Principles of Digital Communications

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Lecture - 29
Companded Quantization - II

Companded Quantization is a non uniform quantization, which is achieved by passing the signal through a compressor followed by a uniform quantizer and an expander. A companded quantizer basically assigns, more number of quantization regions to lower amplitude of the signal which has a higher probability and less number of quantization regions to larger amplitude of the signal which has a lower probability.

We have derived the compressor characteristic which achieves a constant signal to quantization noise ratio and it was given by this expression.

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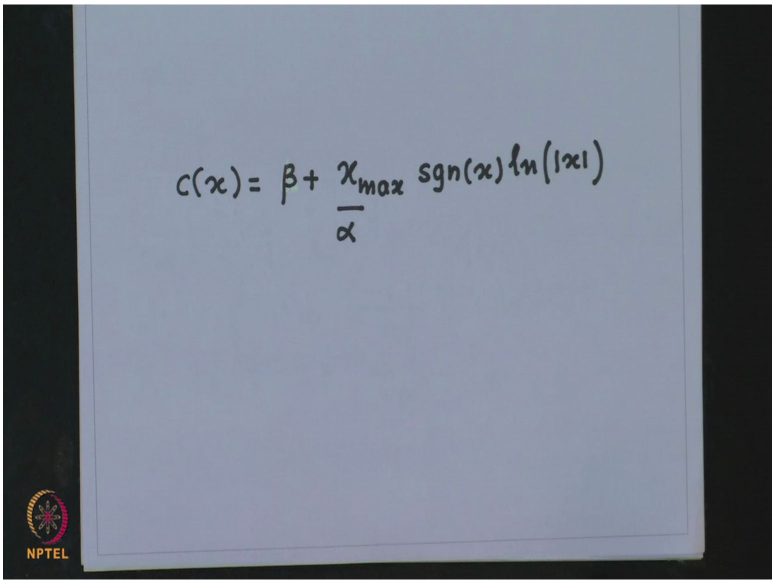

We said that this is difficult to implement in practice because, the value of C x for low values of x is very large and, the other reason is that it does not pass through the origin.

So, in practice this characteristic is approximated by a linear characteristic about the origin, followed by a logarithmic characteristic in practice. There are two popular compressor functions, which are used and this are known as mu law and A law.

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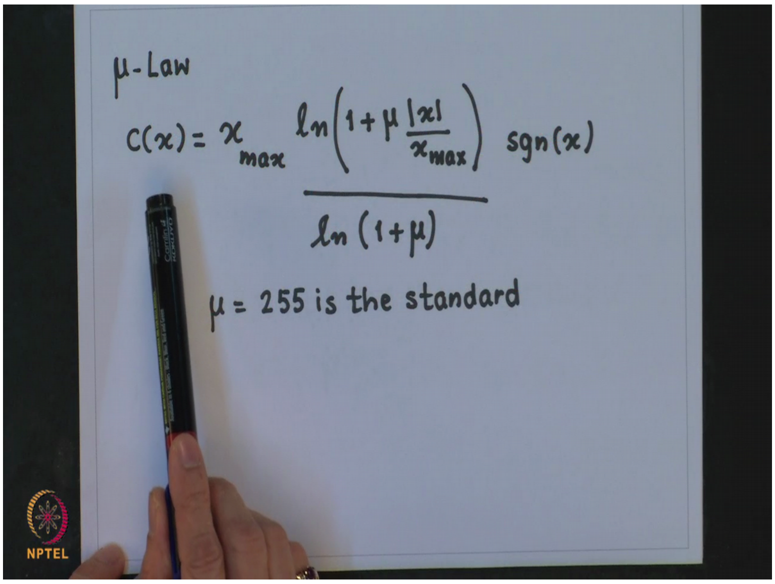

The characteristic for the mu law and this compressor characteristic is used in North America and Japan, the choice of mu equal to 255 is the standard, the expander characteristic which is the inverse of the compressor characteristic for the mu law is also shown on this slide here.

