Principles of Digital Communications
Prof. Shabbir N. Merchant
Department of Electrical Engineering
Indian Institute of Technology, Bombay

Lecture – 32
Delta Modulation

In differential pulse code modulation, popularly known as DPCM, there are different sequence obtained by taking the difference of the information source sampled output sequence and its predictive sequence is also known as predictor error sequence. We would like to make this prediction error as small as possible and this can be achieved if we could carry out the prediction in a efficient manner by reducing the variance of the prediction error.

The quantization noise for the given number of quantization level will also reduce which in turn will provide better reconstruction of the sample values which will provide good prediction of the current sample. So, the role of quantizer and the predictor in a DPCM are quite interlinked. It is desirable to have a predictor which is as simple as possible and quantizer with less number of levels as possible.

Now, we have seen that the optimum quantizer is the one based on conditional expectation which is difficult to implement in practice. Therefore, we use an approximation in the form of nth order predictor. We would like to have this order of the predictor as low as possible. The lowest order which we could have is the first order and as far as the quantizer is concerned, the simplest quantizer which we could have is one bit quantizer that is only two levels.

So, if we use a DPCM which has a first order predictor in the form of a simple delay and a one bit quantizer, then this form of DPCM is known as delta modulation see what happens in delta modulation. So, this is the block of DPCM which we have studied earlier.

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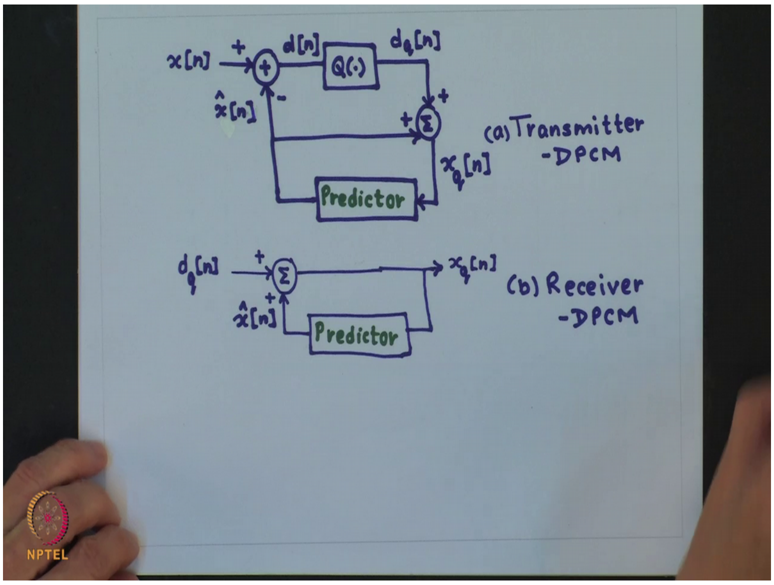

So, in this block, what we are going to do is going to replace the predictor by a simple delay element and the quantizer will be replaced by 1 bit quantizer to form what is known as delta modulation. So, this is the block schematic for both the transmitter and the receiver of a delta modulator. So, this serves as the prediction for the current sample value.

Now, let us look at the operation of this, let us see; what happens at the receiver, the same thing happens at the modulator also or the transmitter end also.

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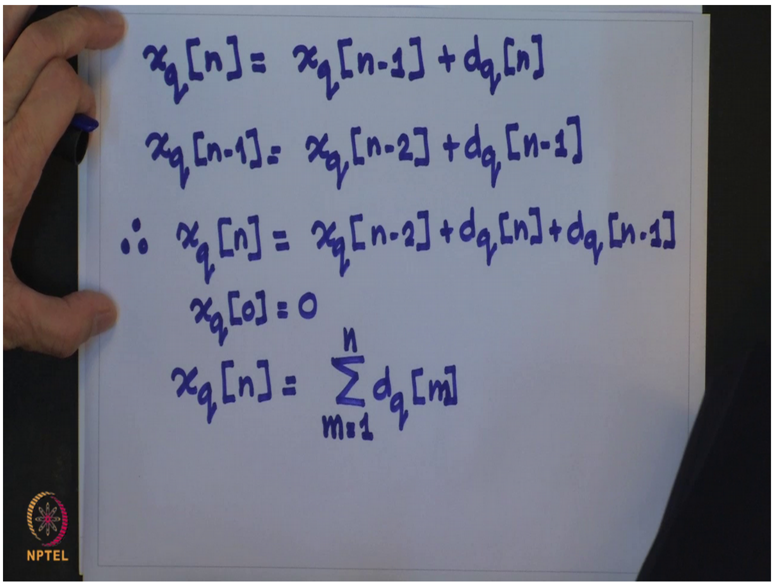

