Principles of Digital Communications
Prof. Shabbir N. Merchant
Department of Electrical Engineering
Indian Institute of Technology, Bombay

Lecture - 39
Pulse Shaping for Zero ISI - II

We studied there it is possible to transmit message pulses at the rate of R b pulses per second, on a channel with a bandwidth equal to R b by 2 hertz without ISI, but to achieved this we have to use for the transmission the pulse P t, which is a sinc pulse. Now, to implement filter, which has the impulse response equal to a sinc function it is difficult, because it is a non-causal function and the only way to achieve is implement in practices to delay it and then truncate it.

Now, this sinc function decays slowly and has quite large values for large T. So, truncation becomes a problem and another problem is that it does not provide any margin of error for the sampling at the receiver this also we had studied. So, the question is there a practical solution to this to get zero ISI. The only solution we will see is that Nyquist has shown that it is possible to get zero ISI provided we use the bandwidth of the pulse P t larger than R b by 2, but it can be less than R b.

So, for such a case we were able to show, that it is possible for me to design PT, which can be implemented in a practical scenarios.

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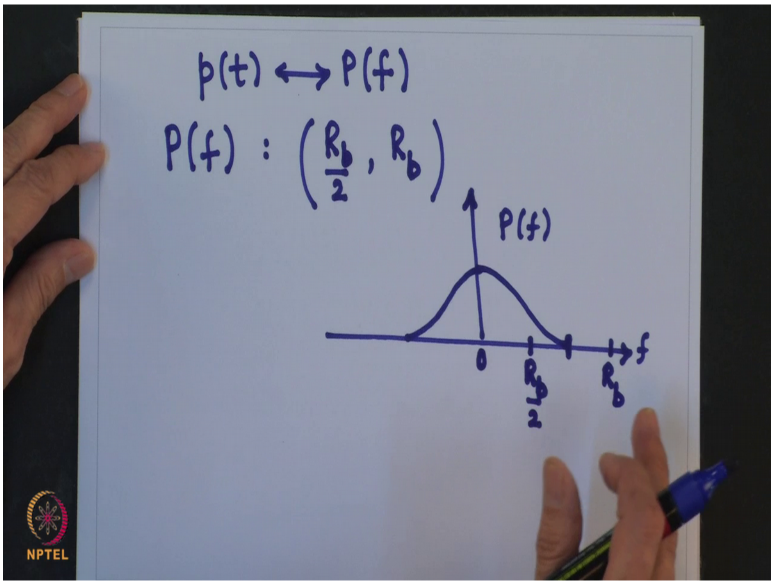

So, let us say that we have a p t with it is Fourier transform pair given by P f and will assume that the bandwidth of P f lies between the range, R b by 2 to R b. So, it would be something of this kind. So, this is going to be this value is going to be larger, than R b by 2, but it would be less than R b.

So, let us see what would be the condition on P f. So, that we get zero ISI. We have looked at the Nyquist first criterion for zero ISI in the time domain, which says that P t should be equal to 1 only for t equal to 0 and for any other sampling instance it should be equal to 0 ok.

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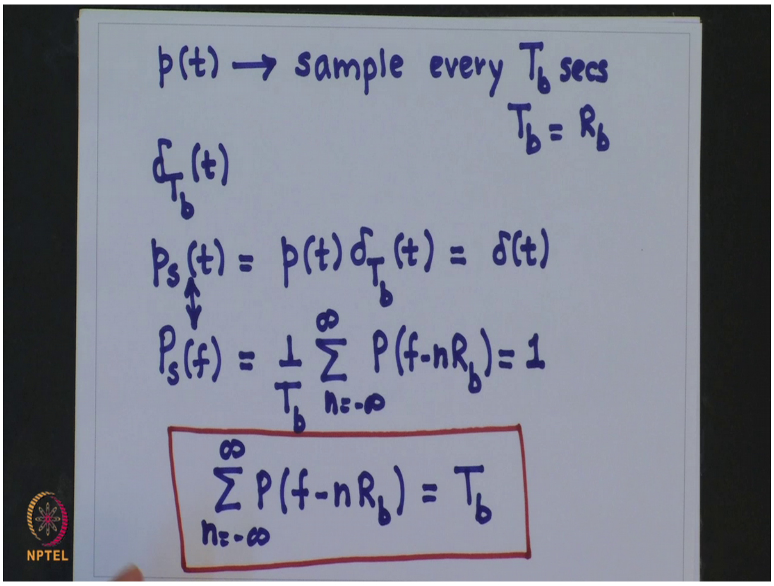

So, given this pulse p t let us sample this pulse every T b seconds where T b is equal to R b.

