Principles of Digital Communications
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Lecture - 41
Partial Response Signaling – I

Given a channel of bandwidth b Hertz then the maximum theoretical rate at which we can transmit the message symbol with 0 ISI would be two b samples per second and this is also known as Nyquist rate. But for this to happen it is important to choose the time characteristic of the message waveform or the pulse as seen between the transmitting filter, and the receiving filter to be off sinc type or in the frequency domain it will have brick wall structure over the frequency band of b Hertz.

Now, implementing this kind of a characteristic is difficult in practical scenarios therefore, for practical realizable transmitting and receiving filters for a channel with bandwidth b Hertz signal design for zero ISI demands that the signaling rate be lower than the Nyquist rate of two b samples per second. But if the bandwidth is precious and if you will still desire for the bandwidth of the channel b Hertz to transmit at the rate two b samples per second and with non brick wall structure in the frequency domain then it is still possible to do it with a technique known as partial response signaling or also known as correlative level coding, but this technique introduces inter symbol interference.

Now, we know that this inter symbol interference is not desirable, but partial response signaling uses this inter symbol interference in a controlled manner. The idea is that the ISI introduced at the transmitter end is known and this can be interpolated at the receiver in a deterministic manner and can be canceled out. So, this is the basic philosophy behind partial response signaling which achieves transmission of message symbols at the rate two b symbols per second for the bandwidth of the channel equal to b Hertz. Let us take an example to understand this.

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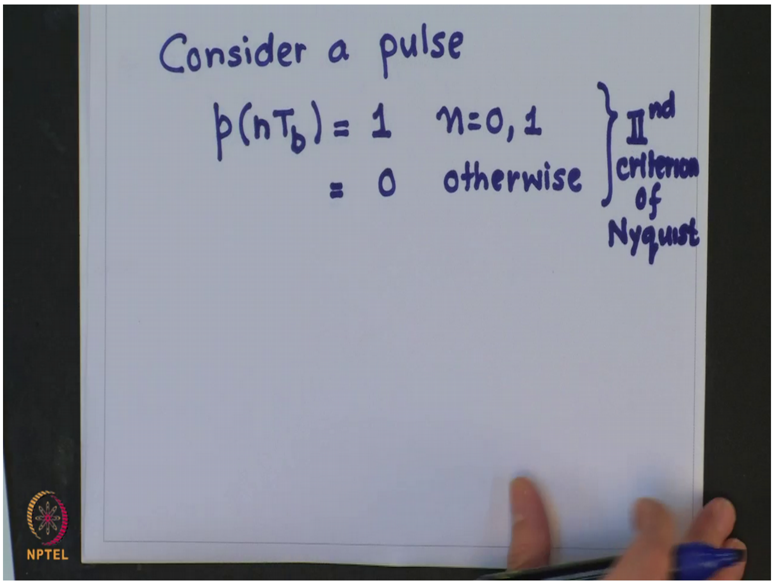

So, consider a pulse such that p n T b that is at the sampling instance this is equal to 1 for n equal to 0 or 1 and is equal to 0 otherwise. This is also known as Nyquist second criterion in the time domain; p t of the form shown in this figure would have this characteristic as specified that at the sampling instant T equal to 0 and at T equal to T b this value would be equal to 1.

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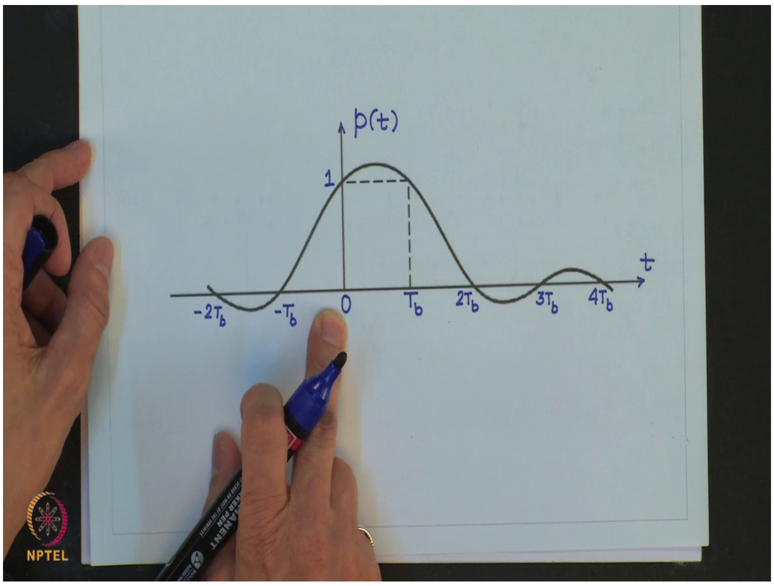

So, known and control ISI from the kth pulse to the very next transmitted pulse is being created.

