Principles of Digital Communications
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Lecture - 38
Pulse Shaping for Zero ISI - I

Power spectral density of a line code is dependent on two parameters: one the autocorrelation of the discrete sequence corresponding to a particular line code and the other is the pulse shape which is reflected in terms of Fourier transform of the pulse p t.

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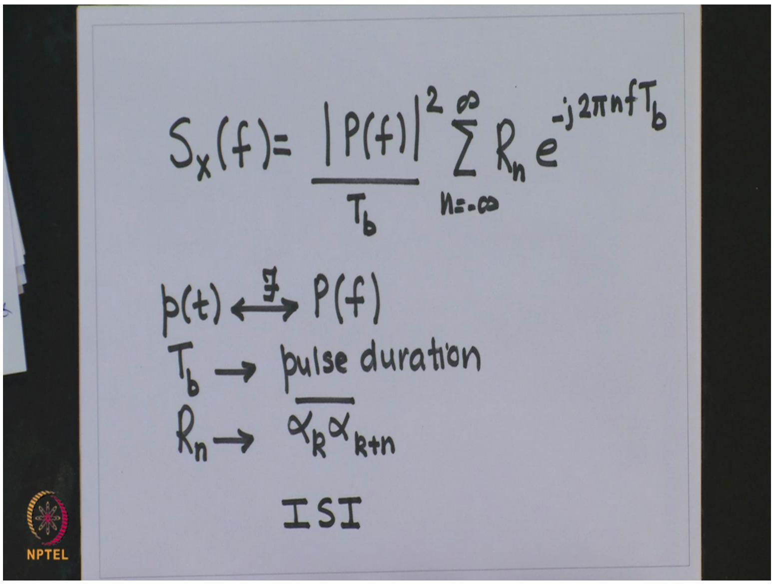

We had derived the relationship for the power spectral density as follows, where p t is the pulse which you choose for your transmission and its Fourier transform pair is P f T b is the pulse duration and R n is the autocorrelation function of the discrete sequence alpha k, which is the amplitude sequence corresponding to the choice of a particular line code.

Now, we see from here that power spectral density is strongly and directly influenced by the choice of p t, because it contains the term P f mod squared. So, the pulse shape is a more direct and important factor in terms of shaping the power spectral density. We have seen that this power spectral density theoretically extends right up to infinity. So, the theoretical bandwidth of a line code is going to be infinity. And, usually what will happen that this, signals are being transmitted on a band limited channel.

Now, because of band limited channel spectral distortion occurs and this tends to spread the pulse. This is also known as Dispersion. So, spreading of a pulse beyond its allotted time interval that is T b will cause it to interfere with neighboring pulses and this is known as inter symbol interference and popularly known as ISI.

So, it is important to know that ISI is not noise. It is caused by non ideal channels that are not distortion less over the entire signal bandwidth. So, ISI is a manifestation of channel distortion. To resolve their difficulty of ISI let us review briefly our problem. So, we need to transmit a pulse every T b interval. So, the k-th pulse is denoted by alpha k p t minus k T b and it is required to detect the pulse amplitude alpha k that is without ISI.

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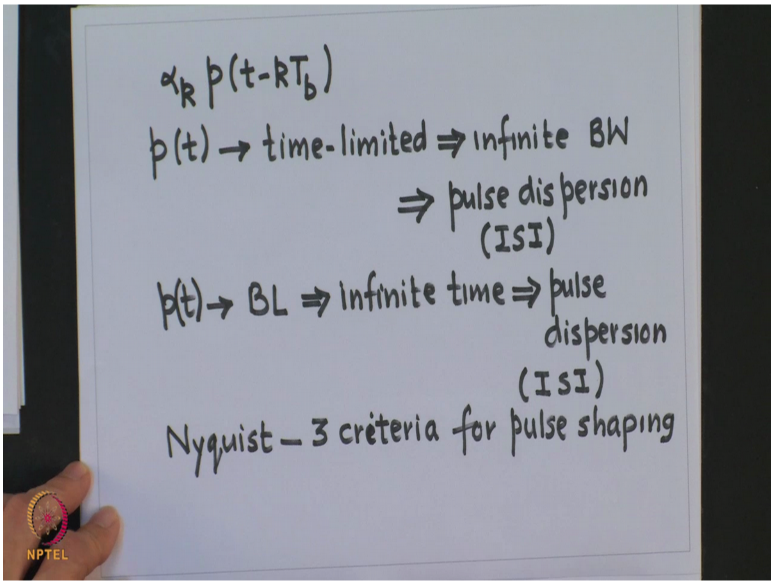