Now, please note that since the our P f bandwidth is going to be larger than R b by 2, when we sample at this rate we are going to get some kind of aliasing, but at this stage we are not bothered about the aliasing, because we are interested in providing non-interference only at the sampling instance.

So, now this p t we multiply by a pulse strain which we denote by this quantity out here. And so, sampling every T b second corresponds to generating a pulse strain, which I call as p s that would be equal to p t multiplied by this delta pulse strain.

Now, they want for zero ISI that this should be equal to delta t, which says that it will be equal to 1 for t equal to 0 and for t other than 0 including the sampling instances it would be equal to 0. So, this is what is the meaning of this quantity? So, your let us try to evaluate the Fourier transform of this. Now, we know from the sampling theorem that the Fourier transform of p t would be given by this expression and a Fourier transform of this is equal to 1.

So, from this the frequency domain criterion, which we get for zero ISI turns out to be this is my frequency domain Nyquist criterion for zero ISI. So, what this shows is that the sum of the spectra formed by repeating P f, which is spaced at R b apart is a constant T b. So, this would look something like this.

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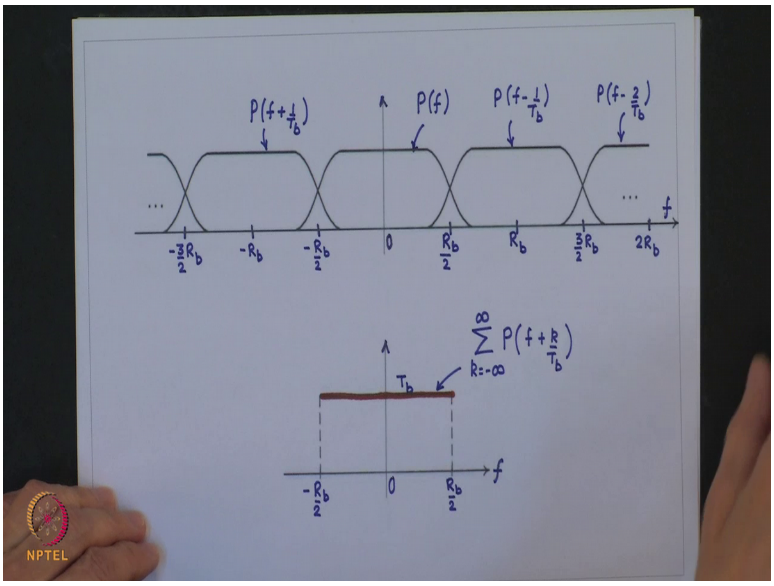

So, you have P f and this is the some of the spectra's, which you will get and I have just shown between minus R b by 2 to R b by 2 this will become a constant is equal to T b.

Now, the bandwidth of a P f lies between R b by 2 to R b. So, over the range from 0 to R b only 2 terms P f and P of f minus 1 by T b will be involved in the summation. So, let us see this is what happens?

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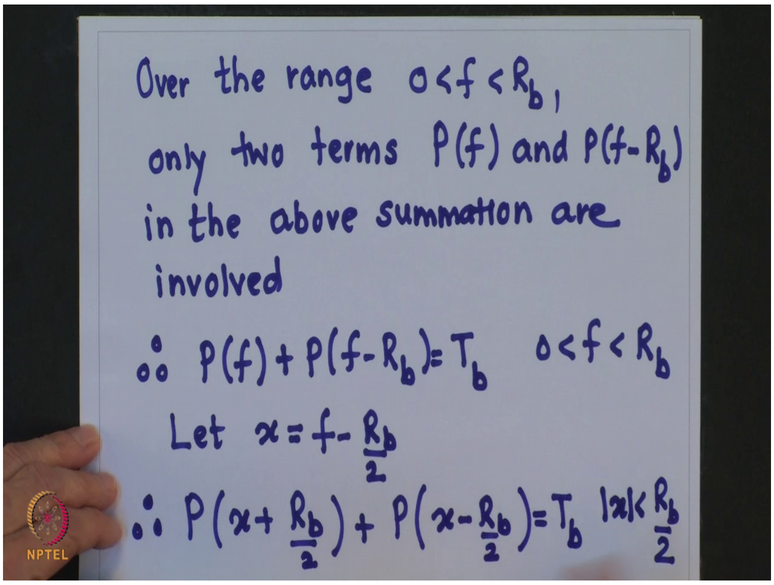