So, let us consider now polar signaling.

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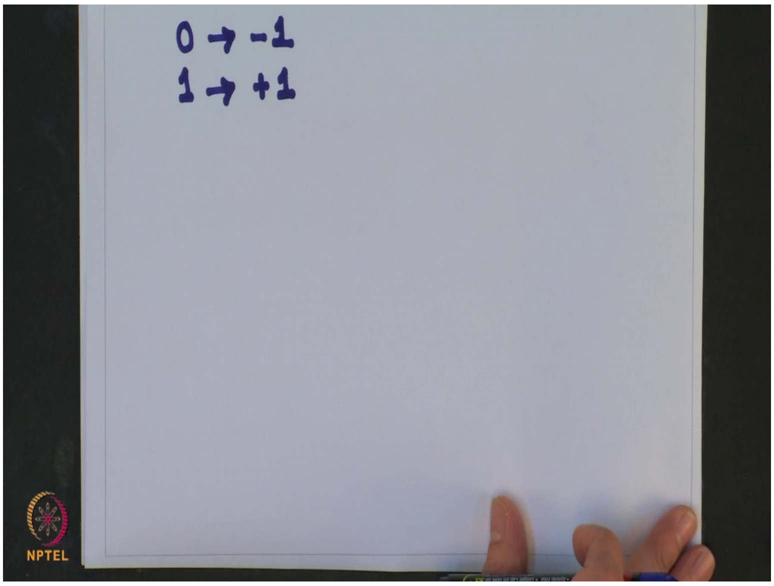

So, in the polar signaling for 0 will have voltage level to be minus 1 and for 1, will have the voltage level equal to plus 1. And consider two successive pulses which satisfy the property as shown on this slide which are located at 0 and T b. It would look something like this.

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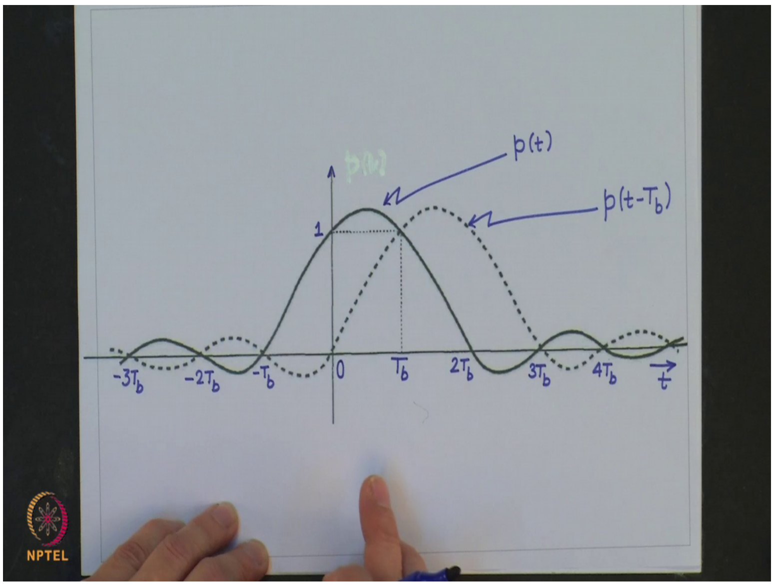

So, this is one pulse which is located at T equal to 0 and this is another pulse which is located at t equal to T b. So, from this figure it is clear that at t equal to T b the value would be equal to plus 2 if what pulses are positive; that means, we are transmitting 1. This value would be equal to minus 2 if both pulses are negative that means, we have transmitted 0, and the value would be 0 if we have opposite polarity, correct.

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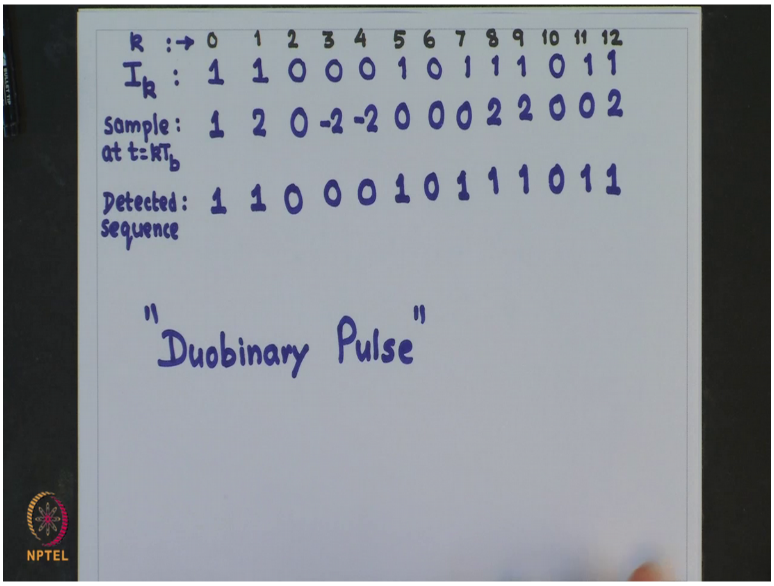