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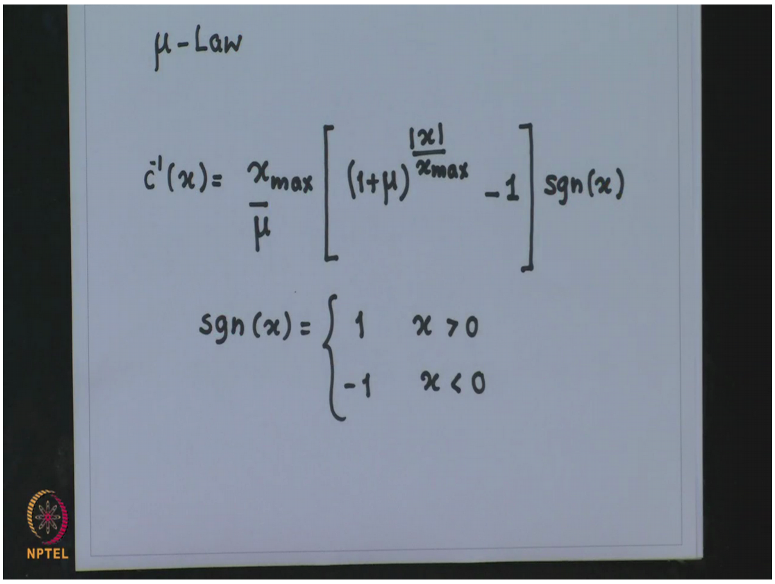

The other characteristics which is used other than this two countries is A laws.

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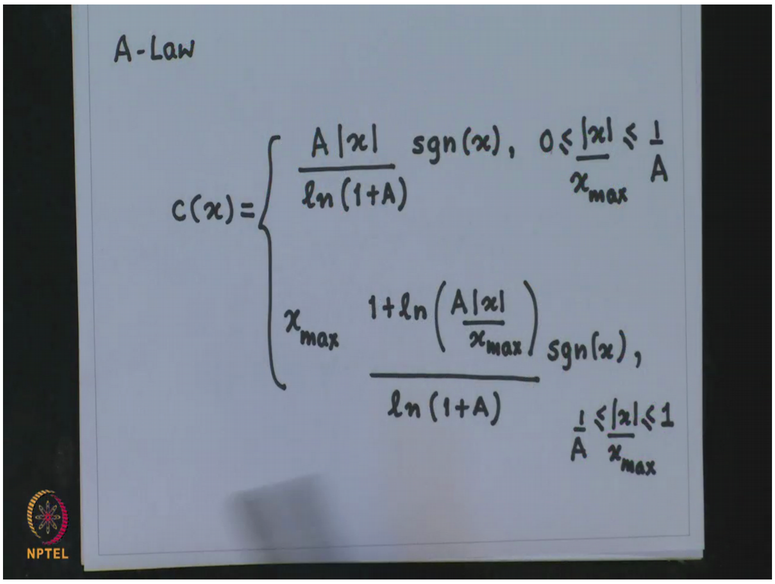

So, this is the compressor characteristic and this is the expander characteristic.

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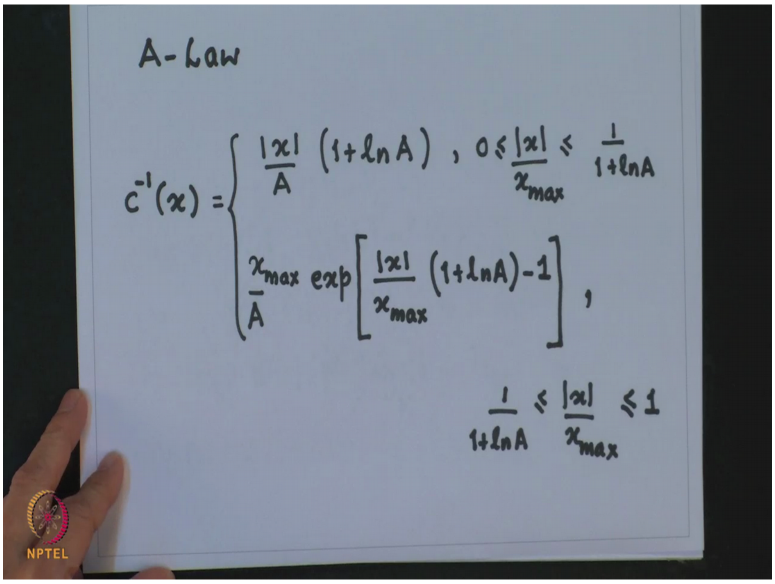

