Principles of Digital Communications
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Lecture - 33
M - ary PCM / PAM - I

In pulse code modulation that is PCM, the output of the quantizer is usually encoded has a binary code word. So, what we get is known as binary PCM. In general the bandwidth required for the transmission of binary PCM is quite high. So, if you want to reduce the bandwidth requirement, then we adopt what is known has a generalization of binary PCM is M-ary PCM, or more popularly known as M-ary pulse amplitude modulation for M-ary PAM.

So, what we do is that? The output of the binary PCM is grouped into k bits. So, from this group of k bits, we form two raised to k equal to capital M messages or symbols which we want to transmit. So, in binary PCM we usually use 2 signal waveforms plus p t to denote the transmission of binary digit 1 and minus p t to denote the transmission of binary digit 0 in whereas, in M-ary PCM M symbols are transmitted by M pulses of the form plus minus k p t.

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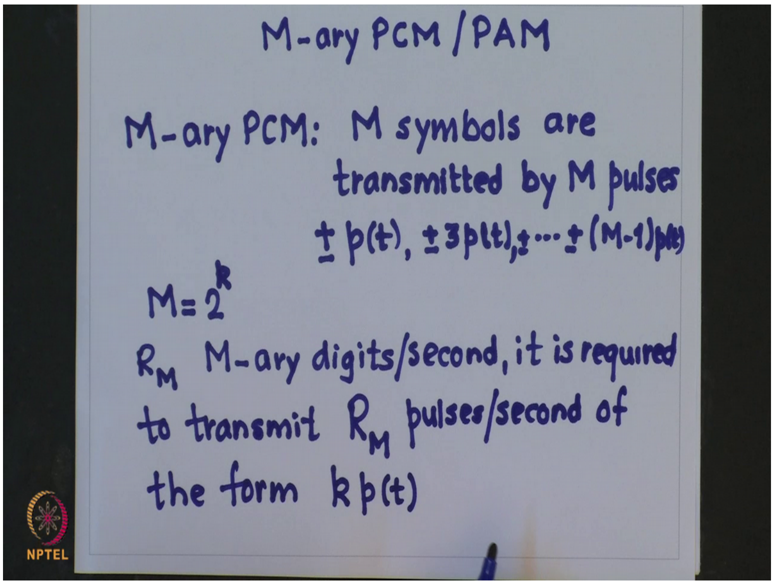

So, we will study today what is known as M-ary PCM, or M-ary pulse amplitude modulation M-ary PAM, M symbols are transmitted or messages by M pulses of the form plus minus p t, p t is the waveform which we choose your M is going to be of the form 2 raise to k, where k number of bits are grouped together.

So, to transmit R M M-ary digits per second, it is required to transmit R M, this denotes the rate at which we are transmitting this M pulses of the form k pt.

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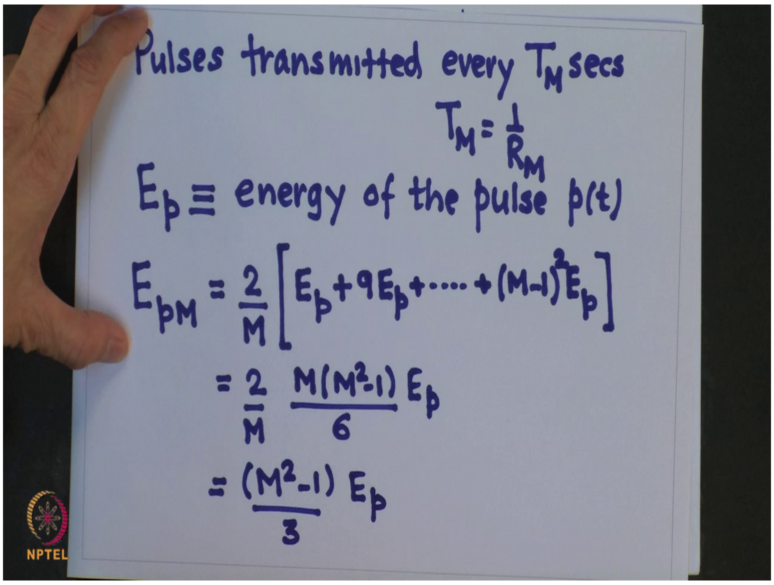

Now, pulses are transmitted every T M seconds. So, T M is equal to 1 by the rate. Let us denote E p as the energy in the pulse p t. Let us also assume that we have equiprobable pulse transmission.