So, over the range we have only two terms P f and P of f minus R b shifted version to R b, which are involved in the summation. So, it means that P f plus P of f minus R b is equal to T b over this range let me make a small substitution x is equal to f minus R b by 2. So, I get that p of x plus R b by 2 plus p of x minus R b by 2 is equal to T b and mod x is less than R b by 2 correct.

Now, using the conjugate symmetry property of the Fourier transform we are assuming that P t is a real signal we can write the equation as follows.

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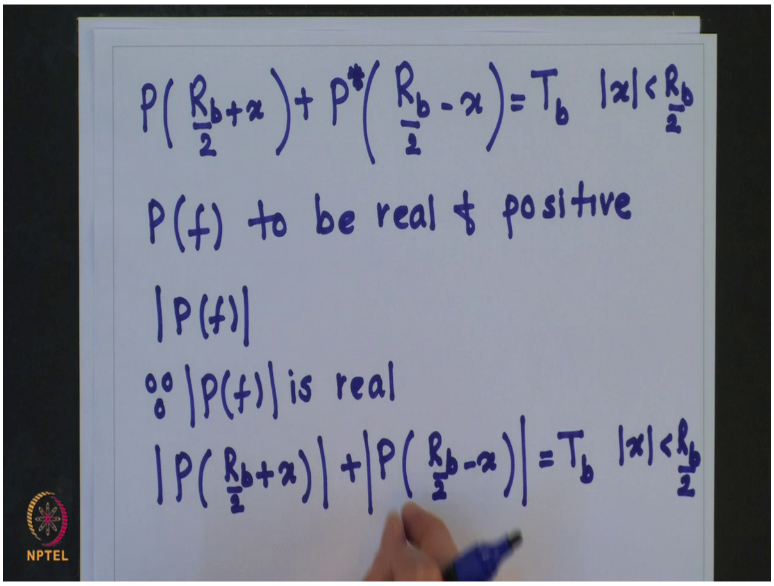

Now, if we choose P f to be real and positive, then mod of P f needs to satisfy this equation. So, because now P f is real from this we get the condition to be now let us see what is the meaning of this equation which we have written.

So, this says that there is some kind of a peculiar characteristic of the choice for P f, which will provide this satisfaction of the frequency domain Nyquist criterion for zero ISI.

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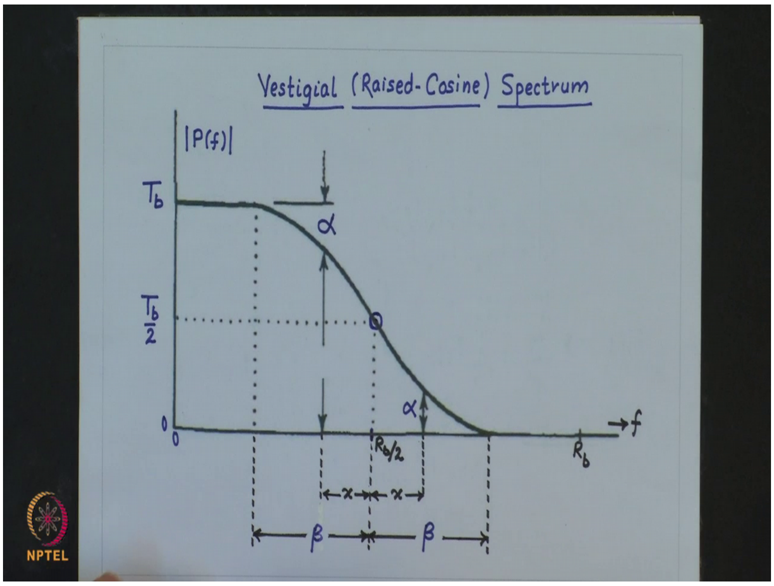

So, if I have my P f chosen something like this which I have shown here. So, this condition will be satisfied if I choose my point R b by 2 this is my frequency axis and this is my mod of P f, which I have drawn for f R b by 2, if you take a distance from here. This is R b by 2 plus x and this is R b by 2 minus x. So, at this point and this point the summation of this and this value should be equal to constant T b correct.

