Principles of Digital Communications

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Lecture - 36
Line Coding - II

The process of converting digital data into electrical waveforms or pulses is known as Line Coding. There are certain properties which are desirable of a line code. To get a better understanding of these properties Fourier representation of a line code would be very useful. Unfortunately, the digital signal based on any particular line code being random in nature or stochastic in nature, we cannot find out the Fourier transform of that digital signal because the Fourier transform is applicable only to deterministic signals.

So, in such cases, where we have sample functions of a random process, then Fourier representation of a random signal is obtained in terms of what is known as power spectral density. Power spectral density can be shown to be Fourier transform of autocorrelation of the random process presuming this random process is at least white sense stationary finding. The power spectral density of the transmitted signal would help us to understand whether that particular signal is compatible to the channel frequency response, because many channels cannot pass DC or 0 frequency owing to ac coupling. And low pass response also limits the ability to carry high frequencies.

So, today, we will study; how to evaluate power spectral density of different formats of line code. But, we will start the study with the evaluation of power spectral density for a generic line code.

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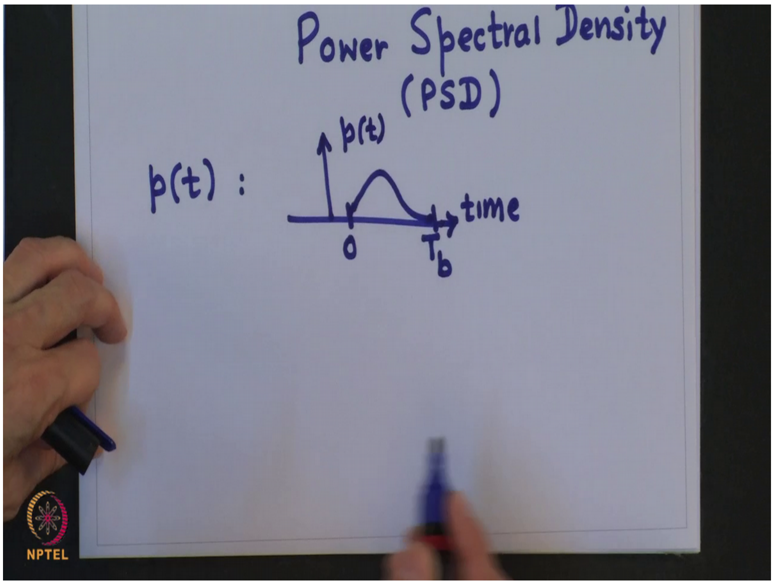

So, we will assume that our digital data is transmitted by using a basic pulse p t. This p t could be of any shape, but for our current discussion, let us assume that this p t is also restricted to the duration T b, correct. This is one form of p t which I have shown here.

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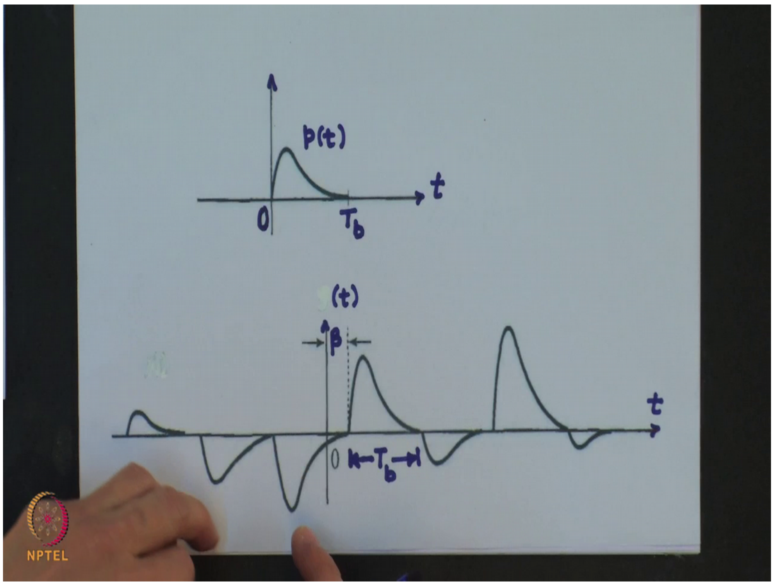

So, successive pulses are separated by T b seconds and the k-th pulse is denoted by alpha k p t; where alpha k is a random variable.