And let us try to calculate the average energy in this transmission. So, this will be equal to twice becausem we have the pulse transmission of the form plus minus and we are assuming that the pulse transmission is equiprobable. So, it is going to be 1 by M and probability of occurrence of each of the pulse and, the energy in each of the pulse is going to be this summation. This is for the plus minus p t, this is for plus minus 3 p t and so, on.

This is for the case plus minus M minus 1 p t, this we can rewrite it as summation here would be equal to which is equal to so, this is the average energy for transmission of any pulse corresponding to a message.

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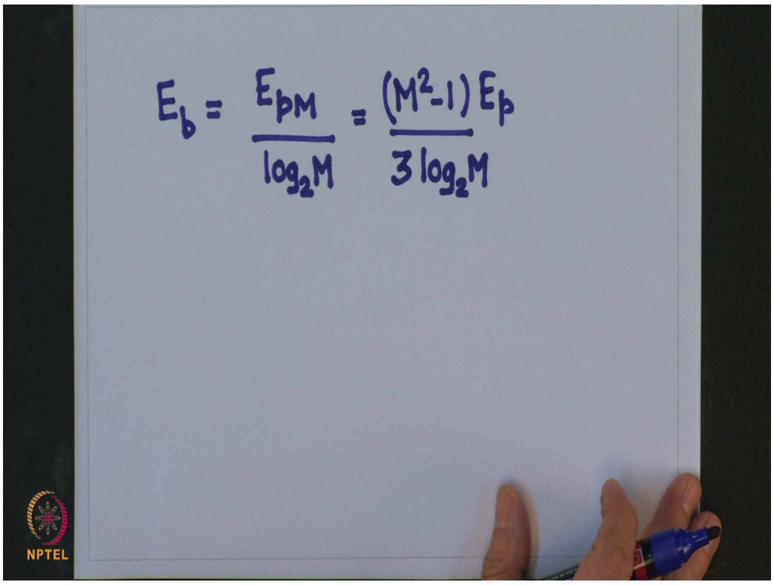

So, from this we can calculate the average bit energy which is equal to the average,energy in the pulse the number of bits which each pulse carry the information is log to the base 2 M all are of equiprobable. So, this is equal to M squared minus 1. Now, it is important to note that for a given information rate M-ary PCM bandwidth is less than that of the binary PCM by a factor of log of M to the base 2. Now, let us try to calculate error probability for this M-ary PCM.

Now for this case it is easy to see that this is a one dimensional problem. So, the signal constellation, we will get the points only on a straight line. The optimum M-ary receiver for this will be a filter match to the basic pulse p t.

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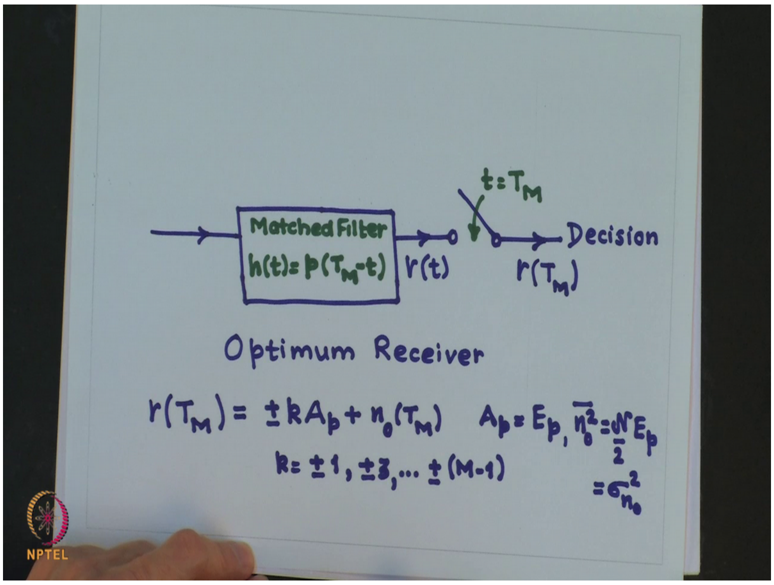

This is what I am going to get my input signal is going to pass through a match filter, with an impulse response given by the mirror image of the signal p t. And since we are going to sample at t equal to capital T M, which is the duration of each of the pulse, you get the impulse response to be of this form. This signal is sampled at t equal to T M. And we get the output to be r at sampled it is in T m.