Now, in our discussion so far we have assumed that our pulse p t is time limited. Now, what this implies that it will have infinite bandwidth. Now, when such a pulse gets transmitted on a band limited channel; the pulse p t itself will get band limited and this will bring about pulse dispersion that is ISI.

So, let us try to solve this problem by choosing a p t which is band limited. So, that it matches with the characteristic of a band limited channel, but then this implies that p t will be of infinite time; which implies that there will be pulse dispersion and again you will get ISI. So, irrespective of the choice of p t either you choose time limited or band limited; on a band limited channel you will always get this ISI.

Now, it is important to note that we are interested in the pulse amplitude alpha k. So, pulse amplitudes can be detected correctly despite pulse spreading, if there is no ISI and the decision making instance and this can be accomplished by a properly shape band limited pulse.

So, to eliminate ISI Nyquist has proposed 3 criteria for pulse shaping and these criteria allow the pulses to overlap. In spite of the overlap they are shaped such a way that they cause 0 or controlled interference with all of the pulses at the decision making instance.

Thus, by eliminating the noninterference requirement only at the decision making instance we eliminate the need for the pulse to be totally non-overlapping. So, we will study only the first 2 criteria; the third is much less useful than the first 2 criteria and therefore, it is not studied here.

So, let us try to understand this ISI problem little more in depth.

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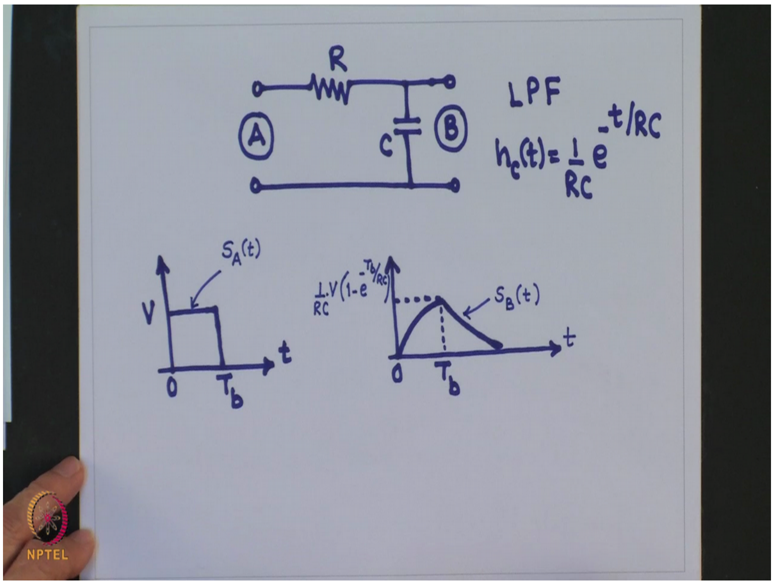

So, let us say that we have a pulse. We choose a rectangle pulse and that pulse is to be transmitted on a communication channel which is of a low pass nature. So, let me model the channel to be a simple R C circuit shown here. So, the output of the encoder and impulse modulator passed through a filter which generates an appropriate pulse of height alpha k is input to the channel at this point, point A and you get output of the channel B.

So, let us see what will happen if I apply a rectangle pulse at point A. I have a rectangle pulse of duration T b of height V and it is easy to see that if I take a low pass filter the output would be something like this.

If we have more than one rectangle pulse in sequence then let us see what will happen and this figure depicts that scenario.

