**Principles of Digital Communications**
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**Lecture - 34**
**M - ary PCM / PAM - II**

M-ary PCM or pulse amplitude modulation is a generalization of binary PCM. We will study PCM system in light of Shannon's channel capacity equation. In M-ary PCM, message signal is quantized in L levels.

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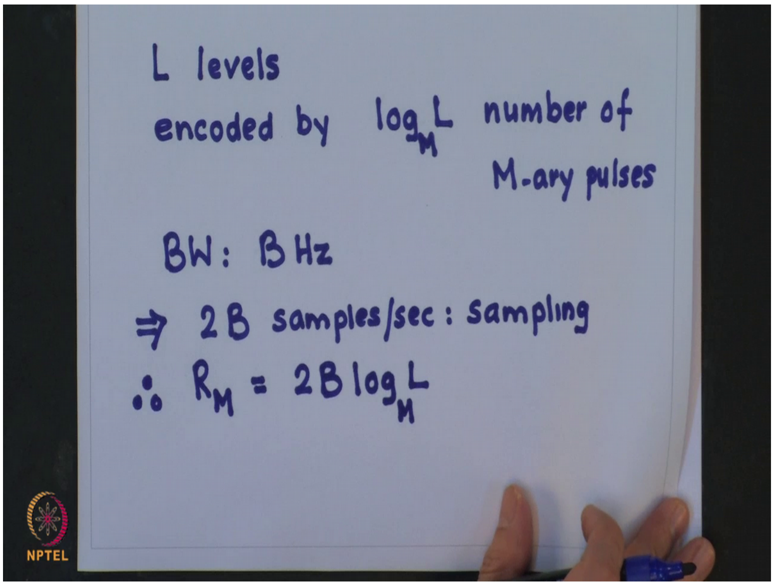


And then each sample is encoded by log of L to the base M number of M-ary pulses.

Let us assume that the message signal bandwidth is B hertz. So, this implies that the sampling has to be done at the Nyquist sampling rate 2B samples per second. Therefore, the number of M-ary pulses per second which I denote by R M would be equal to the number of samples per second multiplied by each sample is encoded by this number of M-ary pulses.

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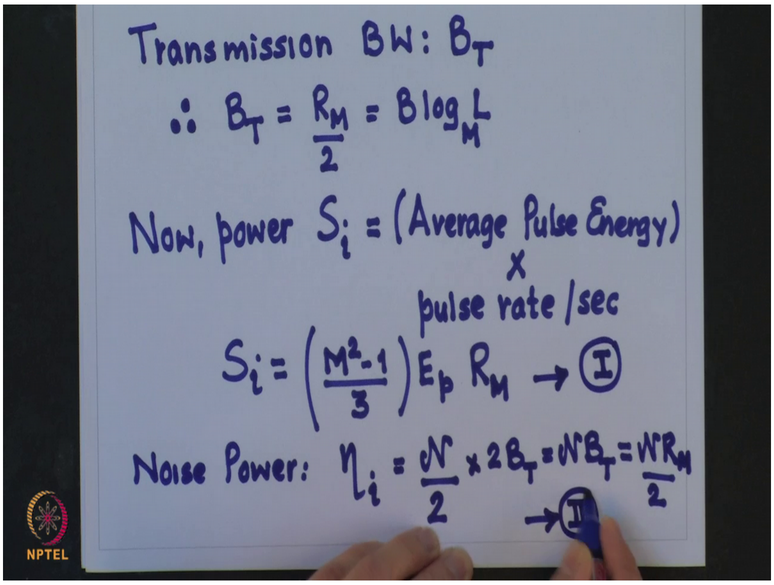


Let us assume the transmission bandwidth to be B T. This is half the number of M-ary pulses per second, remember that with 1 hertz, I can transmit 2 bits of information. So, if I have R M as a number of M-ary pulses per second which I am transmitting then the bandwidth required would be R M by 2.

Now, we know that power which I denote by S i is equal to average pulse energy multiplied by pulse rate per second. So, we get our power to be equal to the average pulse energy for the M-ary PCM; we have shown it to be equal to this quantity where E p is the energy of the pulse B T and this is your pulse rate and the noise power which I denote by eta I would be equal to power spectral density for the White Gaussian noise that is italic N by 2 multiplied by transmission bandwidth and this is equal to using this relationship.