So, x q n at the modulator end is obtained by adding x q n minus 1 plus d q n. Now, x q n minus 1 itself is equal to x q at n minus 2 plus d q at n minus 1. So, from this, we get x q n equal to x q n minus 2 plus d q n plus d q at n minus 1.

So, proceeding iteratively in this manner and assuming 0 initial condition that is x q at 0 is equal to 0, we can show that we can write x q n as summation of d q n sorry summation of d q m where m is equal to 1 up to n. So, what this shows that a receiver or demodulator is just an accumulator or adder. So, if the output d q is represented by impulses, then the accumulator at the receiver may be realized by an integrator because its output is going to be the sum of the strength of the input impulses some of the areas under the impulses.

So, in this case, we may also replace with an integrator the feedback portion of the modulator which is identical to the demodulator. Now, the demodulator output which is x q n is passed through a low pass filter and this yields the desired signal reconstructed from the quantized sample. So, whatever we do here the same thing is going to happen at this part of the modulator or the transmitter end.

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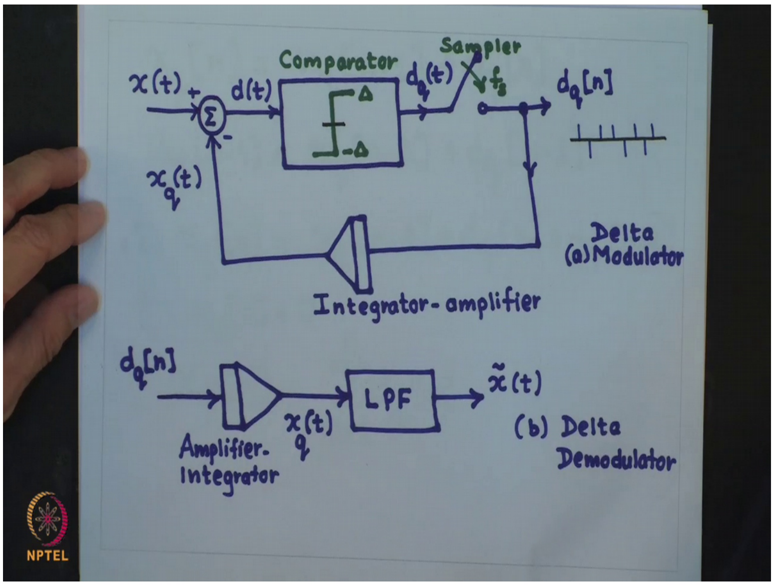

So, the next figure shows a practical implementation of a delta modulator and demodulator the first order predictor which was in the form of a delay is replaced by a low cost integrator circuit. It could be as simple as an RC integrator. So, the modulator now consists of a comparator and a sampler in the direct path and an integrator amplifier in the feedback path correct. So, this block diagram based on the discussion we have had is replaced by this block diagram correct.

Now, let us see the operation of this the analog signal x t is compared with the feedback signal x q which is a predicted signal. Now, the error signal d t is applied to the comparator. Now, if d t is positive the comparator output is a constant signal of amplitude plus delta. So, you will have a square wave the height will be plus delta and minus delta correct and if d t is negative the comparator output is minus delta. So, the difference out here is a binary signal with two levels that is needed to generate a one bit DPCM.

Now, the comparator output is sampled by a sampler at a rate of f s samples per second. Now this f s is typically much larger than the Nyquist rate required for sampling the signal x t. So, the sampler produces a train of narrow pulses which we call it as d q n with a positive pulse when x t is larger than x q t and negative pulse when x t is less than x q. So, this is the pulsed range which we get here is the delta modulated pulse tree. So, the modulated signal that is d q n is amplified and integrated in the feedback to generate x q t which tries to follow x t.