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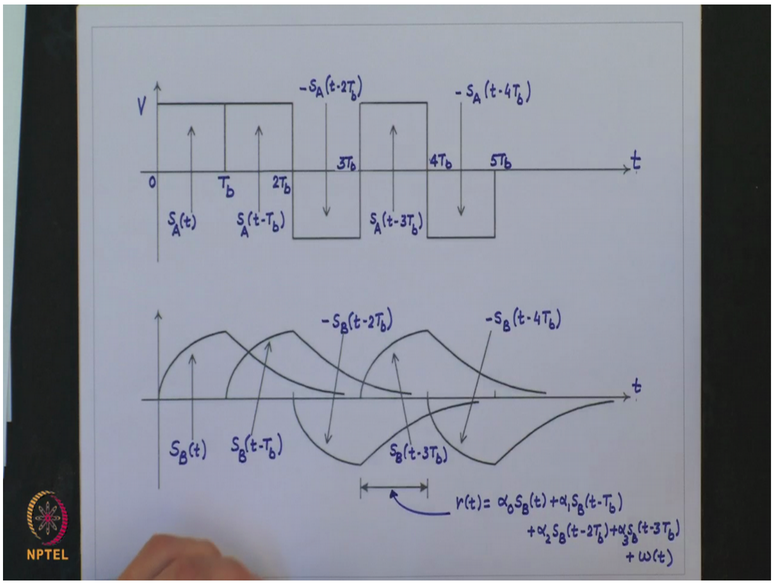

So, here I have shown 5 pulses and this are the pulses which is input to the communication channel and this are the outputs which I get from this communication channel corresponding to each of the pulse. So, if you look at this interval that is between 3 T b and 4 T b; the signal which we receive at that time would be summation of all the previous pulses and the current pulse which has been transmitted at time t equal to T b.

So, if you look at this instant any instant between this to this and if you were to sample the point so, this is your 3 T b. So, if you sample at this point you get contributions from other previous pulses though the contribution from this and this is much lower compared to just the previous pulse. But in principle you get contribution from all the previous pulses at this instant of time. We will try to model this digital communication system as shown in this figure.

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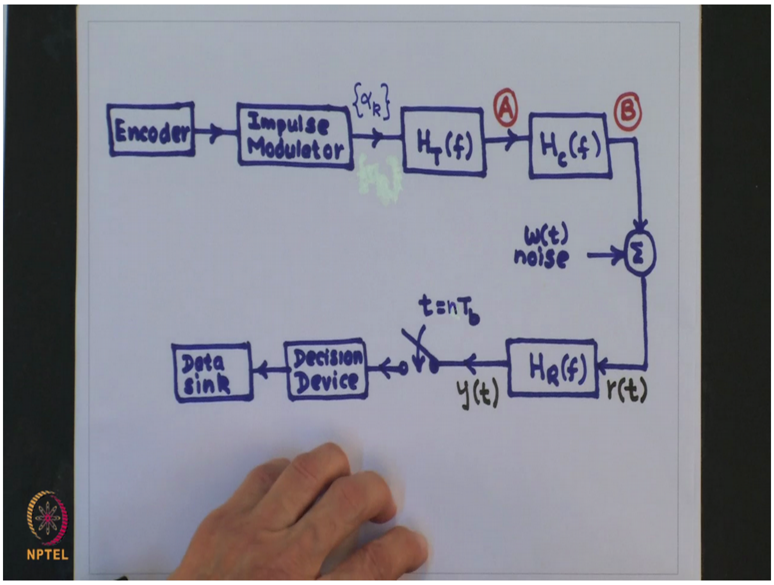

So, I have an encoder the encoder output is usually the binary stream and this binary stream is passed onto the impulse modulator. The output of which are the amplitudes alpha k, these are impulses and then this impulse train basically is fed to a filter H T f. This is known as a transmitting filter.

So, if we choose the impulse response of this filter to be a rectangle pulse whenever I apply a input impulse here; I will get a rectangle pulse as the output of this filter correct. So, this is how you can generate at this point the appropriate line code by choosing appropriate value of alpha k.

So, then this output of this filter goes to the channel; this is the point B which I have denoted and this was the output which I had shown earlier at point p correct. So, this is the same as what you see here ok. Then on the channel we have noise added to it as w t, usually you will choose an additive white Gaussian noise. Then at the receiver we would have a receiving filter.