Now, given this let us take one example to understand how we could exploit the known ISI at the transmitter and remove the effect at the receiver. So, let us take a information bits to be transmitted as shown here and these are their time instances when this bits are being transmitted starting from k equal to 0 up to k equal to 12. So, the sample value at t equal to k T b after we have converted this to polar format would be equal to the first one, this will be 1, because there is nothing previous to that let us assume it to be 0 then this would become 2, this would become 0, this would become minus 2, this and this become minus 2, this will become 0, this will become 0, this would be equal to 0, 2, 2, 0. 0. and 2.

Now, given this values then we can find out what is the detected sequence if we follow the simple logic that whenever the output this is assuming without the noise on the channel. So, whenever the output is equal to plus 2 I know I have transmitted 1 and whenever the output is minus 2 I know I have transmitted 0 whenever it is 0 you will use the polarity of the bit which is opposite to the previous detected bit. So, in this case this is 1, so we start with 1. This is 2, so 2 this is 0. So, it will be of the opposite polarity to the previous one this was 1. So, this is basically 0 this is minus 2. So, it is 0 this minus 2 directly 0 this is 0. So, it is going to be of opposite polarity, so this is going to be 1. Again, 0 opposite polarity; so this is going to be 0. This is 0, so opposite polarity to this previous bit, so 1, this is 1, this is 1, this is would be 0. opposite polarity, this again opposite polarity and this is 1. So, this is the way we can detect.

Now, the problem is that whenever 0 occurs the decision is based on the previous bit. So, if there is an error in the decision then this will accumulate and it will be corrected only when minus 2 or plus 2 occurs again otherwise all the bits will go in wrong ok. So, this is known as correlative or partial response scheme and the pulse satisfying this Nyquist second criteria is also known as a duobinary pulse.

Another important observation from this output is that this scheme has error detecting capability. In the sense you will see that there are even number of 0 valued samples between any two fulls valued samples of the same polarity and odd number of 0 valued samples between two full valued samples of opposite polarity. Take this case, so this 2 and minus 2 they are full valued samples, but opposite polarity. So, number of 0s in between there is only one that is odd. If you take this and this, this is the full valued samples of the same polarity and there are even number of 0s. In this case number of 0s is 0 which is even take this case between 2 to 2 full valued samples, but of opposite polarity. So, the number of 0s in between will be odd. So, this is basically a useful property which can be used to find if there is an error in the detection.

So, let us try to find out the pulse which satisfies the Nyquist second criterion. So, this can be done as follows.

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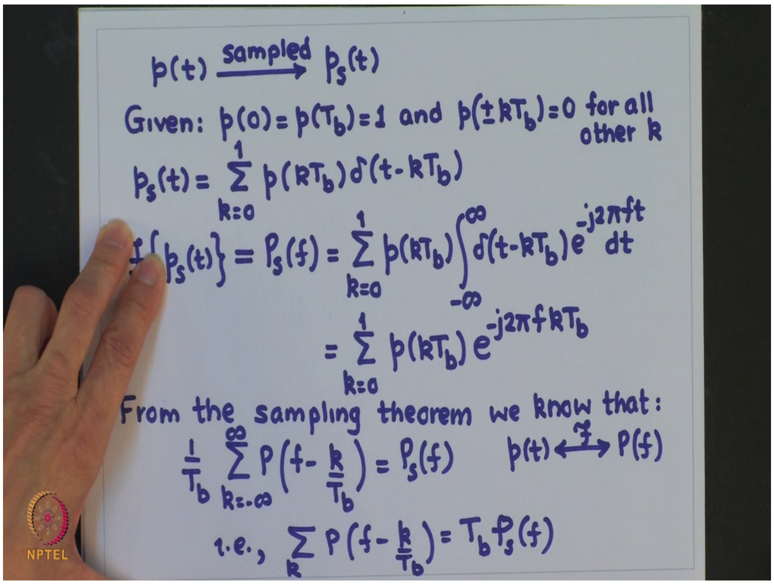

Let me take p t I am interested in finding out the p t. So, the sample value of the p t let me call it as p s t. In this case we have been given that p s t is of the form p 0 is equal to 1 p T b is equal to 1 and for all other sampling instance this is equal to 0 for all other k.