Now, it will be of the form plus minus k times A p, where A p is going to be nothing, but the energy in this pulse. We are studied this earlier and the output noise, which we are going to get out here, will have the variance which is equal to italic N by 2, which is the power spectral density of the white noise multiplied by the pulse energy. We are assuming that the signal is being transmitted over additive white Gaussian noise channel with the 0 mean noise correct.

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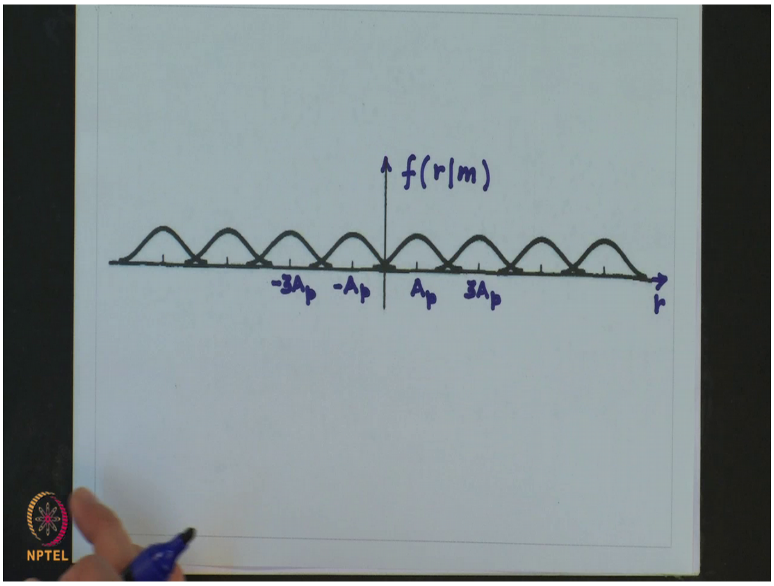

And the signal consolation for this will have is of the form. This corresponds to plus p t, this corresponds to minus p t, this corresponds to the 3 p t, and this corresponds to minus 3 p t. So, signal constellation which is nothing, but the representation of the message signal in the orthonormal basis signal, which in this case is nothing, but the pulse itself p t divided by the root of it is energy ok.

Now, given this we need to calculate the probability of error, to do this basically we will require to find out what is the conditional distribution of receiving the output given that you have transmitted particular message. And that is shown out here, assuming Gaussian noise. This is your if I have transmitted the message corresponding to p t, then the conditional distribution of the output given that message was p t is given here, this is a Gaussian distribute.

Similarly, when my message signal was three p t, then this is my conditional Gaussian distribution. We are interested in calculation of symbol error probability, error probability detecting a symbol.

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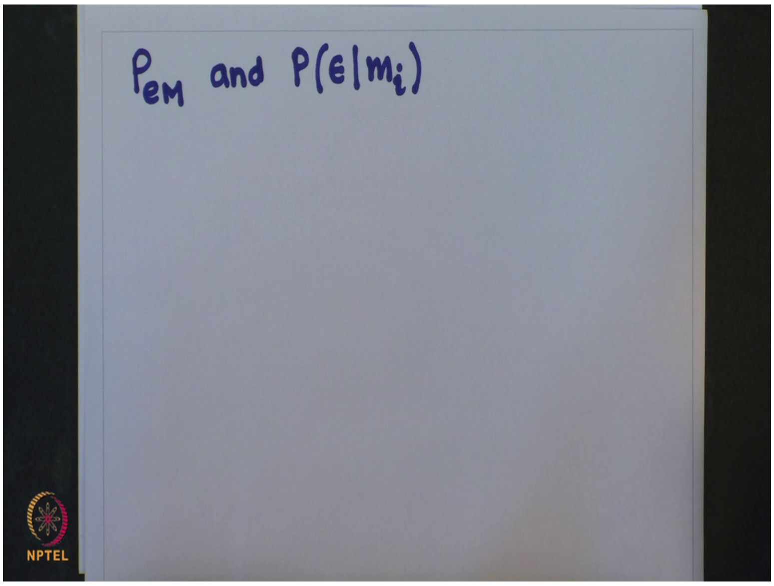

And we would also require to find out the conditional probability of error given that you have transmitted a particular symbol say m i. Now, to do this we should so, to do this observe, that for the case of 2 extreme symbols, which are represented by plus minus M minus 1 p t error probability calculation is similar to the binary case which we have studied earlier, because they have to guard against only 1 neighbor for this it is this side and for this it is this side correct. As for the remaining symbols, they must guard against neighbors on both side for example, if you take this it has to guard against this neighbor also, it has guard against this neighbor also.