And both this laws when plotted they look as shown on this slide.

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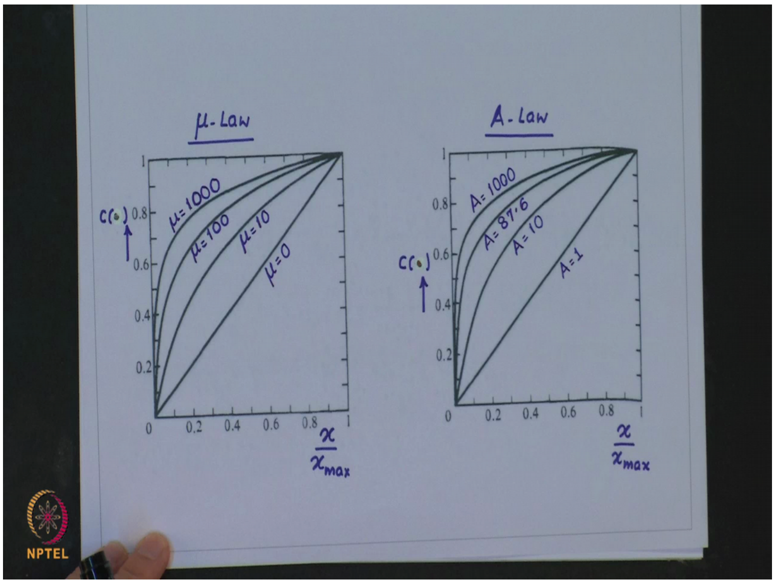

So, mu equal to 0 corresponds to no compounding and as mu keeps on increasing, we see that the slope about the 0 that is the origin also increases. Both these characteristic more or less look very similar form mu law mu equal to 255 is popular and, for A law A equal to 87.6 is quite popular. And in practice mu law is approximated by linear segments for implementation.

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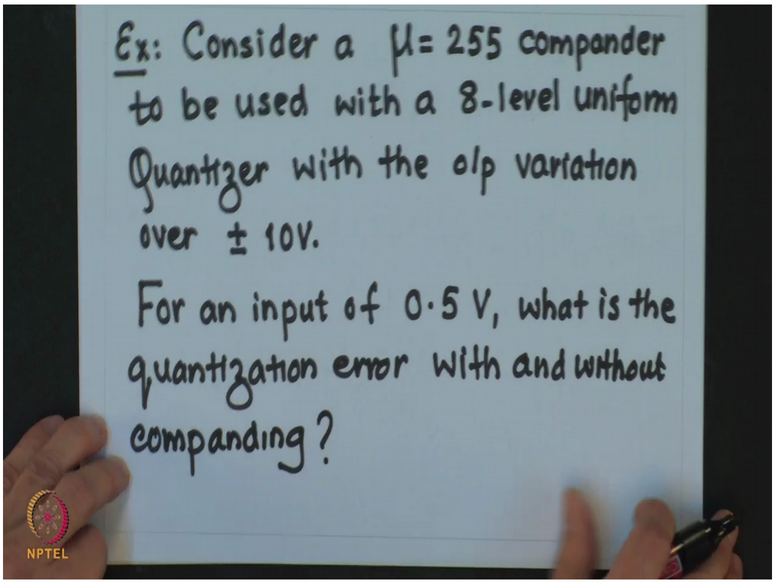

Let us take mu law and see how do we get an advantage, we will take one example. So, let us consider mu equal to 255 compander, compressor plus expander is known as compander. We use this compander with a 8 level uniform quantizer, with the output variation over plus minus 10 volts. Now, consider an input of 0.5 volt and let us evaluate the quantization error with and without companding?