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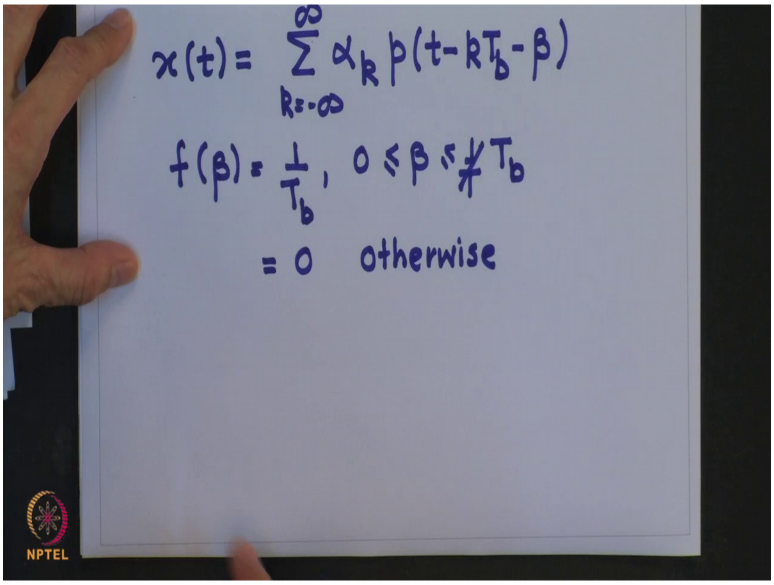

Now to be very generic, we will assume that the pulse train which is being transmitted on the communication channel looks as follows where the distance beta of the first pulse that is k equal to 0 from the origin is likely to be any value in the range between 0 to capital T b 0 to T b, fine.

So, if this is the model which we assumed for our transmission, we can write our digital signal of this form. Now, let us assume that the pdf of this beta is uniform and the value can be anything between 0 to T b is equal to 0 otherwise. Now, before we carry out the power spectral density calculation, we mentioned that is important that for the process to be at least white sense stationary; what this implies that the mean value of the process should be constant with respect to time and the autocorrelation function should only be dependent on the distance between the two samples and not particular value of the time instance.

So, let us try to first find out; what is the mean value of this random process.

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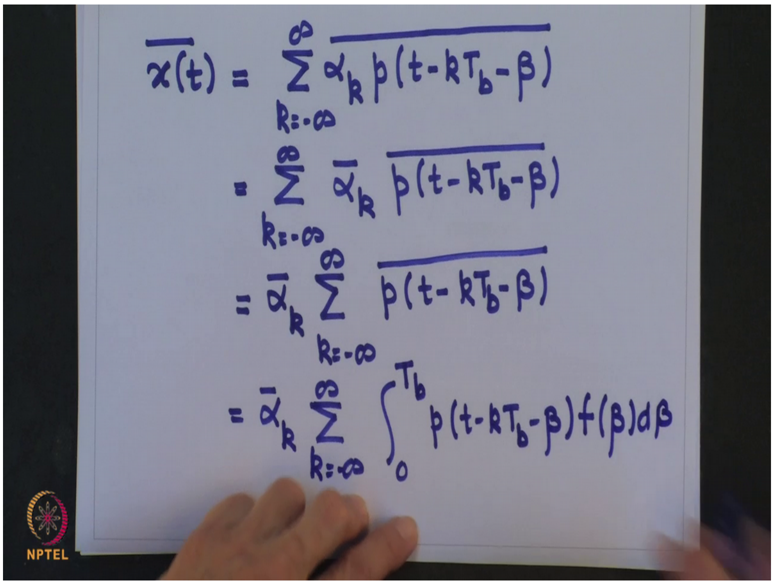

So, to do that; so, we will evaluate this quantity. This bar denotes the expected value this I can rewrite it as please note that alpha k is independent of the random variable beta. So, I can separate it like this; alpha k average value of that I can remove it outside the summation, this can be rewritten as pdf of beta.