So, what this implies that the function P f about this point out here, becomes an odd symmetric function. So, and the this frequency the bandwidth of the P f the difference between that bandwidth and R b by 2, which is the ideal theoretical bandwidth required for transmission of pulses at the rate R b, this excess bandwidth is known as will call it as beta correct. So, since it is odd symmetric about this point from this point to this point also this is beta and here it remain constant.

So, now when you take this kind of a P f you will have a P f at located at R b also something like this. And so, I have just shown the positive side there will be a negative portion also on this side. So, the negative portion, which is their look for the P f located at R b, that and this would add up together to give me a constant value of T b.

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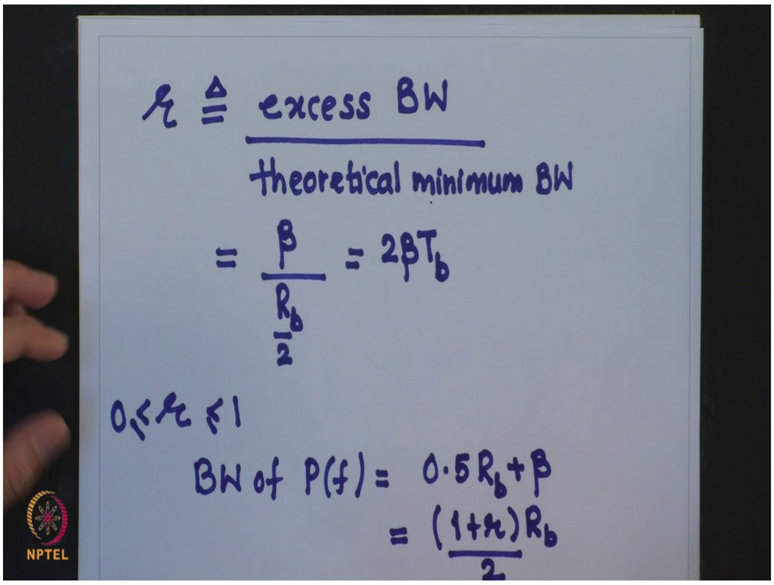

We, define the role of factor as excess bandwidth divided by theoretical minimum bandwidth required for transmission of pulses at the rate R b. So, this is R b by 2.

So, as per our notation excess bandwidth is denoted by beta and this is R b by 2. So, this becomes 2 beta T b. So, your role of factor will be less than equal to 1 and greater than equal to 0, when r is equal to 0 we are transmitting the ideal sinc pulse. So, the bandwidth of P f in terms of the role of factor could be written as this is a theoretical minimum bandwidth plus excess bandwidth. So, I can write this as 1 plus r b by 2. Sometime the role of factor is also express in terms of percentage.

Now, this P f satisfies vestigial spectrum condition, which we have learned in vestigial sideband transmission for am modulation correct. So, this P f has vestigial spectrum and we will see what are the now this will satisfy the Nyquist first criterion for zero ISI this is the frequency domain. Now, we can see that it is not very difficult to see that I have to just choose an odd symmetric function about this.

Now, there are many functions which can satisfy that requirement. So, that I infinite number of P f, which you could have for the given bandwidth given by this 1 plus r R b by 2 correct, but the most popular 1 is what is known as raised cosine spectrum fine. So, before we do that it is important to note again that because this r is greater than 0 and less than 1, the bandwidth of P f is restricted to R b by 2 to R b hertz.