Given this I can represent my p s t in this form simple in form of impulses, and now I take the Fourier transform of this. So, the Fourier transform of this if I try to take it I indicate the Fourier transform to be P s f is the Fourier transform of this e raised to minus j 2 pi f t d t. So, if I do this basically I take the integral inside to each n to the terms in the summation. So, if I do this I get this quantity, and if I evaluate this is simple this evaluation turns out to be this quantity here ok, fine.

Now, we know from the sampling theorem that if you have a signal pt and if you sample it then its Fourier transform basically is a train of P f s. So, that repeats at the interval 1 by T b and the relationship between the Fourier transform of the sampled signal and the original Fourier transform of p t is given by this expression this comes from the sampling theorem.

Now, what we do is basically this I can rewrite it as like this. Now, we take only the P f corresponding to k equal to 0; that means, we ignore all the aliases, correct. So, if I do this and then equate to this quantity I will get the Fourier transform of my pulse p T right, this is what we get from the sampling theorem. So, if I do that I ignore the aliases and consider only k equal to 0 term.

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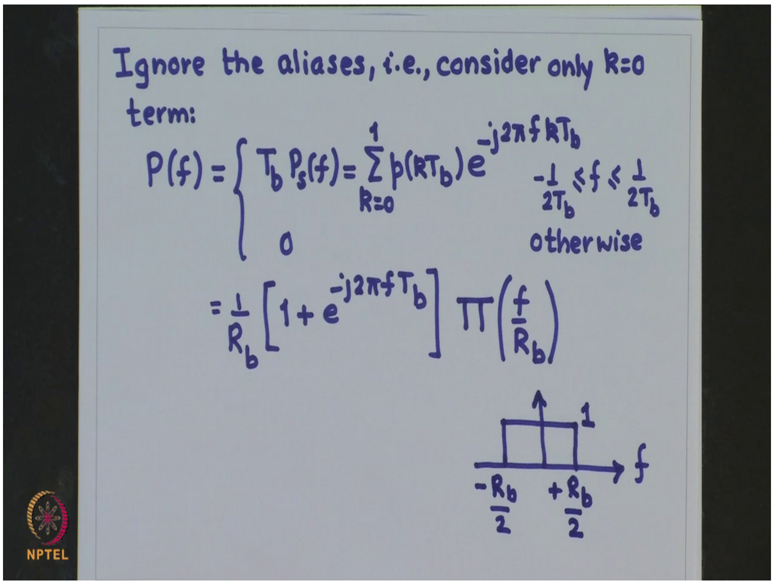

I will get P f which is nothing, but T b, P s f and this is equal to this quantity which we have evaluated here ok, fine. And this is remember it is only over this frequency range because we are neglecting all the aliases, ok.

So, this if you simplify for p k T b equal to 1 for k equal to 0 and k equal to 1 I get this and this is again my notation for my rectangle function what this implies is like this ok, fine. Now, this we can simplify just take e raise to minus j pi f T b outside this, if we do that simple exercise I just remove it out then I can write it in terms of a cos function.

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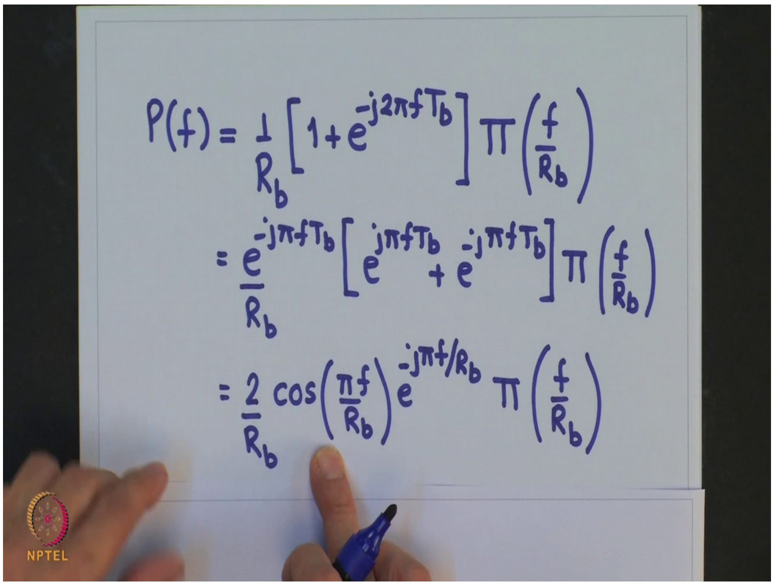

So, I get the P f corresponding to the pulse p t which satisfies the Nyquist second criteria to be of this form which is of the cos type.