So, let me call this as equation 1 and this as equation 2.

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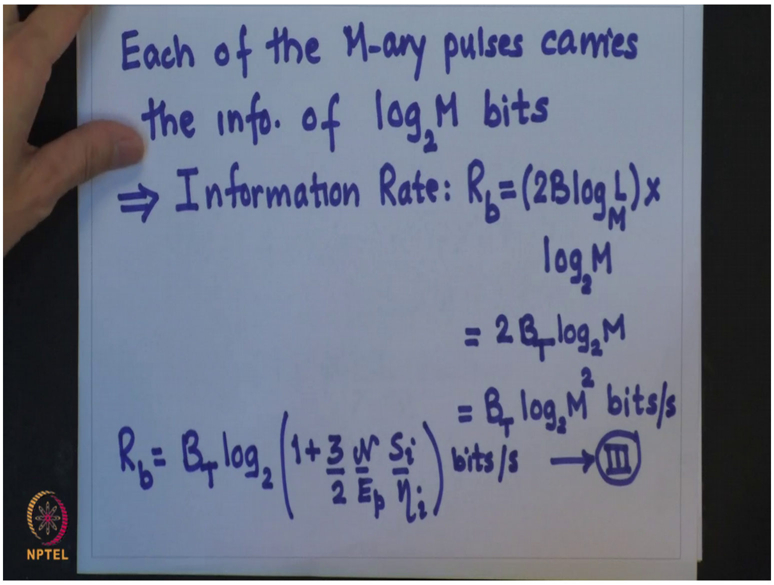


Now, each of the M-ary pulses carries the information of log M to the base 2 bits, we are assuming equiprobable transmission of the messages. So, this implies that the information rate which is denoted by R suffix small b would be equal to the number of pulses which we are transmitting per second, this quantity multiplied each pulse carries this many number of information bits.

So, this is equal to from this relationship which I can rewrite it as so many bits and let me call this as equation 3. We can substitute 1 and 2 and in 3, this will give us R b equals B T for M squared, I am going to use this relationship bits per second. So, information rate should be bits per second please. So, using 1 and 2 for R M, I will use 2 eta i by italic N ok, I get this relation.

So, what it means that we are transmitting the information equivalent to r b binary digits per second over the M-ary PCM.

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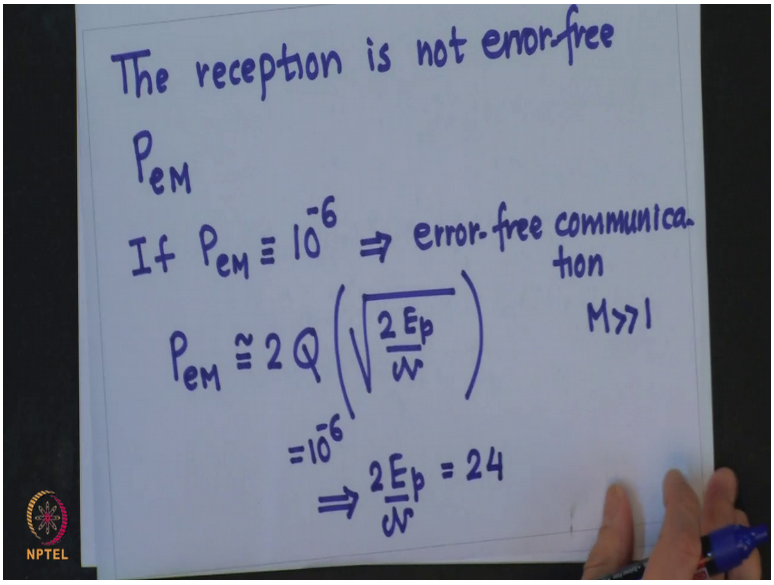


Now, it is important to note that the reception is not error free; however, the pulses are detected with an error probability of P e m, we have derived what is this value for an M-ary PCM system.

Now, if we assume that this value is of the order 10 raised to minus 6, then it is almost error free communication, then let us find out; what is the value of E p by italic N in the formula for P e m. Now, this symbol error probability which we have derived earlier, it is approximately equal to for large M 2 Q of this quantity.

So, if you want 10 raise to minus 6, we can equate this to 10 raise to minus 6 and for this, we can evaluate what is this ratio of twice E p by italic N and this turns out to be 24, if we use this value and substitute it out here, we get the rate to be as follows.

