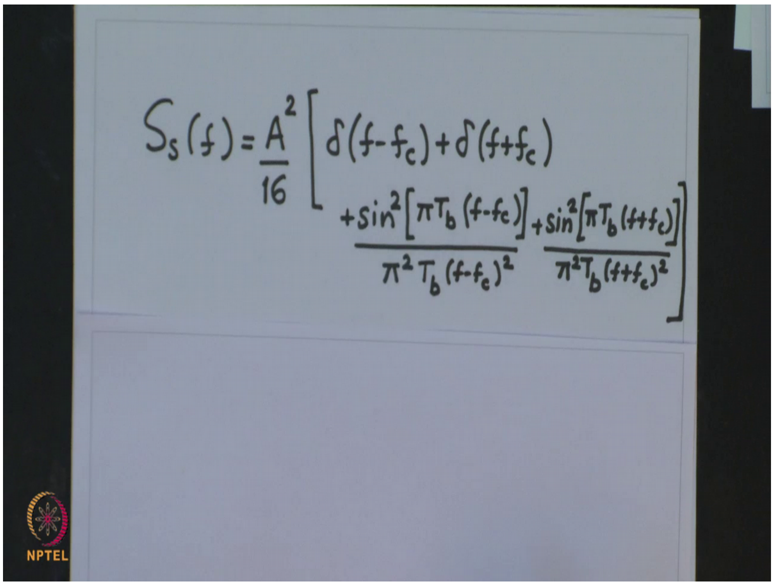

So, if you do that we get this expression, this expression is little bit approximation there would be another term which is sin of this quantity upon pi T b f minus f c square multiplied by sin pi T b f plus f c upon pi T b f plus f c correct, but f c being very large we can neglect that cross product term. So, approximately this would be equal to this and this is very clear I mean what I am saying if it will become very clear; if you look at the diagram for the power spectral density which you will get from here.

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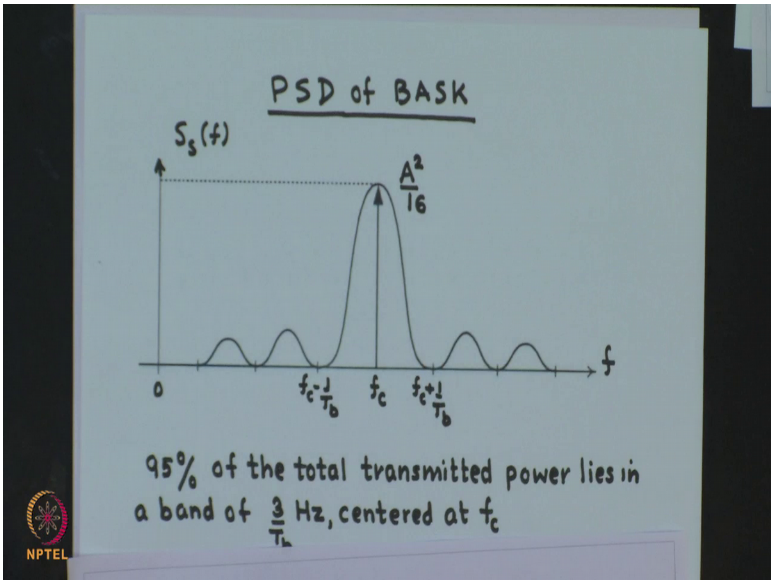

So, I am just showing you on the positive side of the frequency axis the same thing will be valid on the left hand side of this figure. The right hand side this would be the power spectral density, remember this has impulses at 0 1 by T b 2 by T b and all that, but the impulses at 1 by T b, 2 by T b and ahead will be suppressed because of the 0s at the this position 1 by T b, 2 by T b due to the sinc function. So, you do not see the impulses out here and you see only one impulse at f c correct.

So, this is what you will get for the power spectral density for the binary amplitude shift keying; now it looks like an infinite type of thing. So, remember what I was saying about the cross product is something like this because in practice this goes up to the infinity on both the side. So, this could go really into the left hand side and the left hand whatever is here is on there on the left hand side also.