Now, we are interested in finding out the p t which is the inverse Fourier transform of this, right. So, we can evaluate the inverse Fourier transform of this quantity. Again straightforward we evaluate the inverse Fourier transform of the quantity.

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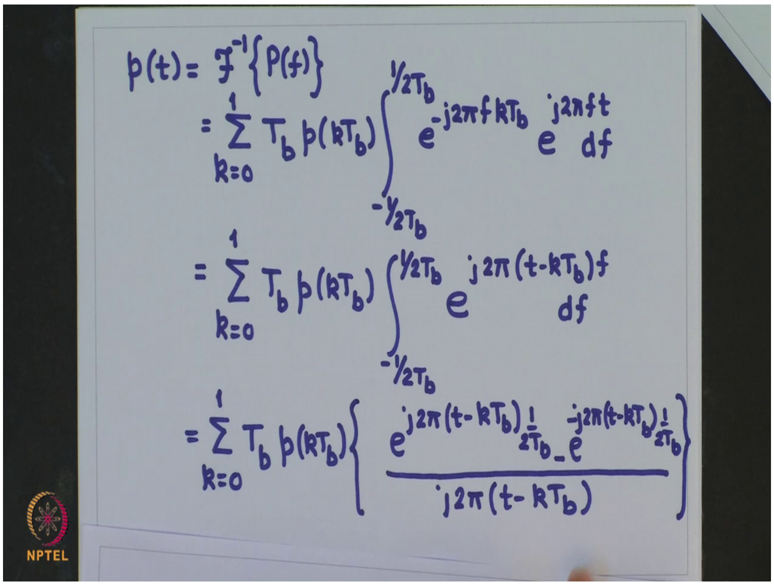

This is corresponding to inverse, I simplify it writing these two terms are combined as one term. I integrate this and substitute the values the upper limit is 1 by 2 T b the lower limit is minus 1 by 2 T b this is the integration this is this, ok. So, this if we evaluate remember this look here this quantity with j 2 is nothing, but of the type sine pi x by pi x, correct. So, this is equal to this quantity out here is simply given by this quantity here.

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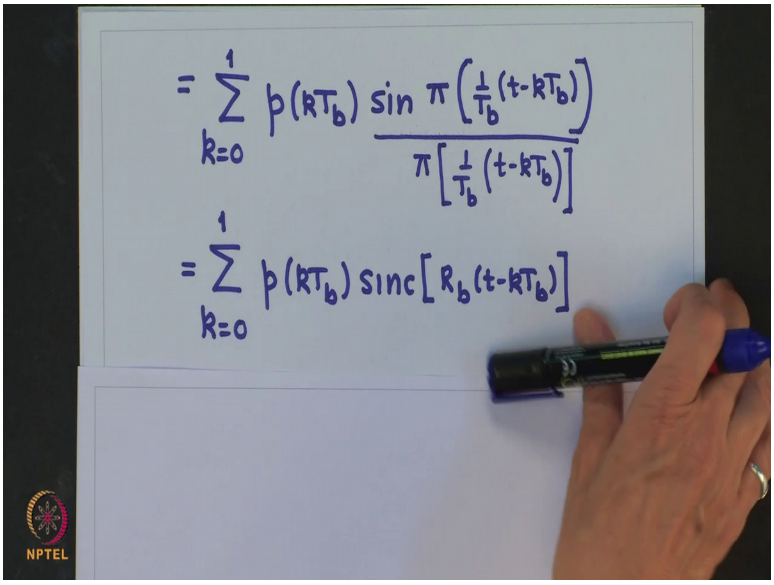

This T b out here is taken along with this term. So, if you take it then I can write as this and this is nothing, but the sinc function ok. So, I get my p t of this form.

We can simplify this expression because we know these values for k equal to 0 and 1 this is equal to 1 1.

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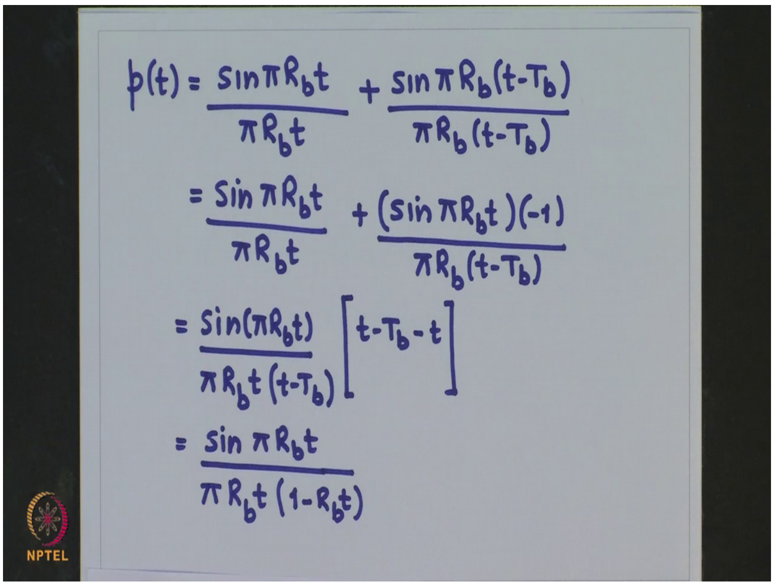