Now, to understand this in a much better way let us try to take a diagram out here. So, in this diagram we see that d q sequence at the input of the integrator gives rise to a step function positive or negative depending on the pulse polarity in x q t correct.

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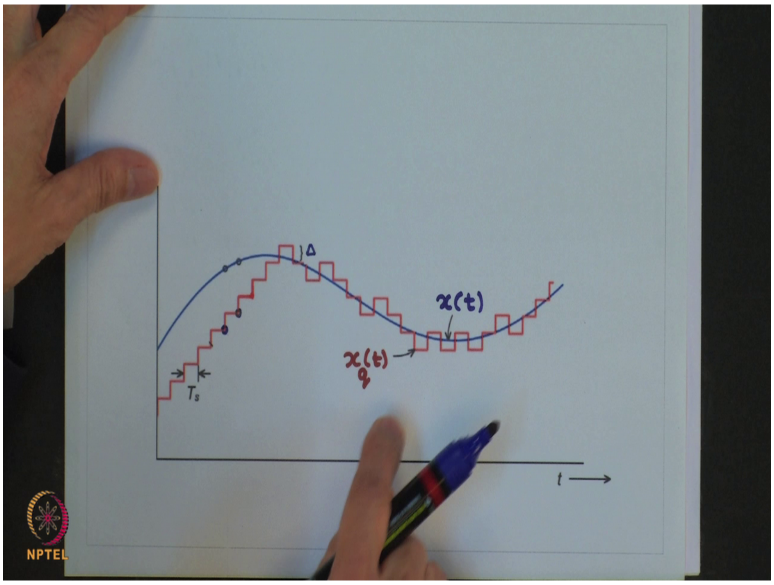

So, x q t; basically, there is a step rise or decrease fine ok.

So, if for example, if x t is larger than x q t, then a positive pulse is generated in this sequence d q n which gives rise to a positive step in x q t. So, this is what we get this is x q t and this is your signal x t and in the process it tries to equalize x q t in 2 x t, in small steps at every sampling instance as shown in this figure correct. So, it can be seen that x q t is a kind of staircase approximation of the signal x t when this x q t is passed through a low pass filter, then the coarseness of this staircase in x q t is eliminated and we get a smoother and better approximation to x t.

So, the demodulator at the receiver consists of an amplifier integrator identical to the feedback path of the modulator followed by a low pass filter. So, remember in PCM the analog signals are samp quantized in n levels whereas, in delta modulation the signal is quantized to just one bit correct and it is also important to understand that in delta modulation we are transmitting the difference between the successive pulse.

And since it carries the information about the derivative of the signal x t the name delta modulation comes. So, in PCM each quantized sample is n bit code word. Whereas, in delta modulation, the information of the difference between successive sample is transmitted by just one bit code word, there are two kinds of errors which could occur in delta modulation.

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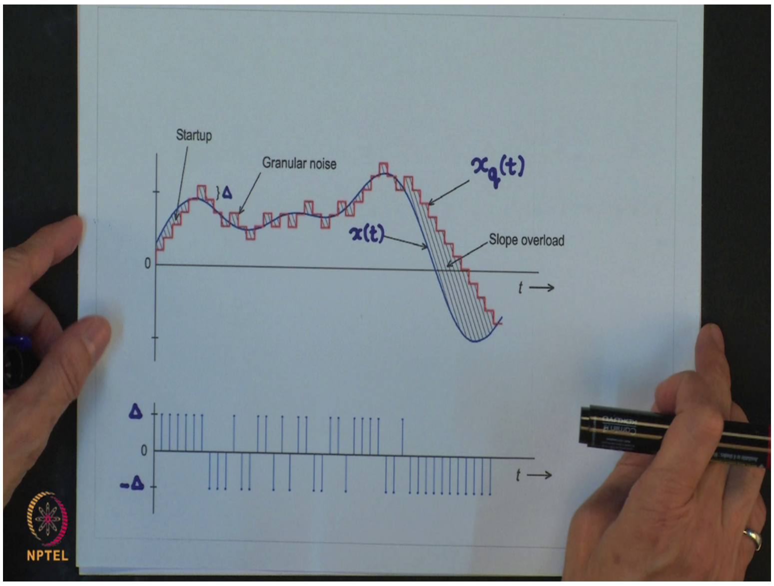

And these are known has granular noise and slope over load. So, let me show you this figure out here a little enlarge figure; what will happen is that when you input x t to the delta modulator s x q t, we start from the 0 value, it will take some time before it catches up the x t.