So, the right portion of that left hand side will also trickle into this portion, but we are saying that this f c frequency is quite high correct. So, the f c frequency is very high by the time it goes in this side of the axis this lobes would have died down ok. So, that is the basically what I meant by was that cross product terms is neglected. So, this is if you calculated you will find that 95 percent of the total transmitted power lies in the band of 3 by T b hertz centered at f c correct. So, this helps us to decide a bandwidth of the binary amplitude shift keying modulation schemes ok

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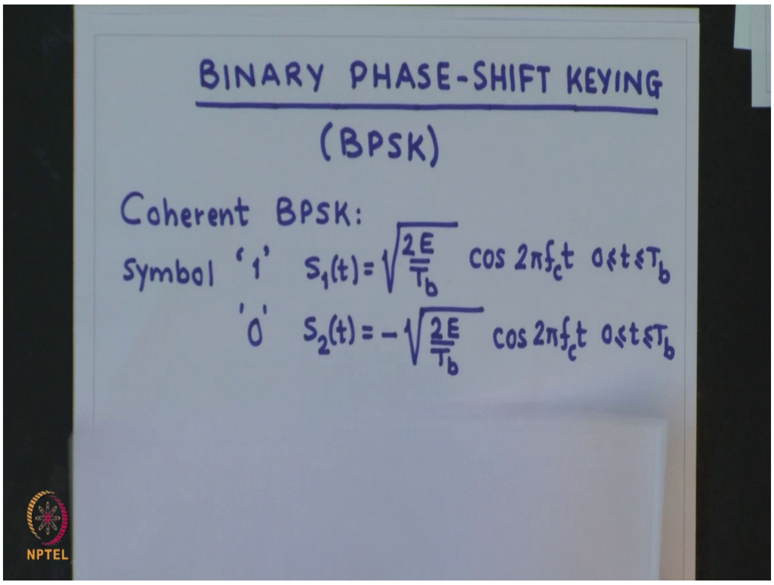

Now, another popular modulation scheme is what is known as binary phase shift keying. And this binary phase shift keying scheme is used in modern communication systems such as satellite links, white band, microwave radio, railway station etcetera and this is very power efficient in terms of signal power and we will see this soon.

So, this is what happens we will be talking about the coherent BPSK; where I assume that I know the phase of my carrier signal and here without loss of generality; I assume that carrier phase to be equal to 0, but it is important to know that we are talking about coherent BPSK. So, for symbol 1 I have this expression and for symbol 2, I have just the negative sorry I repeat; for the symbol 0 I have this S 2 signal given by this which is just the negative of S 1 t. And if you look at the; so, here I show you the how the BPSK will look for the same digital data which we want to transmit.

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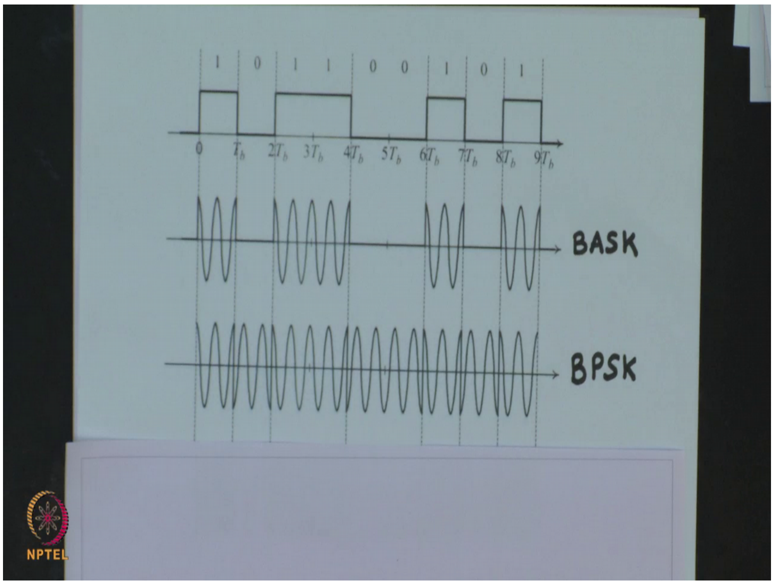

So, earlier we had seen that this was binary ASK. Now, this is your binary PSK; so, for 1 we have this cos waveform going, for 0 basically it changes the phase. So, suddenly it changes like this and then again you have 1. So, again it changes the phase here it comes and then again there is 1. So, it continues with the same thing, so this how you generate the BPSK signal. It is very easy to see that the signal constellation can be obtained with the help of just one basis signal; we will use phi 1 t to be the basis signal over this duration.