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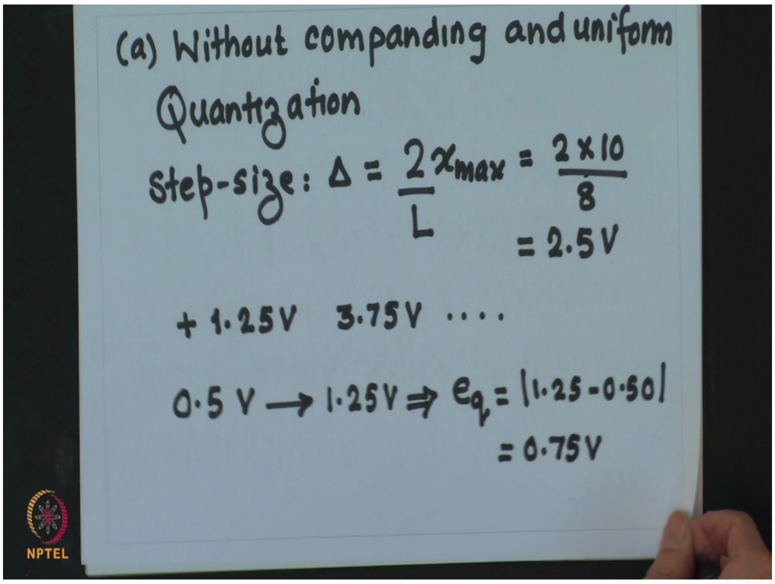

So, first we will solve it without companding and uniform quantization, step size delta in this case would be equal to twice x max, divided by the number of levels which in our case is 2 multiplied by 10 divided by 8, which turns out to be 2.5 volts.

So, the first reconstruction level on the positive side is going to be plus 1.25 volts. Then next would be 3.15 volts and so on. Now, 0.5 volts lies between 0 to 2 0.5 volts. So, this is going to be quantized to the reconstruction value of 1.25 volts, which implies that the quantization error is going to be equal to 0.75 volts ok.

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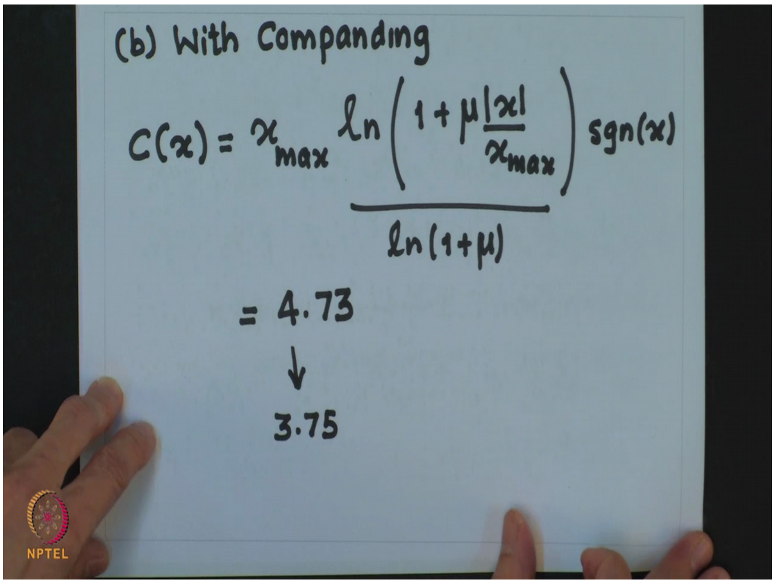

Now, let us consider the same input to a compander. So, we will use the mu law characteristic for the compressor, our x is equal to 0.5, if you plug in that value here, in this equation, you will get it equal to 4.73 and this is going to be fed to the uniform quantizer and this value lies between 2.5 and 5 and the reconstruction level in that interval is equal to 3.75. So, this is the output of the uniform quantizer.