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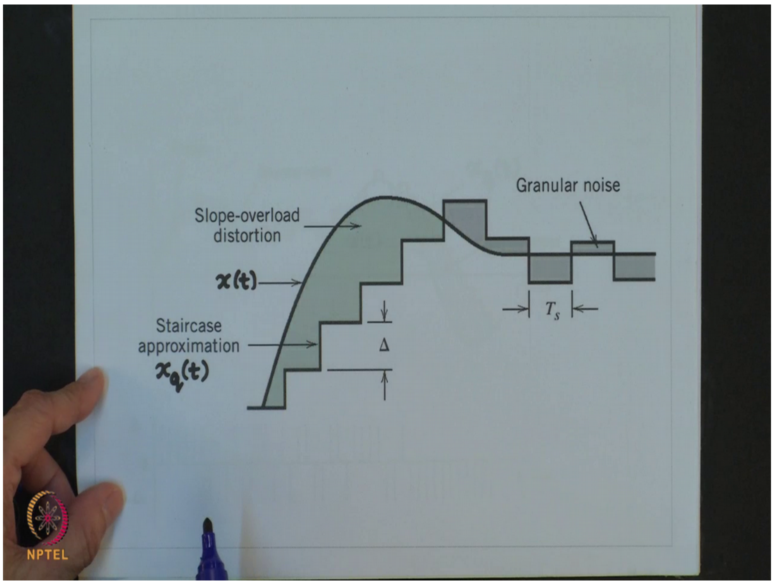

So, in the process you get x q t falls behind x t in this figure out here, for sufficient amount of time and this kind of noise which we get because of the difference between slope between the x t signal x q t signal is known as slope over load distortion where this x q t is almost following the signal x t that kind of noise which we get is known as granular noise.

So, here this is depicted. So, these are your d q sequences in the form of impulses correct and this is the granular noise when your x q t is following your signal x t the difference between x q t and x t is smaller than your step size delta. Whereas, in here; basically you see that the error has increased because x q t is not able to follow the signal x t.

So, let us try to understand the little in more in depth, first, we will talk about what is known as slope over load distortion and this is a major drawback of a delta modulator. So, let us consider case of a tone signal where my x T s of the form am cos 2 pi f m t.

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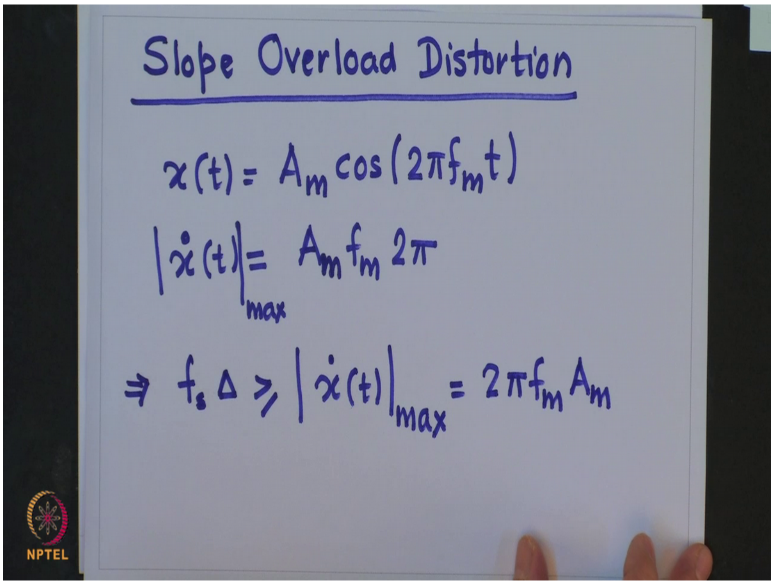

Now, we can find out the derivative of this signal is equal to maximum value of this derivative is going to be am f m multiplied by 2 pi. Now, if you want that there is no slope over load, in that case, the slope of the wake x q t grows should be higher than the slope of your input signal and the slope of x q t is this delta divided by sampling rate duration that is T s.