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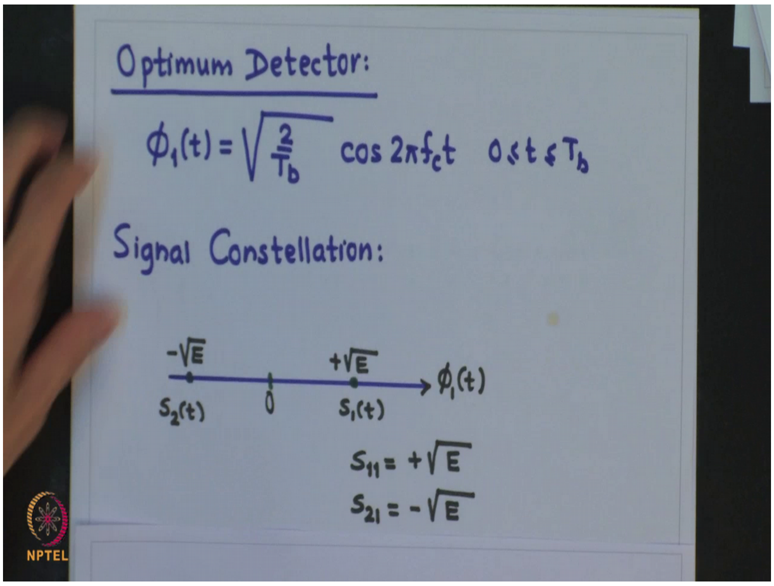

And then using this we can calculate the signal constellation, the signal constellation is as shown here this is your point 0. So, S 1 t is given here and S 2 t has been given here. So, if you take the projections of the signals onto a phi 1 t; S 1 1 I will get plus E and S 2 1 I will get it to be minus E; remember this is a 1 dimensional signal space.

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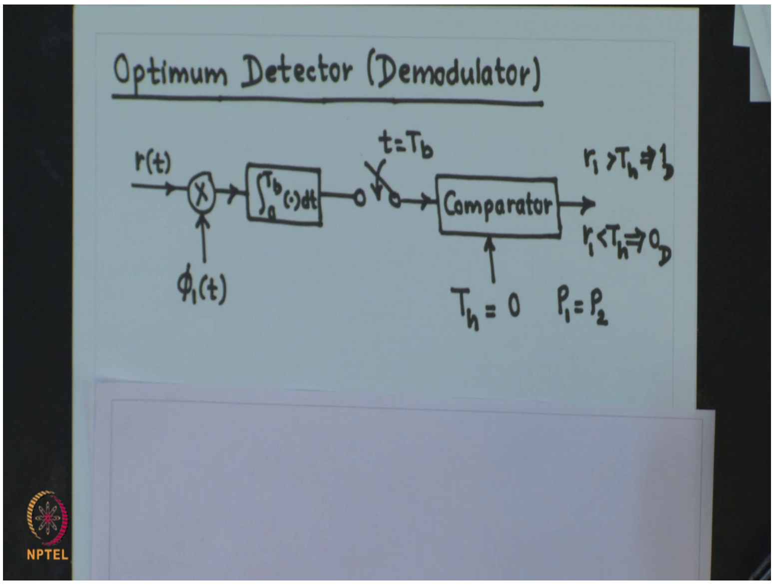

Once we have phi 1 t, we can immediately get the optimum detector again the optimum detector is as shown here in the form of correlation receiver; phi 1 t you correlate with r t, you sample it at t equal to T b.

And then you put it to a comparator the threshold is going to be 0 because the both the symbols are equiprobable. So, we will get P 1 is equal to P 2 when r 1 that is the projection onto and if it is greater than threshold I decide one has been transmitted. So, 1 D means that the decision is in favor of the symbol 1 and if this is a r 1 is less than the threshold which is 0; I decide that 0 has been transmitted; so, 0 D means that decision is in favor of 0.

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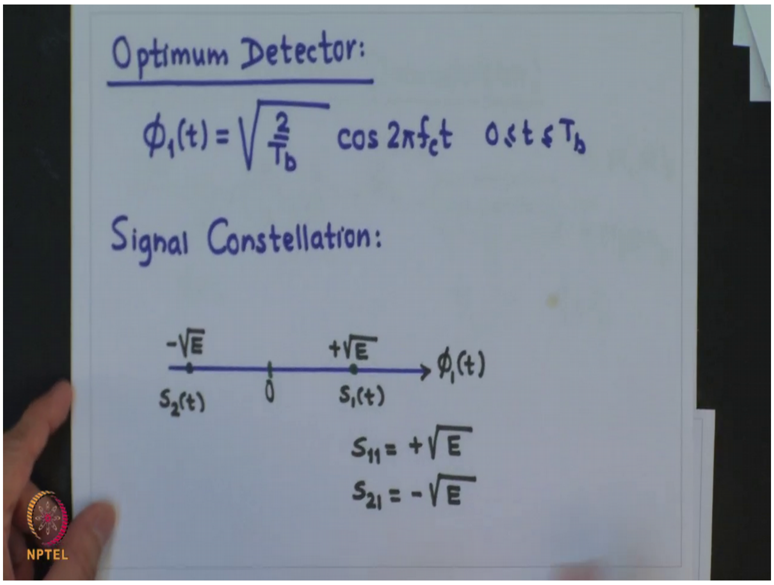