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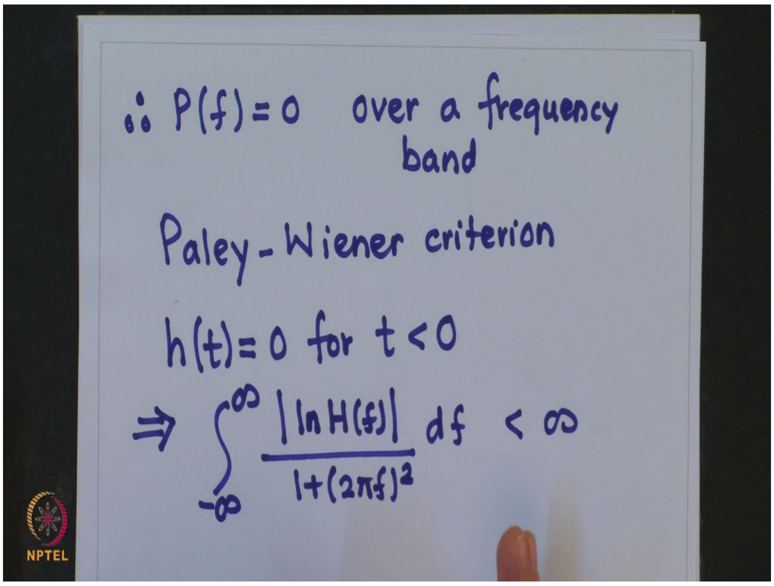

Therefore, your P f will be equal to 0 over a frequency band.

And, consequently this violet us the Paley Wiener criterion, which is essential for making the realizability of P f Paley Wiener criterion says that say example I have a impulse response h t, if you want it to be equal to 0 for t less than 0 then it implies that this integral log of H f mod of that 1 plus 2 pi f square integral of this should be less than infinity. Now, do you see that this is will not be satisfied by P f, because over a frequency band this is equal to 0. So, this quantity will become infinite right, but the vestigial role of characteristic being gradual, it can be more closely approximated by a practical filter.

So, the difference between this vestigial spectrum for the P f and the sinc pulse, which has a rectangular characteristic with a brick wall type of characteristic is that, because of the brick wall the in the time domain your pulse P t has lot of oscillations. And this oscillations decay slowly with large t and we have seen that this decay is 1 by t. Whereas, when you have the role of gradual than in the time domain this decay becomes faster that is basically the idea by moving over from the sinc pulse to a pulse which has vestigial spectrum ok.

And, it is important to note 1 more thing, that I can obtain this kind of characteristics by taking a basic pulse say even function pulse in frequency domain with a bandwidth less than beta. And convolve that with my rectangle pulse in the frequency domain of bandwidth R b by 2. If I do the convolution of the 2 I would be able to generate this kind of a vestigial spectrum, which satisfies the frequency domain Nyquist criterion for zero ISI. Let me take 1 example to make you appreciate this.

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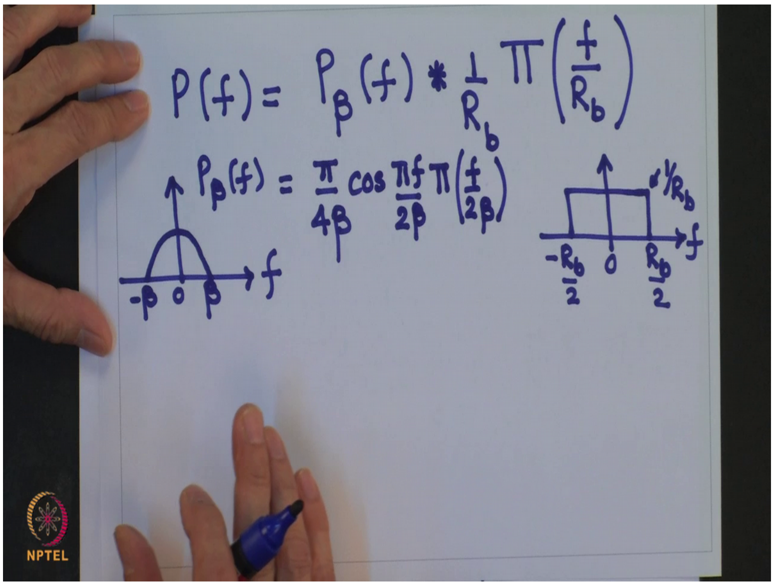

Let us say that we choose so, my claim is that I can obtain my vestigial spectrum P f by choosing P beta, which is an even function and real correct this can happen by choosing my P beta t to be real and even so, this will be also real and even. And, then convolve this with a rectangle function this denotes the rectangle function of with R b about f is equal to 0 and the height of that rectangle would be denoted by 1 by R b. So, it looks something like this symmetric.