So, in this case the conditional probability of error given that we have transmitted any of this signals, would be twice then what we obtain for the extreme signals in here ok. So, using this fact let us try to calculate the probability of symbol error before we do that.

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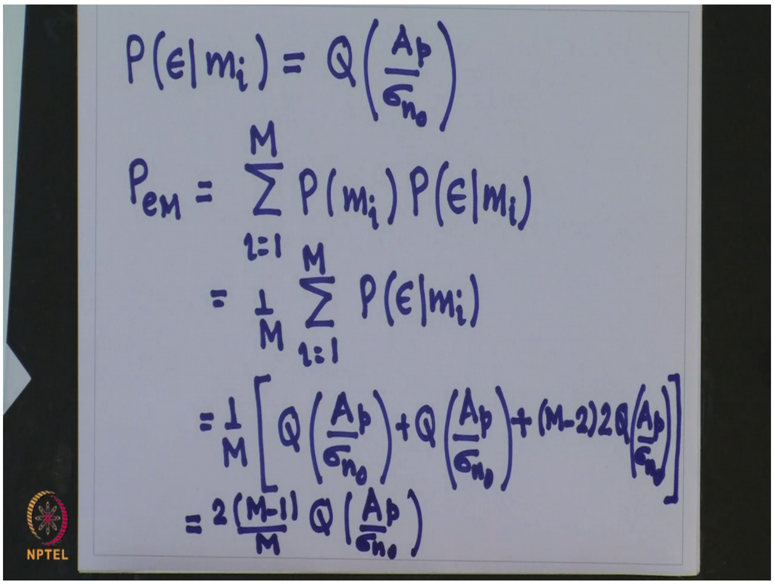

Let us try to calculate the conditional probability of error given a message m i. Now, we know that this is equal to for the extreme cases, for the signal located at the extreme end, this is the Q function of that.

So, this would be equal to what is the probability of transmitting a particular message multiplied by the conditional probability of the error for that message. I will be equal to from 1 to capital M. This is equal to 1 by M. Based on the discussion we had just now for the extreme 2 symbols, this would be the conditional probability of error and, for all other it will be multiplied by twice. So, this would be equal to M minus 2 multiplied by twice Q of A p by there are M minus 2 symbols in between. So, we get this quantity, this we can simplify to be 2 M minus 1 divided by M.

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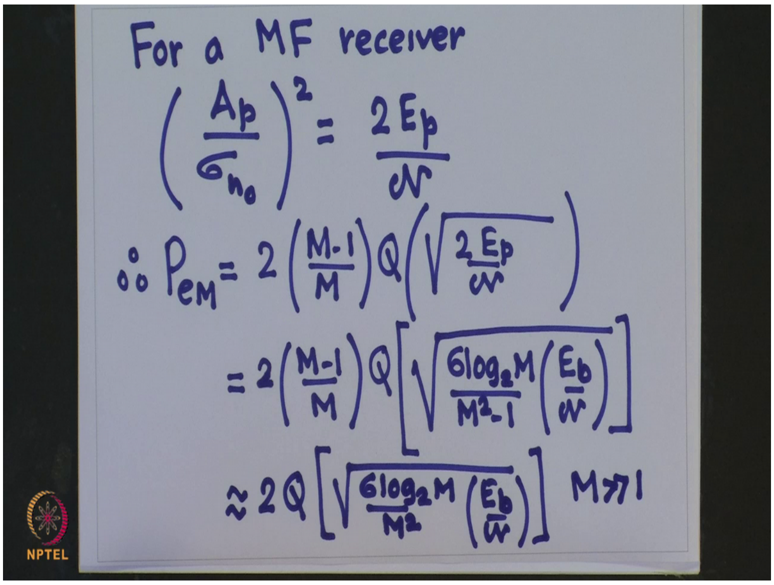

Now, let us calculate this quantity we know for a match filter receiver. This ratio squared is equal to twice into the energy of the pulse italic N italic N by 2 denotes the power spectral density of the white noise. Using this relationship, we get the probability of symbol error to be, using this relationship for the average energy and the energy of the pulse, we can rewrite this as.