And if I do that then I would get on expansion this is one, this is one. So, I just solve it expand it using the trigonometric function, sine of this quantity cos of that. So, cos of sine pi R b t b is going to be cos of pi which is minus 1 and the other term on expansion will become 0. So, I get this and I get finally, this quantity out here. So, this thing I have derived looking from the point of view of frequency domain.

Now, the same thing can be derived from the time domain point of view let me just quickly do it. So, I will give you another way of getting the expressions for p t and p f for that is the duobinary pulse and this we will do it from the again from the sampling theorem perspective, but in the time domain.

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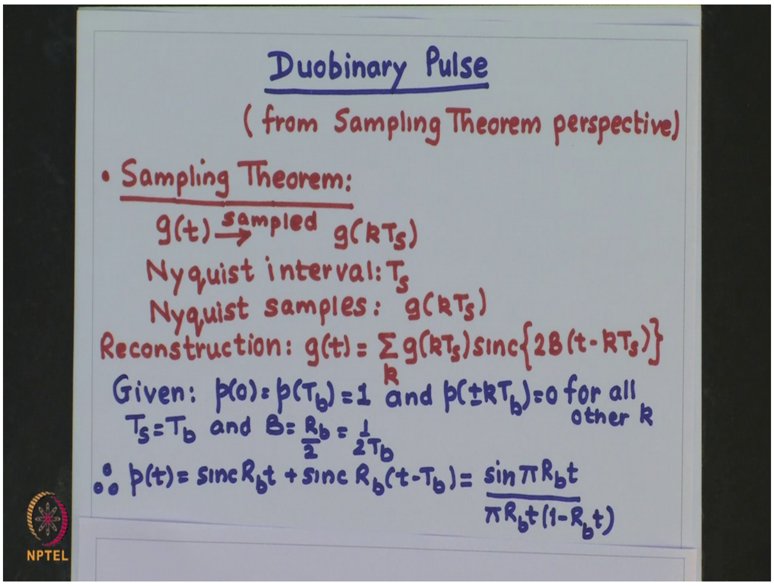

So, we know that if I have a g t sampled and its sample value are g k T s then in this case Nyquist interval we know is T s define, their liquid samples are j g k T s then we know that the ideal reconstruction filter would give us the output equal to this quantity, correct. This relation comes from the sampling theorem in the for the ideal reconstruction of the sample signal, correct. So, you will sample it and then reconstruct the signal back, correct.

So, if we take this as a starting point then also we can find out p t and p f very easily. In our case p 0 p t b is equal to 1 and all other terms are equal to 0. So, T s is equal to T b, bandwidth is equal to R b by 2 which is nothing but 1 by 2 R b, in our case the reconstruction filter bandwidth is R b by 2. And if we take this and plug into this we immediately get this p t we get sinc R b t plus sinc R b t minus T b from this formula and this is nothing, but this expression, correct.

So, duobinary pulse transmits binary data at a rate of R b bits per second or R b samples per second and has the theoretical minimum bandwidth of R b by 2 Hertz, but it introduces control ISI and the transmitter end which is taken care of and receiver sight, ok.

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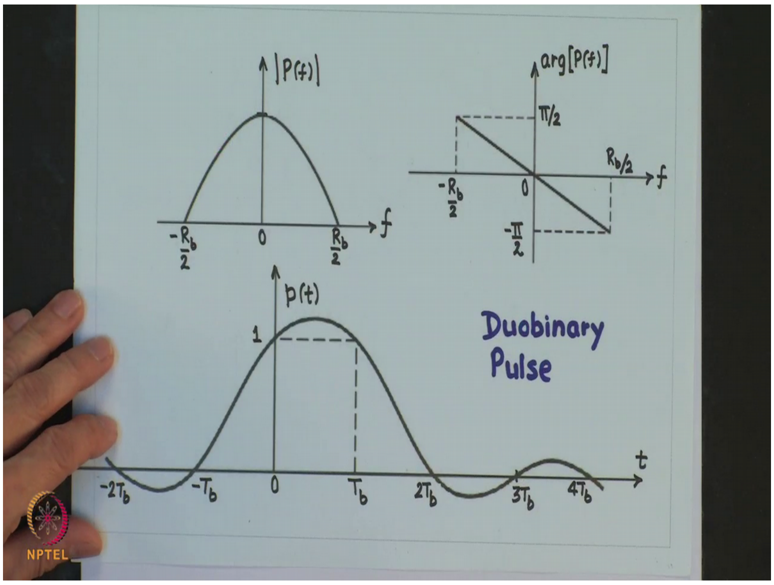

And let us look at the pulse we get from this the pulse which we obtained from this are shown here for p f and p t. So, the p f magnitude response is shown here look here this is non brick wall structure, and the p t turns out to be this basically is composed of tune same pulses which are time displaced by T b the impulse response has only two distinguishable values at the sampling instance.