Given this now again very simple looking at the signal constellation, we can immediately write down the probability of error; remember in this case also the probability of error is symmetric, whether I transmit S 2 t or whether I transmit S 1 t; the probability of error is going to be the same conditional probability of errors are same. So, the conditional probability of errors are same, the probability of error would be equal to the conditional error probability of any one of the signal.

So, it is easy to calculate the probability of error in this case.

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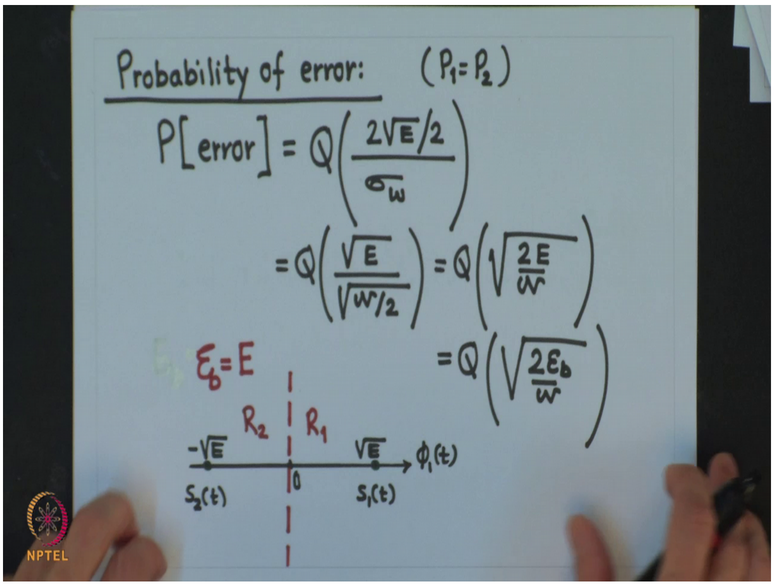

That would be given by looking at the signal constellation; the distance is 2 root E. So, 2 root E distance divided by 2 standard deviation of the noise; remember the noise is being projected onto the orthonormal axis phi 1. So, the variance of the noise remains the same that is italic N by 2. So, the standard deviation is the square root of this, I calculate. In this case the bit energy turns out to be the same as the energy E of the each of the signal S 1 t and S 2 t correct equiprobable and then I get this.

Now, and the last thing we have to do is basically calculate the power spectral density for this. So, for the power spectral density we use the same expressions as we did for the binary ASK case.

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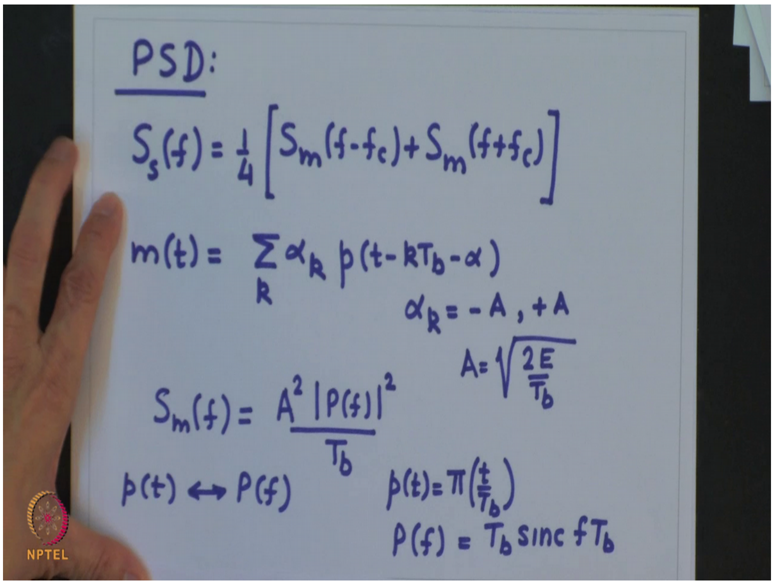

The only difference is that I have to evaluate now power spectral density for m t signal. In this case my m t signal value of alpha k in m t signal will be equal to minus A and plus A, where A is given by root of 2 E by T b.