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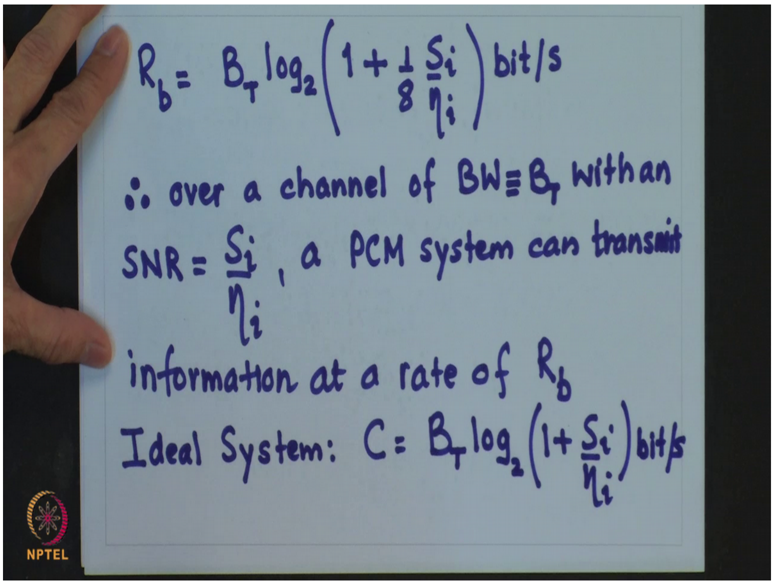


So, what does it means that therefore, over a channel of bandwidth equal to B T with an S N R equal to S i by eta i, a PCM system can transmit information at a rate of R b bits per second given by the expression here.

Now, we know that for the ideal system from the Shannon's capacity equation, C is equal to B T log to the base 2 of 1 plus signal to noise ratio. So, from this, it implies that PCM uses roughly 8 times as much power as the ideal system. So, it corresponds to about 9 dB 10 log of 8 to the base 10.

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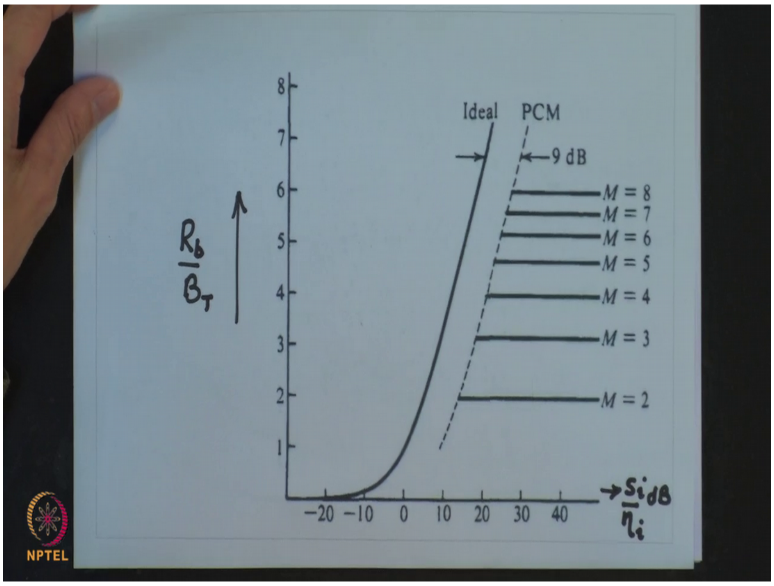

So, this figure out here shows R b by B T ratio as a function of signal to noise ratio both for the PCM and for the ideal system for the ideal system R b by B T which is nothing, but this quantity is also plotted here.