So, in the absence of say H C f, like if you choose H C f to be equal to 1 then your H R f for we have shown that for identity white Gaussian noise; this would be a match filter to the pulse which is coming out at this point. And then we sample it and then we use a decision device to take the decision of the amplitude alpha k and then accordingly we find out which message has been transmitted. So, we will assume that here we have this is your y t output and this we can call it is the r t. So, this r t is what I have shown here, ok.

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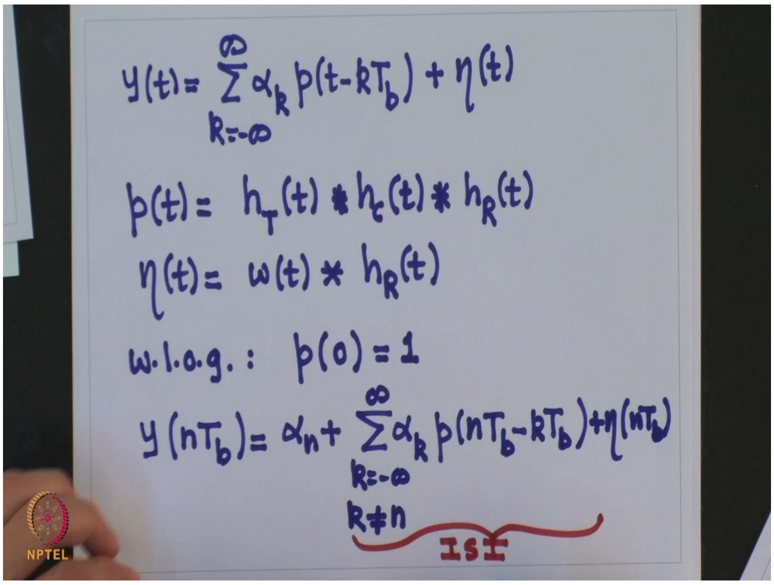

So, in general let us write down the received signal at the output of the receiver filter. So, y t would be equal to summation alpha k p t minus k T b; k is equal to minus infinity to plus infinity plus eta t; pt is basically the convolution of the impulse response of the transmitting filter with the communication channel impulse response convolved with impulse response of the receiving filter.

So, this is the overall response of the system due to a unit impulse at the input and your eta t noise is w t convolved with the impulse response of the receiving filter.

Now, without loss of generality let us assume that p 0 is equal to 1. Now, at the sampling instant I am going to sample this. So, at the sampling instance n T b I am going to get the output from here equal to alpha n plus k not equal to n plus eta sampled at n T b.

Now, this portion out here this portion corresponds to your ISI; Inter Symbol Interference. Now, we look into the conditions on the overall transfer function which is given by P f is equal to H T f multiplied by H C f, channel transfer function multiplied by receiving transfer function. We will try to choose this P f in such a way that we get ISI to be 0, correct ok.

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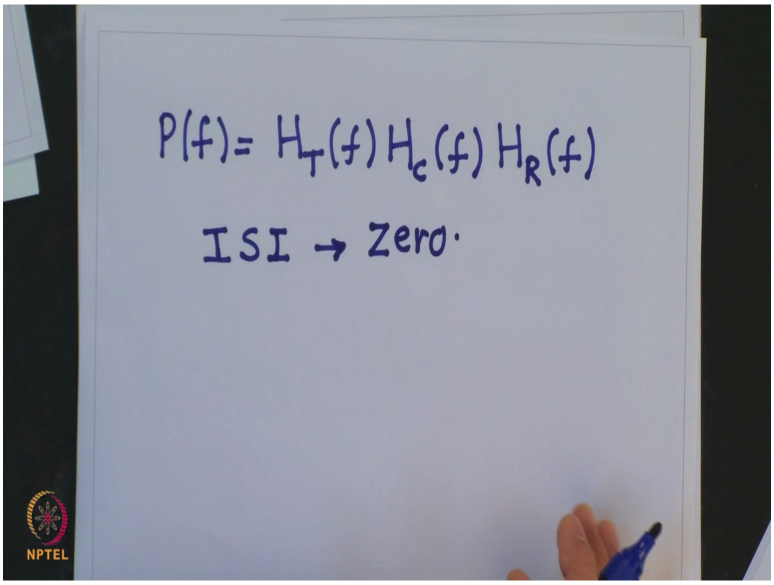

So, let us try to understand then we will get ISI to be zero and we can do that with the help of this figure out here.