Now, the response to any one input pulse is spread over more than one signaling intervals. So, in any one of the interval the response is only partial and hence the name partial response signaling comes correct. And duobinary pulse also could be considered because we are using two same sinc pulses to obtain this duo binary pulse.

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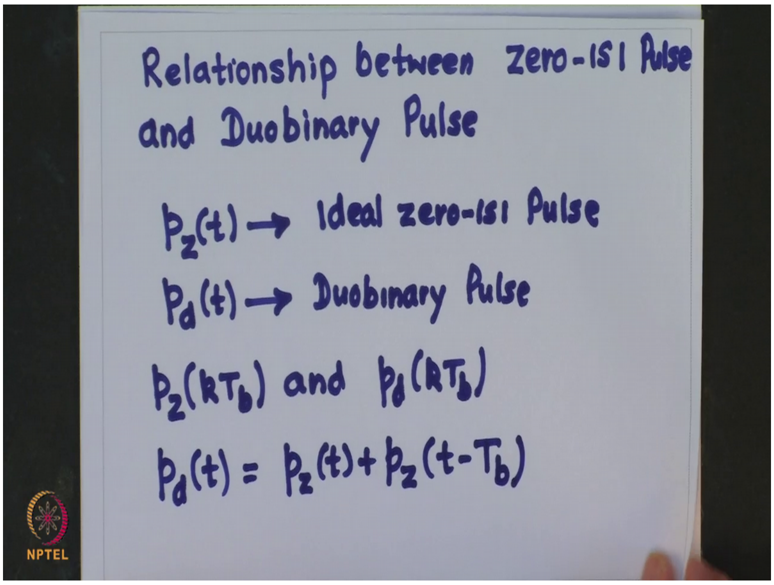

Now, quickly let us look at the relationship between zero ISI pulse and duobinary pulse. So, if p z t denotes the pulse which satisfies the Nyquist first criteria and p d t denotes the pulse which satisfies the Nyquist second criteria that is also known as duobinary pulse, this is ideal 0 ISI pulse. So, we see that the difference between p z k T b and p d k T b they differ for only k equal to 1 otherwise they are same. Therefore, it is very simple to see that I can get my p d T from p z by this relationship ok, so from this relationship.

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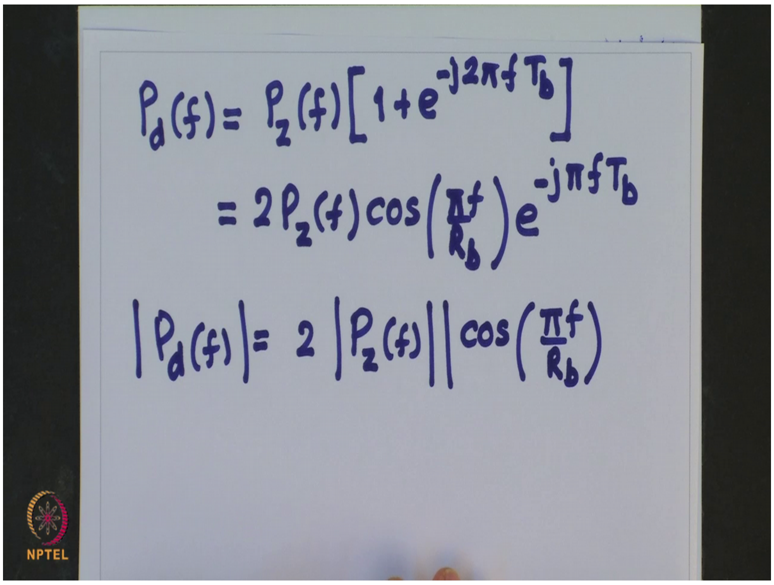

We can see that I can immediately calculate my Fourier spectrum for the duo binary pulse in terms of the Fourier spectrum of the ideal 0 ISI pulse as follows. And this is nothing but 2, so your this would be a brick wall structure, ha ok.