Now, this E p we can rewrite in terms of bit energy, which we have derived earlier correct. So, will write this E p in terms of the average bit energy, if we do this I get this of this form and, this I can approximately write as 2 Q for M much larger than 1. This is the symbol error which we get; in this case it is also possible to calculate the bit error rate, if you follow a particular strategy of coding the symbols as follows.

So, if you look at this conditional probability distribution, then the type of errors that predominate are those in which a symbol is going to be mistaken for it is immediate neighbors. So, in that case what we could do that assign M-ary symbols binary code words that differ in the least possible digits. And the gray code is suitable for this purpose. So, in the gray code what we do is that if you have four messages, then you can use 0 0 0 1 1 0 1 1, but this four are placed in such a way that if you go from one symbol to another symbol the difference is only 1 bit. So, for example, it would be something like this.

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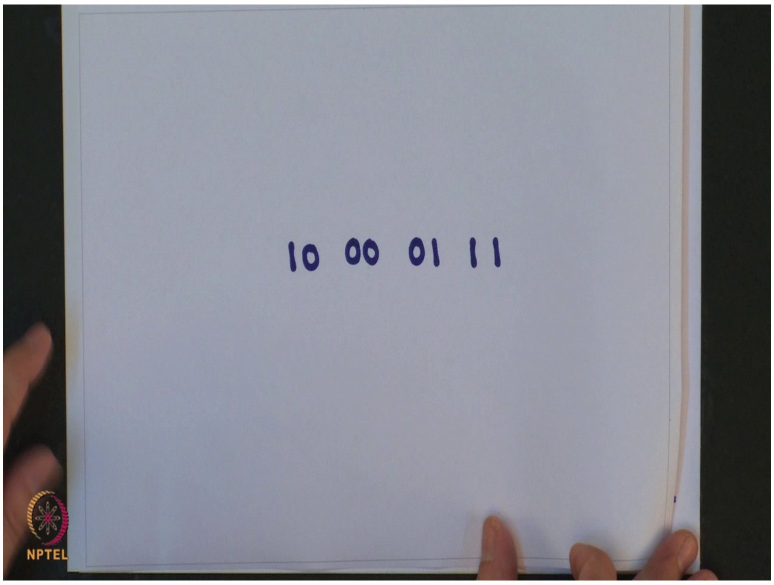

I have 0 0 this could be 0 1 this is 1 0 correct and the next 1 is a 1 1. So, difference if you take the adjacent difference, there is a difference of only 1 bit. So, that happens basically if you assume, then in this case the probability of bit error would be equal to symbol error divided by log of M to the base 2 fine. And if you do this and, if you plot this is the kind of curve which we get.

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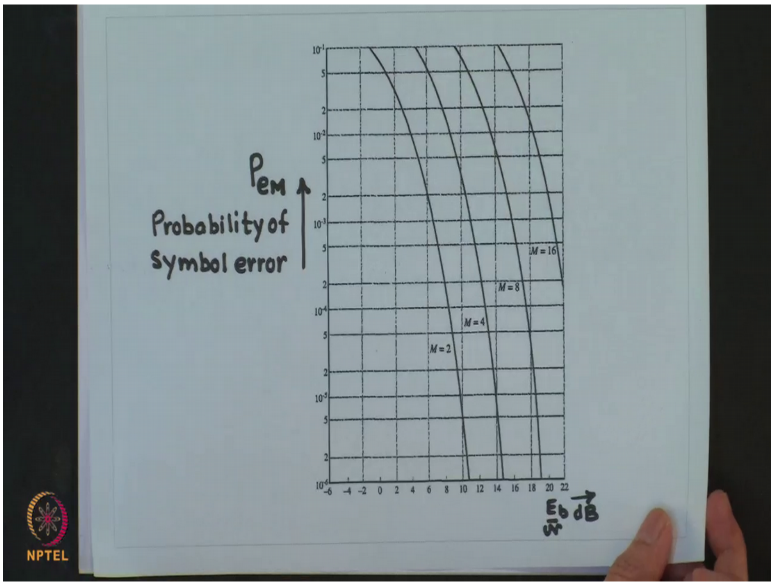

So, this curve clearly shows that an M keeps on increasing correct for a particular value of symbol error, the E b by italic n ratio also keeps on increasing, or for the same value of E b by italic N. The higher the value of M the probability of symbol error also increases ok.