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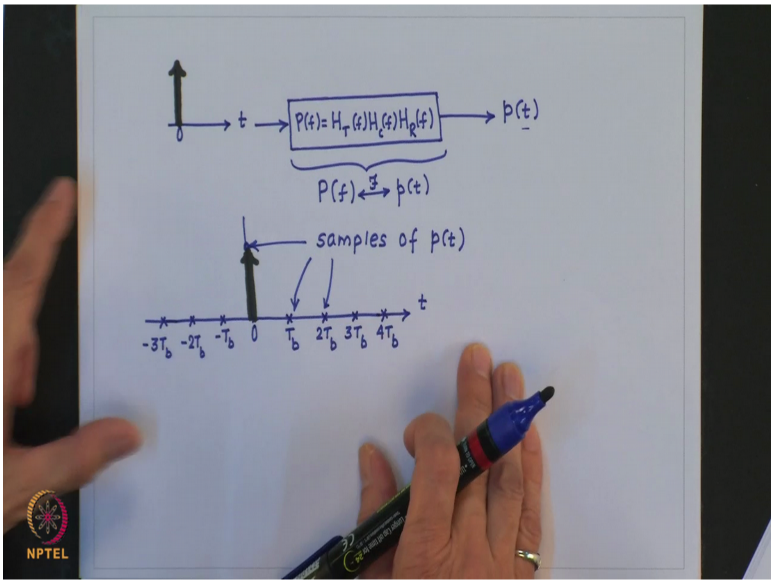

So, here I have shown P f which is the overall system transfer function composed of these three filters. If I apply input impulse of unit strength, then the output I will get will be p t and the Fourier transform of this p t is P f ok.

So, what we expect that if there is no inter symbol interference then when I apply the impulse at t equal to 0. I should get output at t equal to 0 and all other sampling instances; this output should be equal to 0's right from minus infinity to plus infinity, for all values of T b n multiplied by T b except for n equal to 0. This value should be 0.

So, if p t is such that this is the case then the system will experience no ISI. Since, an impulse say at t is equal to m T b will produce a non-zero value only at that time t equal to m T b and 0 at all other sampling points correct. So, this is the time domain condition for no ISI and this is known as Nyquist First Criterion for Zero ISI.

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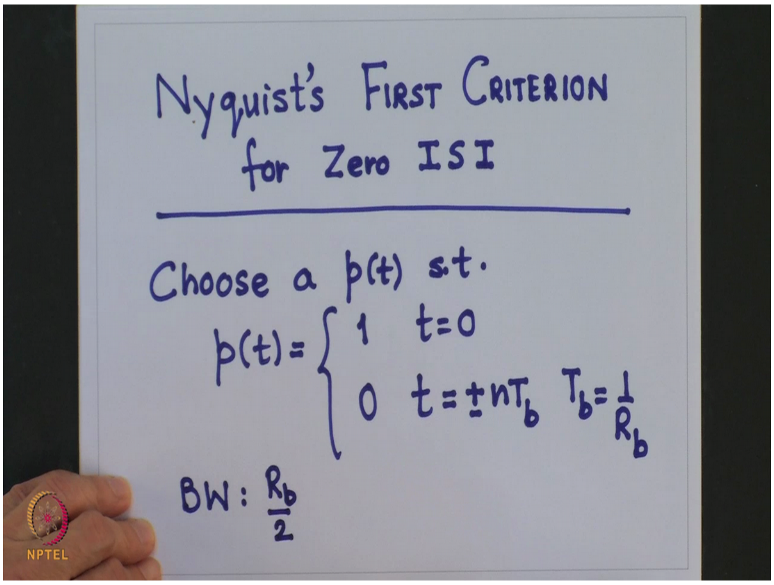

So, it says that choose the pulse p t such that p t is equal to 1 for t equal to 0 and it is 0 at all sampling instances which is denoted by plus minus n T b where your T b is equal to 1 by R b; the rate at which we are transmitting the pulses.

Now, we know that for transmission of R b samples per second, we require a theoretical minimum bandwidth would be equal to R b by 2. So, it would be nice if the above p t had this minimum bandwidth of R b by 2.