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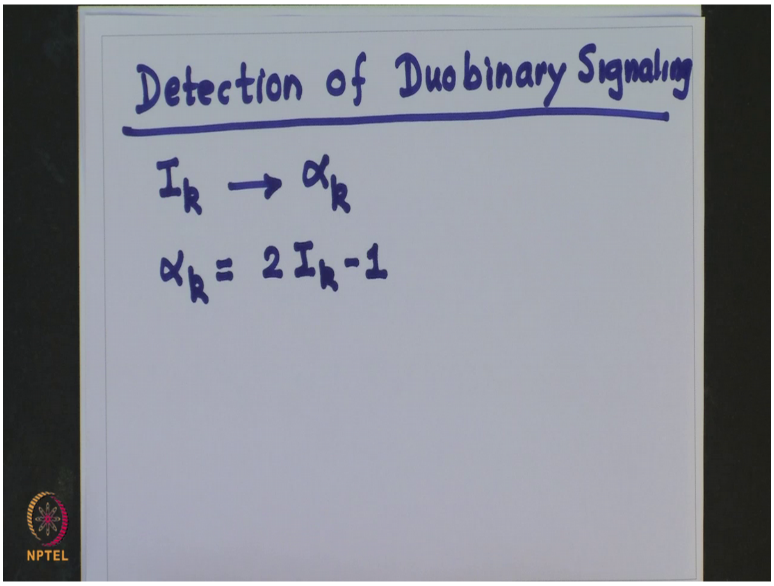

Now, let us quickly look at the detection of the duobinary signal. So, we will have we have I k, message bits this gets converted to the polar bits let us call it as alpha k. So, alpha k you can write it as equal to 2 I k minus 1. So, for I k equal to 0. I will get minus 1 and for I equal to 1 I will get plus 1, correct and then this basically goes to the duo binary pulse generator.

Now, we have seen the relationship between the duo binary pulse and the 0 ISI pulse. So, from this relationship we can immediately show that this block diagram will give me the at this output will give me the duo binary pulse.

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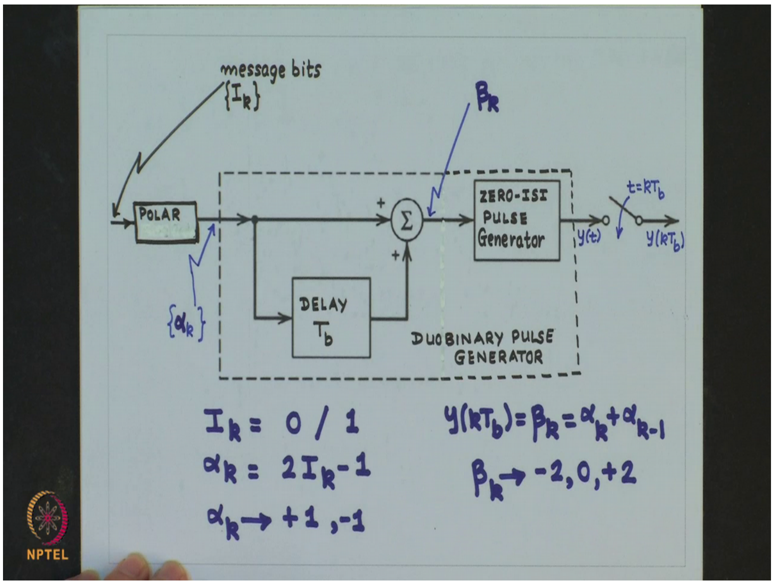

So, if I take this as my diagram therefore, generating my duobinary pulse. So, from this figure it is very clear that the output out here which is nothing, but d k and it is related to alpha k as alpha k plus alpha k minus 1 and this goes to the 0 ISI pulse generator and then I get the output, from this point to this point basically this block acts like a duo binary pulse generator. So, this duo by the impulse generator has been put in the form of 0 ISI pulse generator, correct.

So, we see here basically that for I k equal to 0 1 alpha k r plus 1 and minus 1 there are only two levels and this are basically uncorrelated because occurrence of I k and 0 and 1 are independent. And as whereas, y k T b or b k you get 3 levels 2, 0 plus 2 and this are basically correlated sequence which you get correct. So, that is why basically this is also known as correlative level coding, correct.

But now we have seen that this kind of detection there is a problem because if you make whenever we get 0 then your decision is based on the previous bit, and if you have made an error in the previous bit then there will be accumulation of the error till you receive another symbol where the output becomes minus 2 or plus 2. So, in this case we will require to do some kind of pre coding before we transmit. So, what we will do is that at this end I k, before we transmit it correct if we cannot carry out some kind of a pre coding on this message bits such that their output becomes directly related to the message bit rather than related in this form indirectly, correct. And this we will study next time.

Thank you.