So, delta by T s which is f s delta is the slope of this x q t that has to be larger than the derivative of x t to avoid slope over load distortion. So, what this implies is that your f s delta should be larger than this quantity always and the maximum you can have is 2 pi fm a m for the tone signal correct.

So, let us assume now that we have chosen our delta and f s such a way that there is no slope over load. So, in that case. you will get only granular noise. Now let us try to evaluate the distortion due to the granular noise. So, let us assume that we have negligible slope over load then your different signal is x t minus x q t and that signal maximum the mod of that should be always less than equal to delta, if you want to avoid the slope over load.

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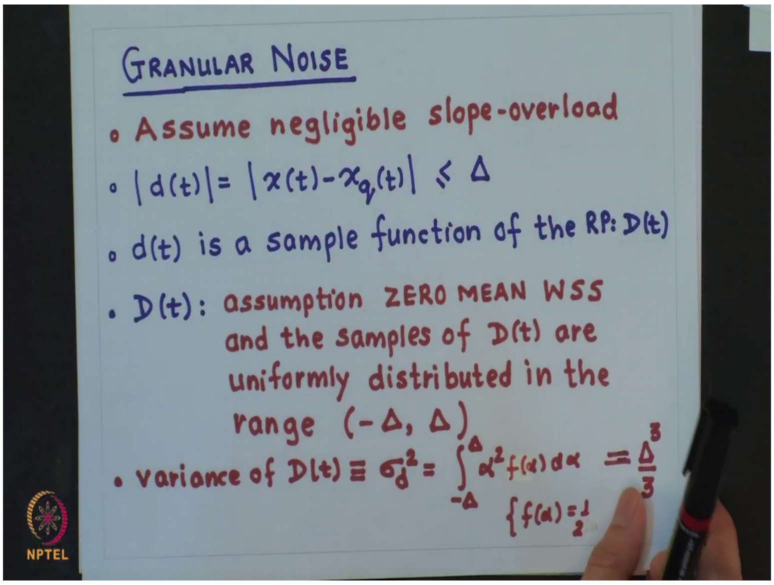

Now, let us assume that this different signal is the sample function of the random process which I call it as capital D t and we also assume that this random process is a 0 mean white sense stationary process and the samples of this process are uniformly distributed in the range between minus delta 2 plus delta given this, we can evaluate the variance of any sample of this sample function d t is simp is equal to this, we have seen earlier also is the integral given here and your step size in this case, basically becomes two delta for positive, it is plus delta for negative it is minus delta.

So, this becomes delta q by 3 and remember that because of the uniform distribution f alpha is equal to 1 by 2 delta fine ok.

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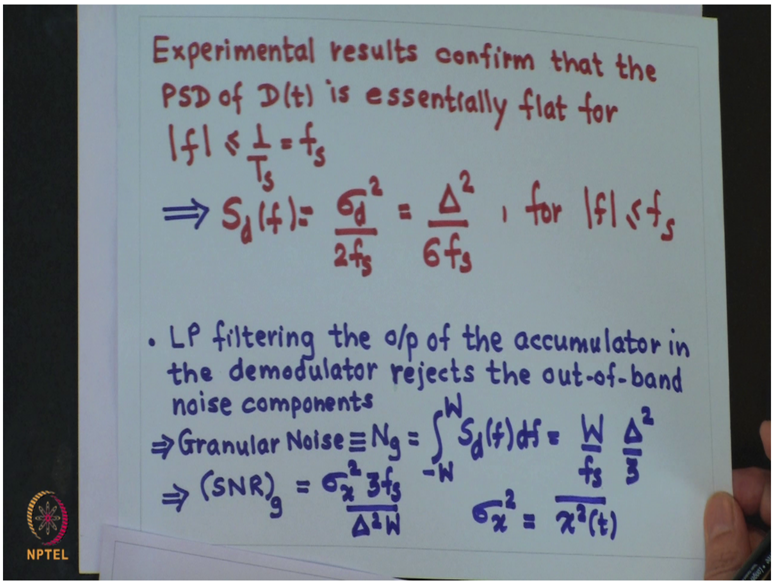