So, this function is shown here. Now, P beta f I can choose for example, if I choose something like this, which is even function and real. Let me choose this to be of the form pi 4 beta cos pi f by 2 beta rectangle. So, this is this function and if I convolve this function with this function it is easy to see that my bandwidth, now will be R beta by 2 plus beta because when I convolve to this 2 functions like this I will get the total bandwidth to be that of my P f.

And, about R b by 2 this function is going to become odd symmetric if I choose this to be even function correct. So, this choice which I have written out here is a very popular 1 and the P f, which I generate out of this is known has raised cosine spectrum.

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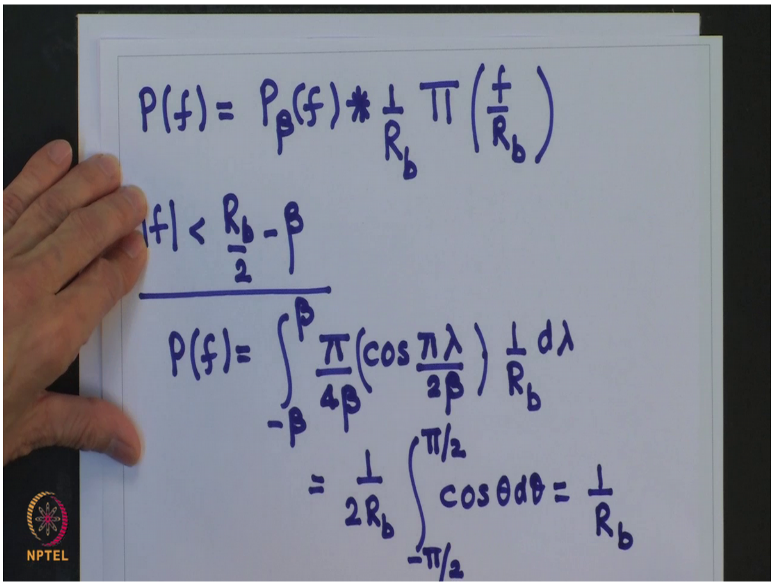

So, if I do this convolution let us do it quickly P f is equal to P beta f convolved with my rectangular function, that is 1 by if I do this we can calculate what is that P f. So, this can be broken up into 2 part this part I will get my P f is equal to and rectangle function it would be just constant, I integrate this and convolving the 2 function and this is nothing, but equal to I get this.

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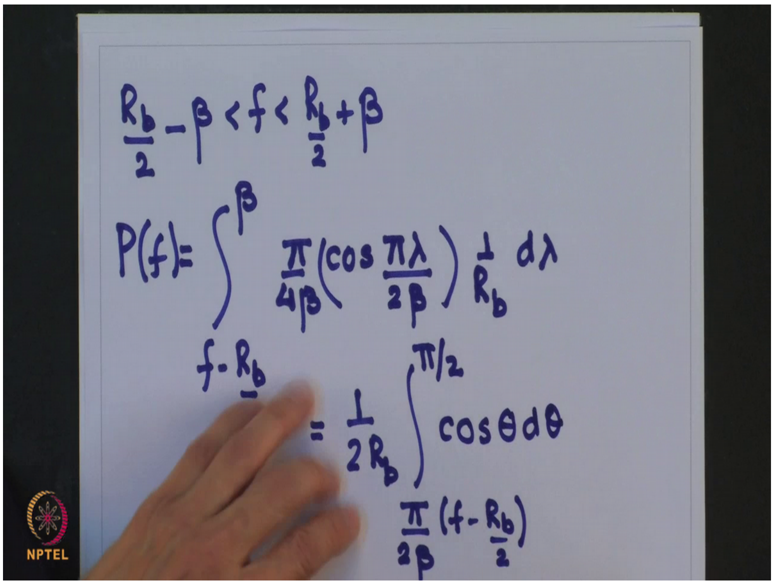

And for the range R b 2 minus beta for this range, I can show my P f is equal to this is equal to this I can rewrite it as, which I can rewrite it as and this can be simplified to this quantity.