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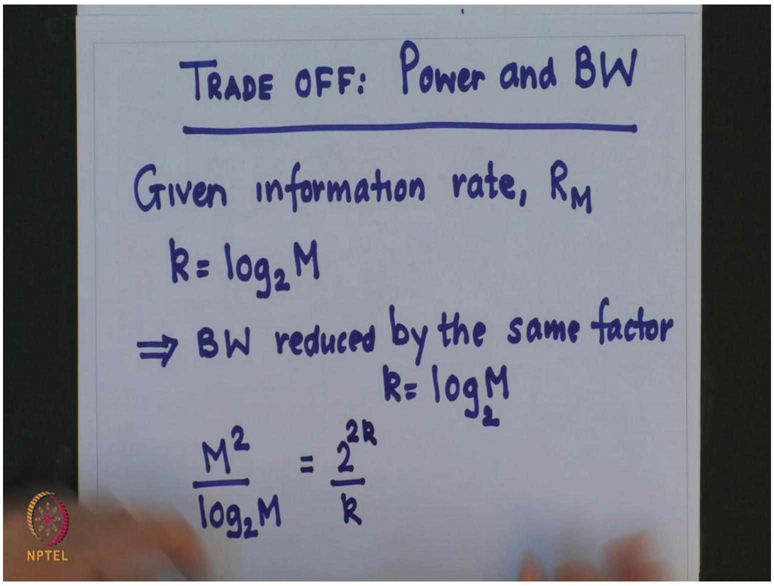

Let us try to quickly understand. What is the tradeoff between the bandwidth and power for this generalized PCM system? So, for a given information rate so, given information rate the pulse transmission rate, which is R M in the M-ary case is reduced by the factor, k is equal to log of M to the base 2. So, what this implies that bandwidth is reduced by the same factor, that is log of M to the base 2, but if you want to maintain the same probability of error, the power transmission per bit which is proportional to the energy per bit also has to be increase look from this expression.

So, that increase is going to be M squared by log of M to the base 2 this quantity. This if you want to keep this same Q is a decreasing function. So, as you keep on increasing this will fall; so you will have in the M-ary case, you will have to increase the energy in the bit it means basically the power has to be increased. And this will be equal to 2 raised to 2 k by k.

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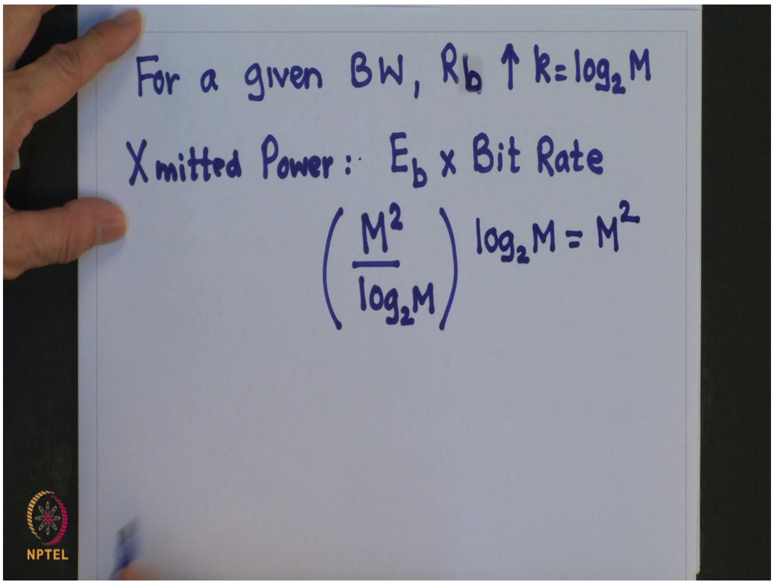

For a given bandwidth the information rate in the M-ary case is going to be increase by the factor k is equal to log of M to the base 2.

So, in this case the transmitted power, which is E b multiplied by bit rate has to increase by the factor M squared by log of M to the base 2, this is the increase in energy, if I want to keep the performance to be the same. And there is increase in the bit rate because, I have kept the bandwidth to be same and the increase in the bit rate is going to be this quantity. So, this means it is going to be M squared. So, this is an exponential increase.

So, the conclusion is that in a high powered radio system such a power increase may not be tolerable. So, M-ary PCM is very attractive when bandwidth is very costly. We have come to the end of our module on the study of PCM and, we have only one final thing to be done and that is basically to judge the performance of PCM in the light of Shannon's equation which we will do next time.

Thank you.