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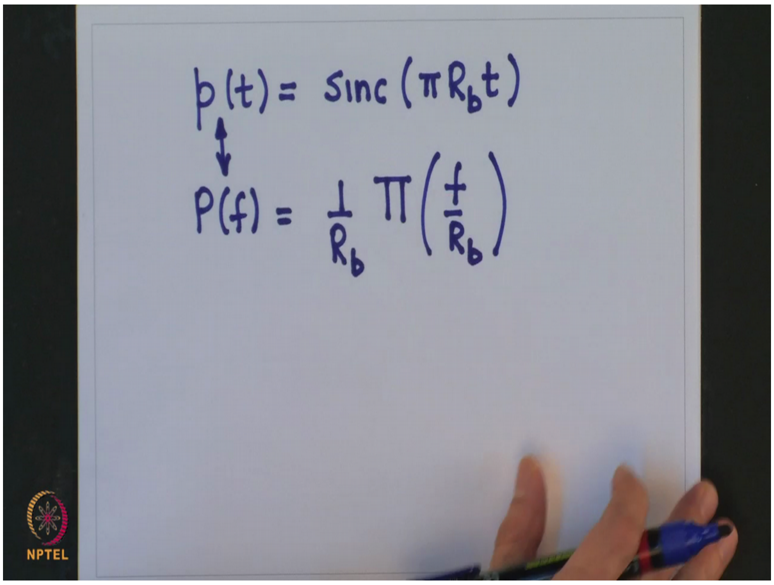

So, the question is can you find such a pulse and the answer is very simple the pulse which will satisfy this is the sinc pulse and the P f for this; what this means is a rectangle function of a duration R b about f equal to 0.

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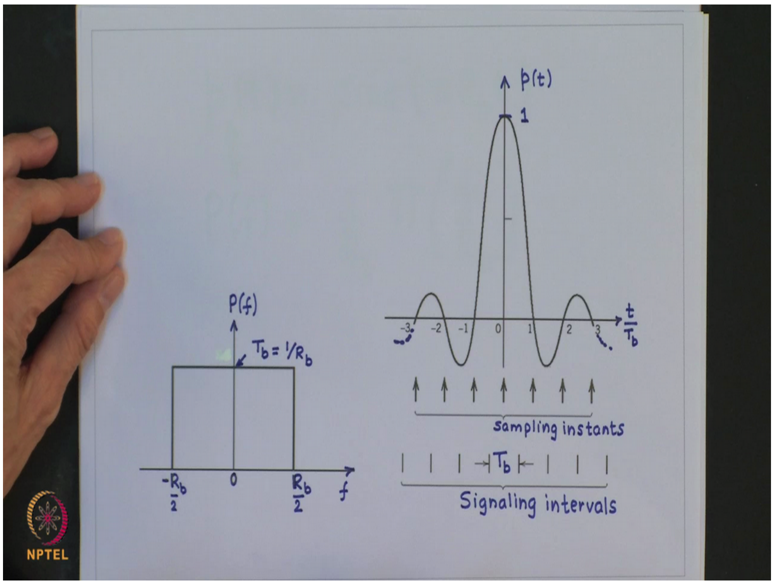

So, the function P f is shown here in this figure it is a rectangle function and the time domain signal corresponding to this P f p t is a sinc function correct. So, if you choose this then we see that only at t equal to 0 you get the value equal to 1 whereas, at all other sampling instances the contribution from this pulse would be equal to 0. So, there is no ISI in this case.

Now, there are two problems with this, with the choice of this pulse one the above p t is impractical why; because it is non-causal, it is existing from minus infinity to plus infinity.

The other problem is that when we are sampling when you transmit such pulses and when you do the sampling at the signaling intervals, the pulse amplitude will not vanish at the other pulse center; if the sampling is not done exactly at plus minus n T b. So, if you have a small deviation there it will contribute to other pulses correct.

So, let us see this.