Now, experimental results have confirmed that the power spectral density of this different signal is essentially flat for this frequency range our x q t signal, here our pulses staircase approximation the form of pulses the duration of each is T s. So, the bandwidth of this signal is going to be approximately equal to fs. So, what does this imply that I can write my power spectral density to be equal to the variance which we have got divided by the bandwidth this is equal to delta squared by 6 f s for mod f less than equal to f s, correct.

Now, low pass filtering the output of the accumulator in the receiver will reject the out of band noise components. So, the granular noise which will get at the output of the low pass filter can be obtained by integrating the power spectral density over the bandwidth of the input signal which is W. So, this is equal to W by f s sigma squared by 3.

So, the signal to noise ratio due to the granular noise which I denote by this notation is nothing, but the signal power divided by the noise power. So, we get this equal to 3 f s by sigma squared W where sigma square x is the power in your input signal we also assumed that input signal is also 0 mean random process.

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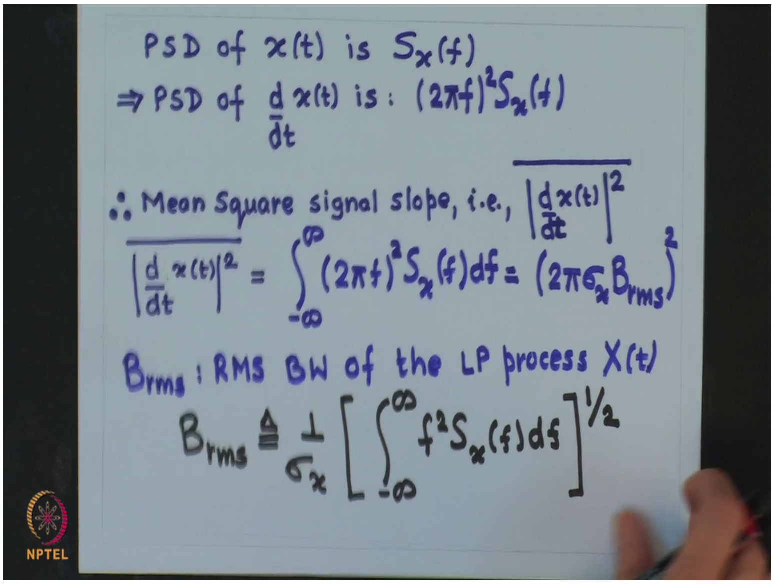

Now, once we know the power spectral density of the input signal, then we can find out the power spectral density of its derivative, it is very simple a derivative operator is nothing, but a linear system whose transfer function is 2 pi f.

So, if I have a random process input to the linear system, then the output power spectral density is equal to the input power spectral density multiplied by the magnitude response of the filter squared. So, in this case, your h f that is a filter corresponding to the derivative operator is 2 pi f. So, this will be equal to 2 pi f squared multiplied by S x f

So, the mean square signal slope which I denote by this quantity can be very easily written as power spectral density integration, you will get the power and this we will write it as 2 pi sigma x; B rms where B rms is the RMS bandwidth of the low pass process x t and it is defined as.

So, with this definition now we introduce the so called slope loading factor, we define another term which is slope loading factor which is nothing, but the maximum delta modulation slope which you can have is f s multiplied by delta and this is the RMS signal slope which we have evaluated here, correct.