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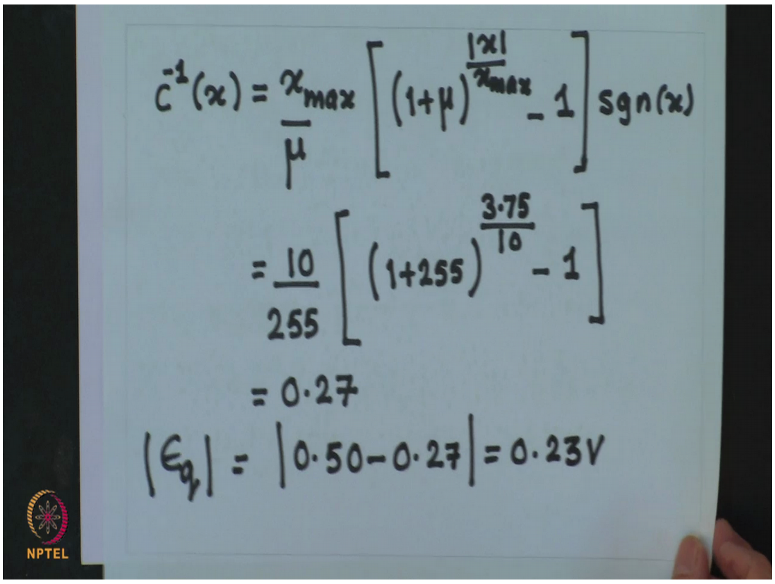

And this output of the uniform quantizer will be fed to the expander and it is characteristic for the mu law is as follows. So, this is equal to this expression out here our case x is 3.75 which is the output of the uniform quantizer, this is equal to 0.27. So, the quantization error which we get from here is equal to 0.23 volts, which is substantially less than we got without the companing ok.

Now, the next question is can we evaluate the signal to quantization noise ratio say for mu law. Let us do that we will substitute the mu law function in the binit integral, which we have derived earlier.

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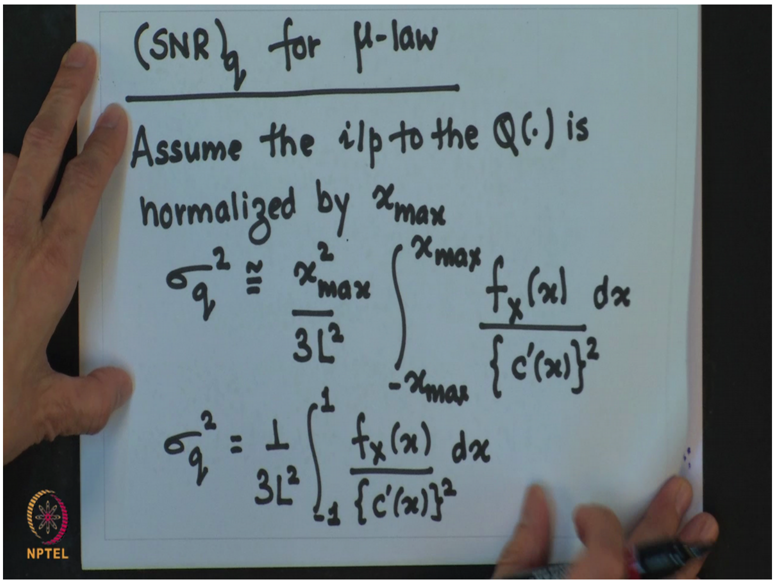

So, we are interested in the calculation of signal to quantization noise ratio for mu law. So, we assume the input to the quantizer is normalized by x max, this is done without loss of generality, it will help us to simplify our calculations ok.