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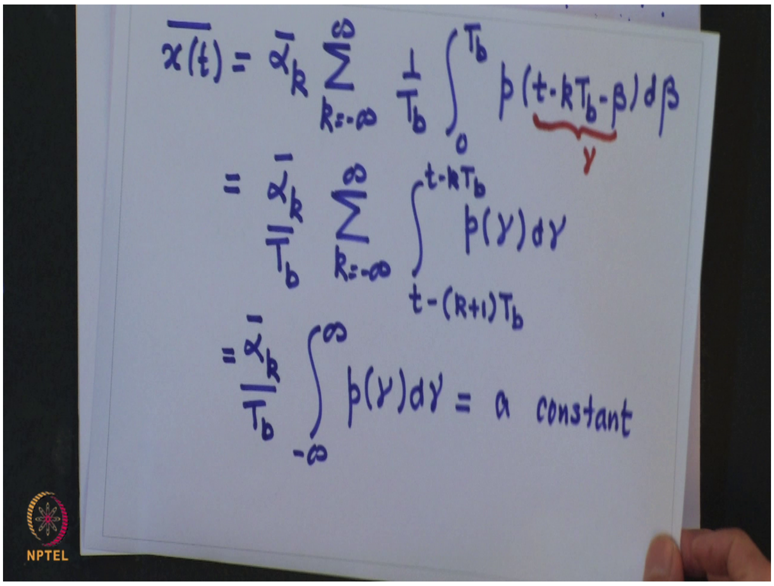

We know the pdf of beta to be constant 1 by T b, this I can rewrite as follows, I will make a simple substitution that t minus k T b minus beta equal to say gamma, then if I do this I can rewrite it as p gamma delta gamma and then substitute the limits note that when I take the derivative of this with respect to beta, I will get minus sign.

So, delta of gamma is going to be minus and then that is why this limits again change back as shown here and this can be rewritten as which is equal to a constant. So, we have shown that the mean value of a; such a random process is a constant. So now, we will proceed to find the autocorrelation function of this random process.

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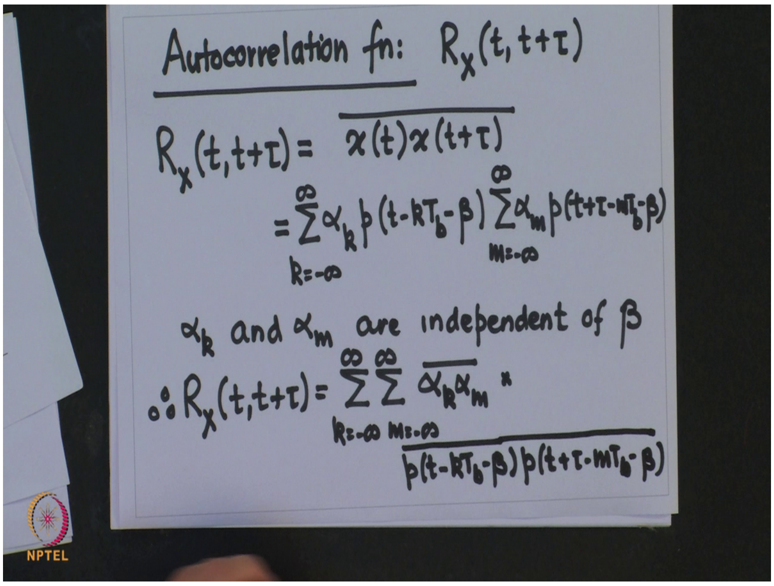

So, we are interested to calculate this quantity and let us see what happens, this is equal to this, I can rewrite it as remember alpha k and alpha m are both independent of beta.

Therefore, we can rewrite the above expression as follows multiplied by and write the quantity below expectation of this quantity.

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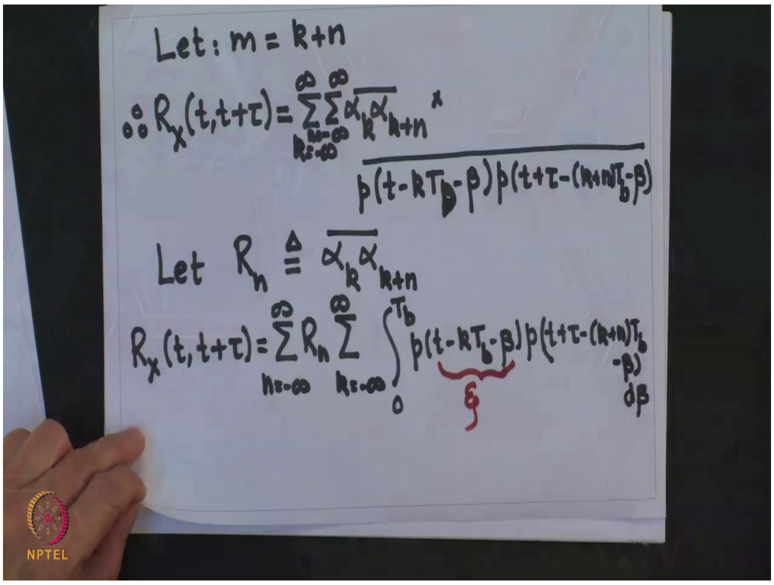