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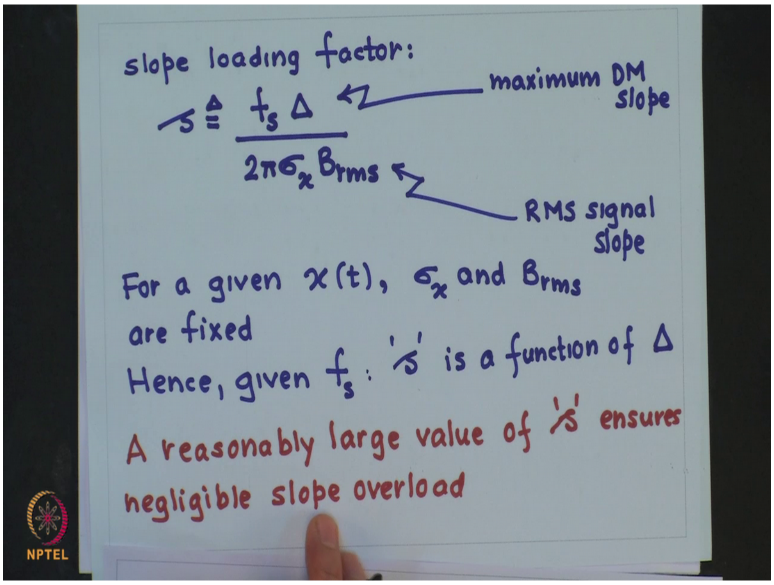

Now, for any given x t sigma x and B rms are fixed, these are the property of the signal this is the square root of the variance of the noise process x t and B rms is the RMS bandwidth of the low pass process x t. Hence, given the sampling frequency, this loading factor is a function of only delta. Now, it has been shown experimentally that a reasonably large value of loading factor ensures negligible slope over load.

Now, taking this into consideration from this relationship we try to express delta in terms of the loading factor. So, this is what I have done.

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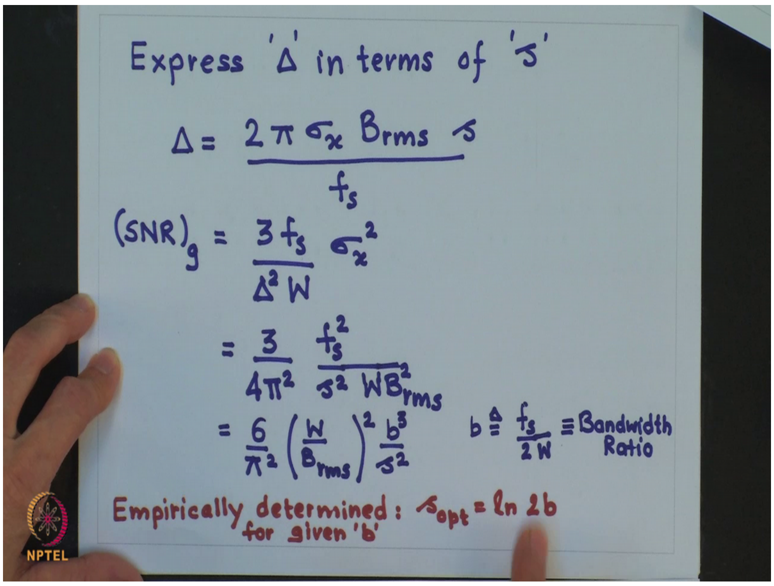

I expressed delta in terms of the loading factor and now let us recalculate the signal to noise ratio granular noise assuming there is no slope over load. So, if we do this basically we know that we had just shown that signal to noise ratio granule is equal to this quantity, I am going to just substitute for the value of delta obtained from this above eq equation, if I do this, I get this quantity out here and we define one more parameter, what is known as bandwidth ratio and I write this expression in terms of this bandwidth ratio is all this is simple algebraic manipulation nothing very difficult ok.

So, here this B is by definition the ratio of f s by 2 2 W and this is known as bandwidth ratio. So, sometime in the literature this thing is also known as the over sampling ratio now for a given b for that is bandwidth ratio empirically it has been determining that s optimum turns out to be lon 2. So, this all this information helps us to find out the output granule signal to noise ratio, we will get given the input signal given the sampling frequency and it also helps us to decide the value of delta from this s optimum, right.

Now, in PCM, the output of the quantizer is usually encoded as a binary code word and in general the bandwidth required for transmission of this PCM is quite high. So, to reduce this bandwidth there is another form of a PCM which is known as M-ary PCM are also known as M-ary pulse amplitude modulation and we could say this is a generalization of binary PCM and this we will study next time.

Thank you.