And we know that this is a polar line code, we have evaluated the power spectral density for a polar line code is given by this expression. And we choose p t to be rectangle pulse, given a rectangle pulse I get my P f from this and using this relationship; I just plug into this equation, I get my power spectral density for the binary phase shift keying scheme.

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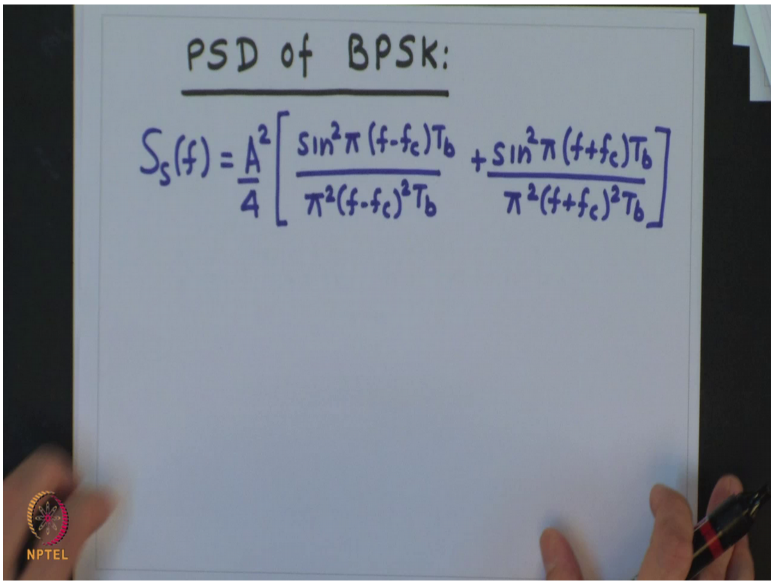

And if you plot this; it will look as shown in this figure.

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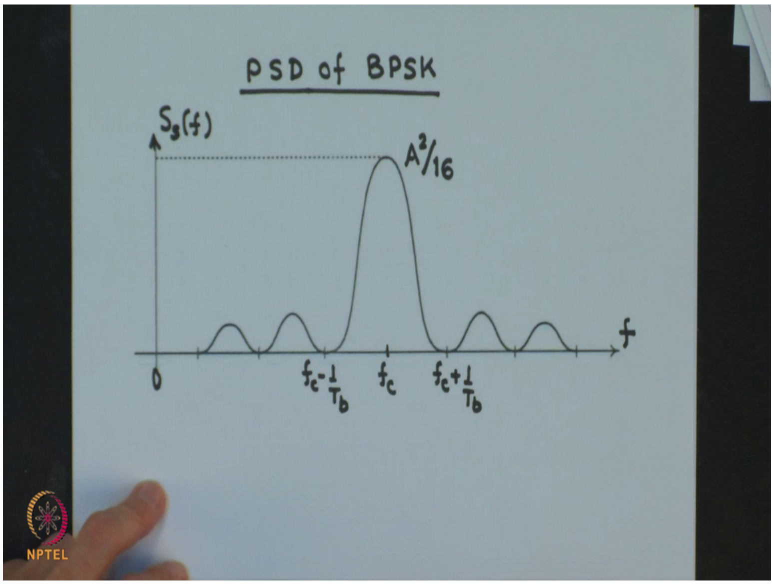

Again I am just showing the right side of the frequency axis, the left hand side has not been showed. So, this is 0 exactly the same thing will be there on the left hand side, the difference between BPSK and BSK is that; in BPSK it is an absence of the carrier at f c; that is the only difference. And again just for the sake of completion, I would like to point out that we have neglected the cross product term in this case also and it is not difficult to see from where that cross product term would come. So, it is because this trickles on to the left hand side and this thing is also there on the left hand side.

So, the lobes from the left hand side could trickle into the right hand side and that also has to be accounted when we plot this, but if f c is very high then this lobes will die out correct and. So, again we can find out the bandwidth for this signal BPSK which is as same as for the BSK case. Now the next parameter of the carrier wave which we can change is the frequency. And if you do that basically we will get what is known as binary phase shift key and this will study next time

Thank you.