Now, we make a simple substitution, let m is equal to k plus n, if I do this, then I can write my autocorrelation function as follows both k and n are being sum over minus infinity to plus infinity, again, this quantity I rewrite it here sorry this is this is d.

Now, let us define R n to be expectation of alpha k and alpha k plus n, if I use this, then I can rewrite my autocorrelation function to be as follows. This is a integral of these two terms. Now, again we make a small substitution, let us write this quantity to be equal to eta, if we do this, then this expression can be rewritten as follows.

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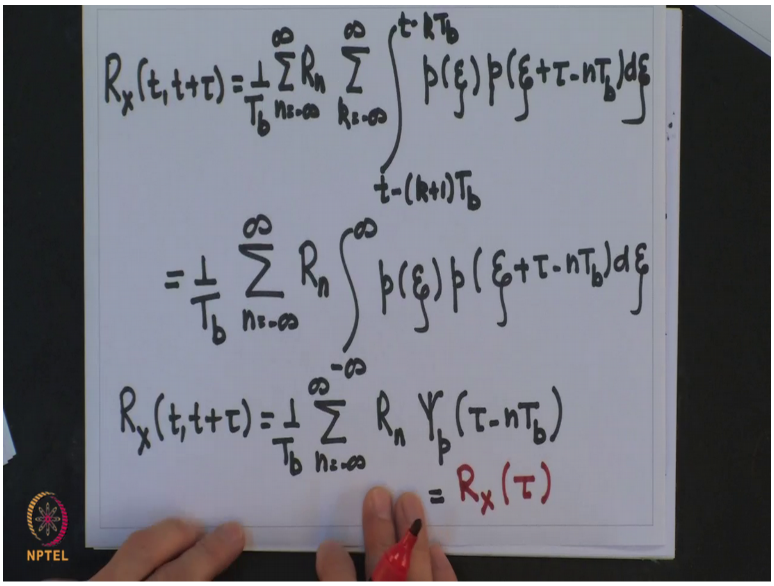

I missed out one term here, this should be multiplied by the pdf of beta; when I am travel trying to evaluate expectation, I have to put this f beta and this f beta remember is equal to uniform pdf which is 1 by T b. So, that can be removed outside, this is n is equal to minus infinity to plus infinity and k is equal to minus infinity to plus infinity.

And the limits for this integral would be as follows and this expression, I can rewrite it as now the integral on the right hand side is the time autocorrelation function of the pulse p t with the argument tau minus n T b, right. So, what this implies that my autocorrelation turns out to be the following expression where R n as defined earlier is equal to this quantity and this by definition is to time autocorrelation of the pulse T.

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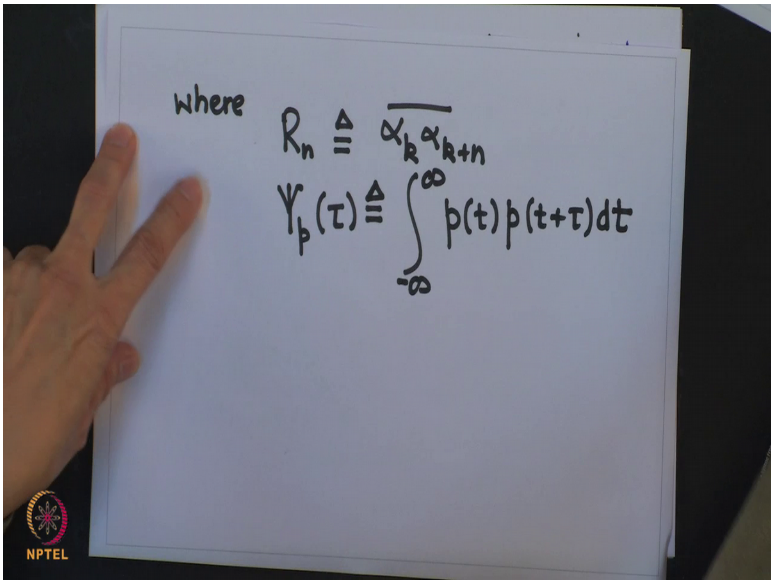

Now, with this definition, what this shows that this quantity out here is equal to Rx tau because it is not function of t.