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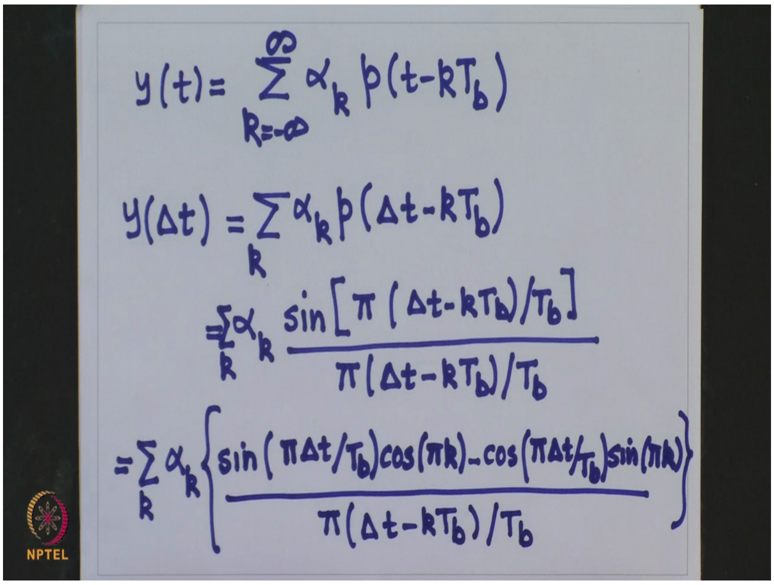

So, we know y t is equal to summation alpha k p t minus k T b; k equal to minus infinity to plus infinity. Let us say that instead of sampling at t equal to 0 I do the sampling at y is equal to delta t where delta t is small.

Now, what will happen this would become this expression here? So, k is equal to minus infinity to plus infinity. So, I will not write it here it is presume. So, let us try to expand this we have chosen the sinc function.

So, this would be equal to sin of pi I am just substituting the value in the sinc function. This I can rewrite it as summation over all case alpha k, let us expand this minus cos of sin of pi k whole thing divided by pi delta t minus k b ok.

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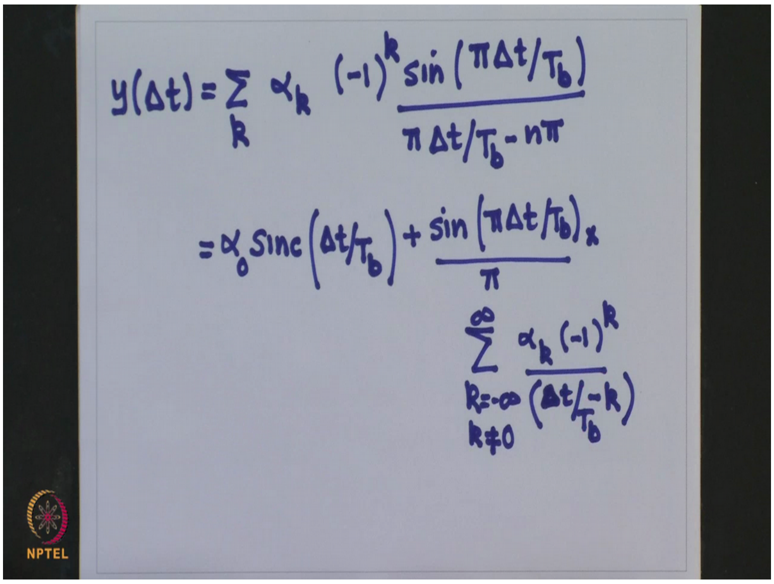

Let us see what we get; the other term here we will go to 0 because of sin pi k. This I can rewrite as and this whole this quantity gets multiplied by the quantity which I am writing below here. So, this is a k equal to minus infinity to infinity, but k not equal to 0 because this has been taken here. So, this is equal to alpha k delta t divided by T b minus k ok.

Now, this term the last term out here; this is not absolutely summable. So, because of this the pulse decays, because this pulse decays only as one by if you look at this pulse out here. The decay of this pulse is 1 by t and if you have this miss timing of the sampling; then this is the cumulative interference at any pulse center from all the remaining pulse and this does not converge correct.

So, now the next question is it possible to find p t which satisfies Nyquist criterion for zero ISI, but decays faster than 1 by t. Now, Nyquist has shown that such a pulse exists which has a bandwidth lying between R b by 2 and R b. And, we will study this case in the next class.

Thank you.