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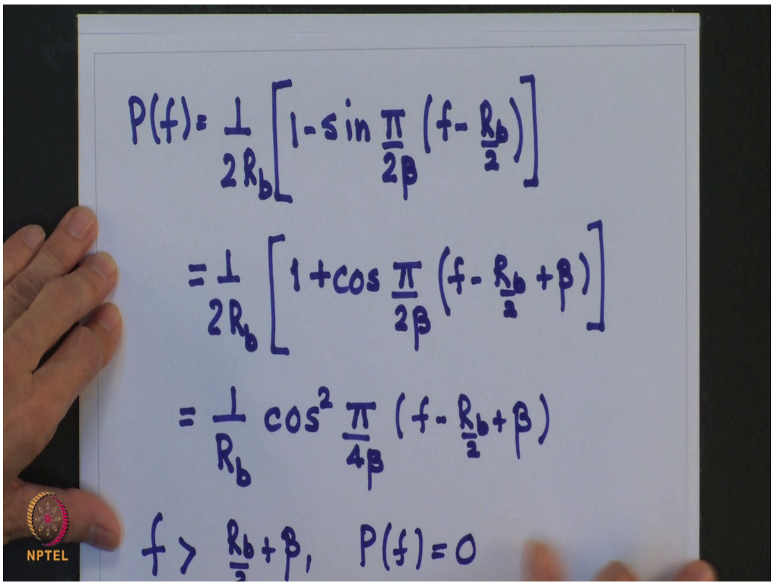

And for f this quantity your P f will be equal to 0.

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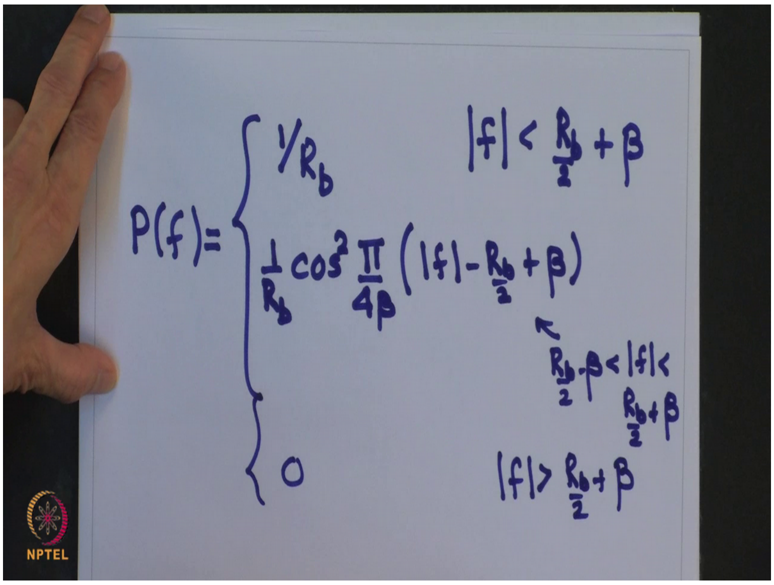

So, since P f has even symmetry.

So, since P f has even symmetry I can immediately write my this vestigial spectrum, which is popularly known has raised cosine, spectrum for the reason, which will become very clear now this is valid for R b by 2 minus beta is equal to 0.

And, you can see if you want to find out the time domain pulse for this, this will be equal to nothing, but the multiplication of p beta t by sinc function.