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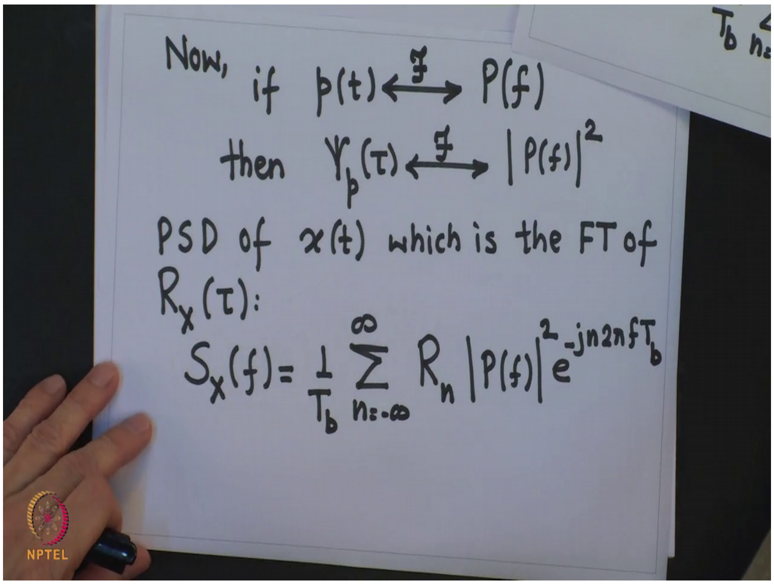

Now if P t has Fourier transform pair P f, then we know that its time auto correlation will have the Fourier transform pair which is P f mod squared. So, from this, we get the power spectral density of x t which is the Fourier transform of R x tau from Einstein Wiener Khinchin theorem can be written as we are taking the Fourier transform of this. So, Fourier transform of this quantity will be P f mod squared and because of n T b, there will be a phase shift given here the final expression we get power spectral density of the random process x t is given by the following relationship.

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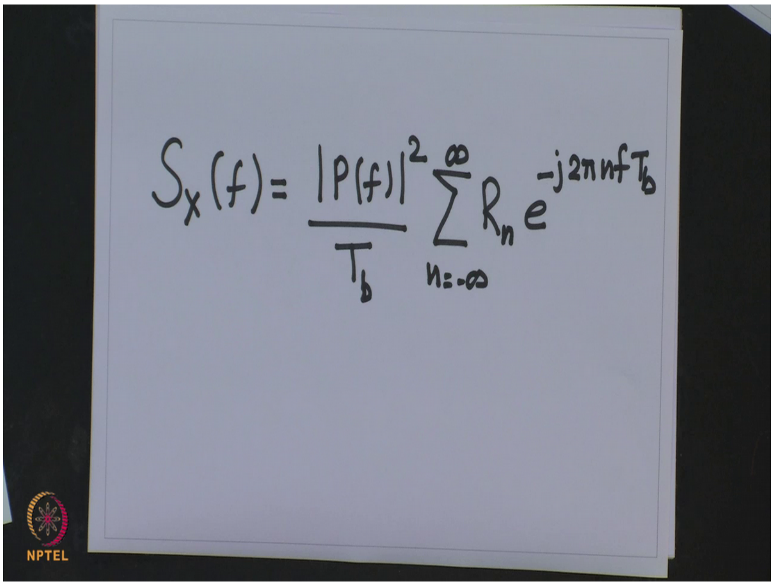

So, we see that the power spectral density is heavily dependent on the choice of the function P t, this we have derived power spectral density for a very generic line code, we will try to evaluate this power spectral density for some specific line codes like polar bipolar. And this, we will do next time.

Thank you.