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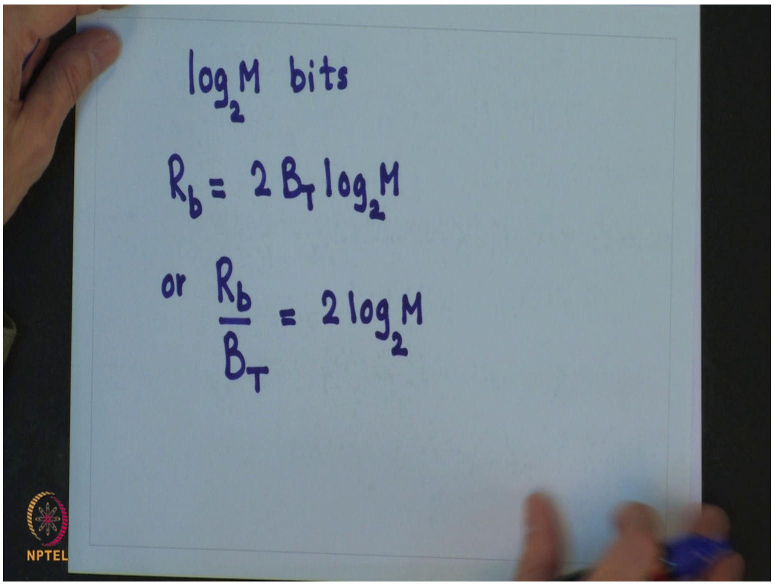


Now, when PCM is in saturation, it means that the detection error probability approaches 0 and in that case each M-ary pulse transmits log of M to the base 2 bits. Hence, we will get R b equal to twice B T log of M to the base 2, from this, we get R b by B T equal to twice log of M to the base 2 and this is clear from this figure out here also. So, for example, M equal to 2 after certain value of signal to noise ratio, this ratio of R b by B T saturates.

So, if you want this ratio to be higher, we have to move over to the higher value of M M equal to 3 4. So, these are the saturation levels for different Ms, summary PCM system roughly uses 9 dB as much power as the ideal system. So, with this we come to the end of our study of the module on PCM, in this module, our focus was on quantization specifically scalar quantization, we began our study with uniform quantizer which is an optimum quantizer for input which has uniform pdf.

We also showed that when the quantizer levels are quite high, then the improvement in signal to quantization noise ratio which we get for every additional bit of quantization is approximately 6 dB, we also studied uniform quantization for the case of non uniform pdf and for unbounded pdf, the solution to this problem was quite messy and then we moved over to the study of non uniform quantization, we studied Lloyd Max quantizer which is an optimum quantizer in the sense that it minimizes the mean squared quantization error.

We saw that the reconstruction level in any quantization interval is centroid of the pdf mass in that interval and the decision boundaries are midpoints of the adjacent neighboring reconstruction levels, we also mentioned that Lloyd Max quantizer performance deteriorates very rapidly whenever there is a mismatch due to the wrong choice of variance of the input signal or due to the wrong choice of the pdf of the input signal and we also mentioned that implementations of Lloyd Max quantizer in practice is quite difficult companded quantization is a mean of implementing non uniform quantization.

In companded quantization, the signal is passed through a compressor followed by an uniform quantizer and an expander in this process of quantization, more number of quantization regions are allotted to smaller amplitude of the signal which have a higher probability and less number of quantization regions are allotted to a higher amplitude of the signal which have lower probability, we also studied mu law and a law companders which are popularly used in practical applications and with the help of an example, we showed that this quantizers are quite robust to the variation in an input pdf and power levels.

Now, in real signals there is a high degree of correlation between the samples of the signals and this fact is exploited in the design of DPCM that is differential pulse code modulation, we saw that the operation of the quantizer and the predictor in DPCM are interlinked, the goal in the design of a predictor is to minimize the prediction error which will reduce the quantization noise which in turn will provide better prediction because of better quality of reconstructed samples.

The goal in the design of predictor is to minimize the prediction error so that this will reduce the quantization noise for the given number of quantization levels and this in turn, we will provide better prediction. We also learned that for linear prediction the prediction error is minimized in the mean square sense when the prediction error is orthogonal to all those samples which are used in the linear combination to form a linear predictor, then we moved over to the study of a generalized binary PCM in the form of M-ary PCM and this M-ary PCM is also known as M-ary PAM that is pulse amplitude modulation we showed that for a given information rate the bandwidth requirement of an M-ary PCM goes down by a factor of log M to the base 2 , but the requirement of power goes up by the factor of M square by log M to the base 2.

We also saw that for a given bandwidth the information rate can be increased by a factor of log M to the base 2, but there is a requirement for increase in power of the factor M squared because the voice channels of a telephone network have a fixed bandwidth. Therefore, multi amplitude signaling is a very attractive method of increasing the information rate and this is exactly what happens in wise band computer modems to achieve high data rates. So, with this, we end our module on PCM.

Thank you.