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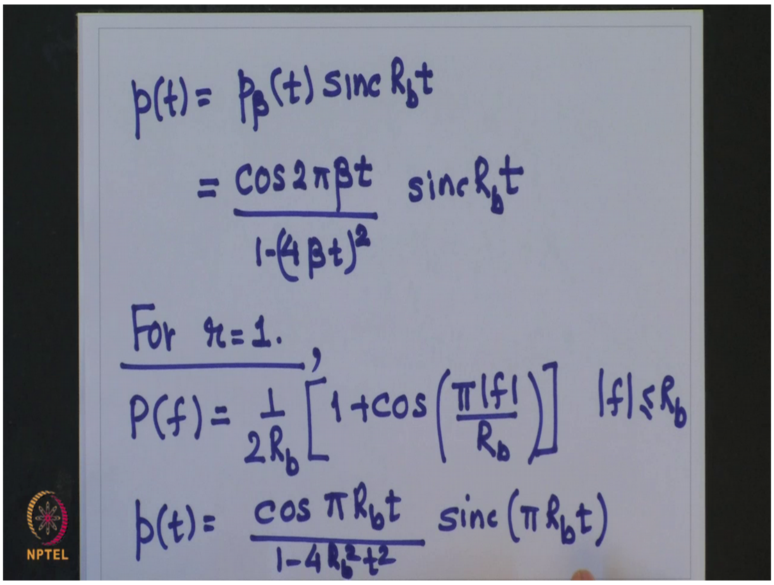

So, you can evaluate this you have to find out just the inverse of this p b beta f, that turns out to be this quantity. Now, for specific case for r equal to 1; that means, where beta is equal to R b by 2 this turns out to be P f turns out to be 1 by 2 R b 1 plus cos of and your p t for this turns out to be minus 4 R b squared t squared sinc pi R b t.

Now, this term out here basically is a factor characterizing the ideal Nyquist channel this ensures 0 crossing of p t at the desired time instants t equal to k t b. And, this factor decreases with time and this you can see it is a proportional to 1 by t squared. So, this reduces the tail of the pulse considerably below that of the ideal Nyquist condition and that is why this is preferable compared to the earlier ideal sinc pulse.

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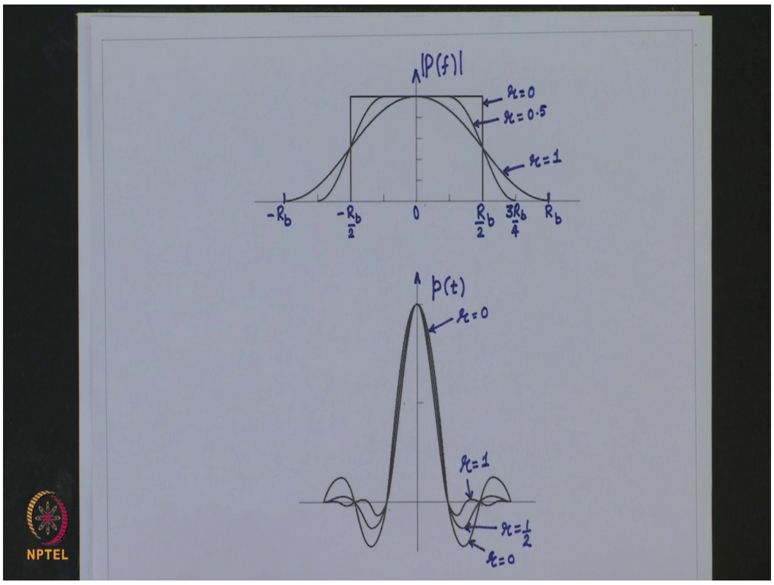

Now, if you plot this vestigial spectrum for different values of r you would get something like this. I have shown here this corresponds to r equal to 0 this is your ideal Nyquist condition r equal to 0, r equal to 0.5 and r equal to 1 corresponding to this we get the pt time domain p t for this.

And, from here we see that for the case when r is equal to 1 the bandwidth of this pulse becomes R b hertz and you will find for this case that there is 0 at all signaling instances and also at midway this corresponds. So, also at midway you will find there are 0’s and this decays rapidly as 1 by t cube.

So, this basically implementation of this becomes very useful in a practical scenario. And if you look these characteristics looks like a raised cosine and that is why this is known as the raised cosine spectrum. So, for different values of r for r equal to 0.5 also you see here basically this looks like a raised cosine. So, this is a reason why this vestigial spectrum is popularly known has raised cosine spectrum, which is used in a practical scenario ok.

Now, having done this we have shown that your P f, which is equal to transmitting filter transfer function multiplied by channel, transfer function multiplied by receiving, filters, transfer function. And, the overall P f should satisfy Nyquist condition for zero ISI correct. Now, the question is that in a practical scenario usually HCF, that is the channel characteristic remains more relaxed fix; so how do you choose the transmitting filter and the receiving filter correct, this we will study next time.