We have also evaluated the quantization noise in general to be approximately equal to this value remember, C dash x is the derivative of the compressor characteristic C x. So, since we are assuming it to be input I repeat since we are assuming our input to be normalized by x max, the expression for quantization noise in our case will reduce to this form.

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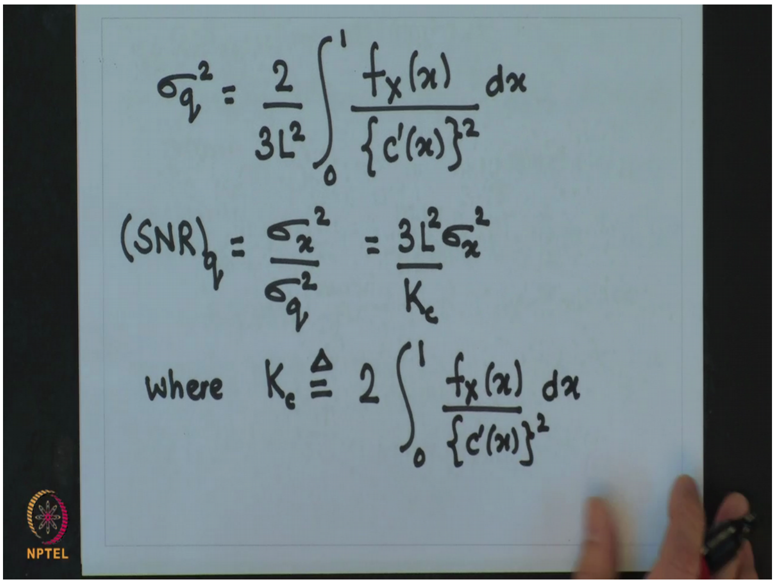

Now let us assume the symmetry both for the pdf and the C dash x in which case I can write my quantization noise to be equal to twice by this quantity. So, I have replace the integral between minus 1 to 1 by 0 to 1 and multiplied by 2 ok.

Now, the signal to quantization noise ratio would be equal to in this case, where I define my K c to be twice the integral of this quantity.

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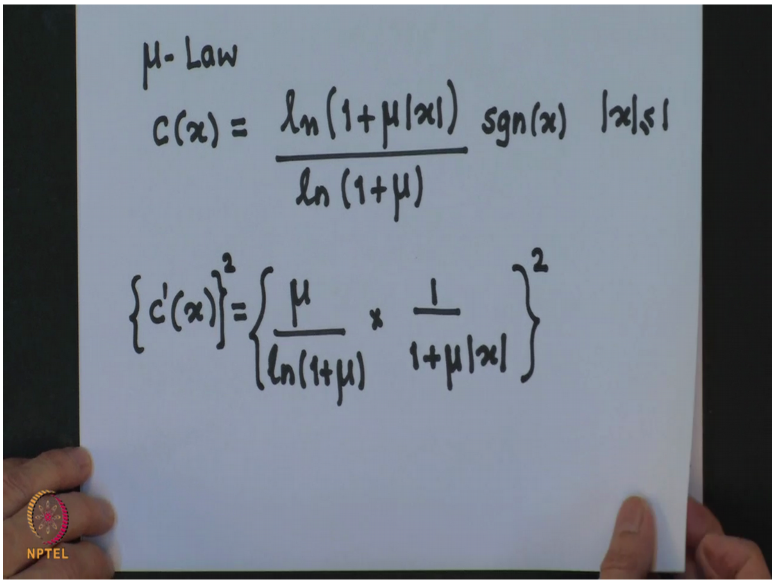

We will use mu law, for which the compressor characteristic for the normalized input would be given by this expression. Now, we need to calculate C dash x derivative of this which is simple to do this. This is equal to this expression fine. And now we will use this expression so, square of this we require it.

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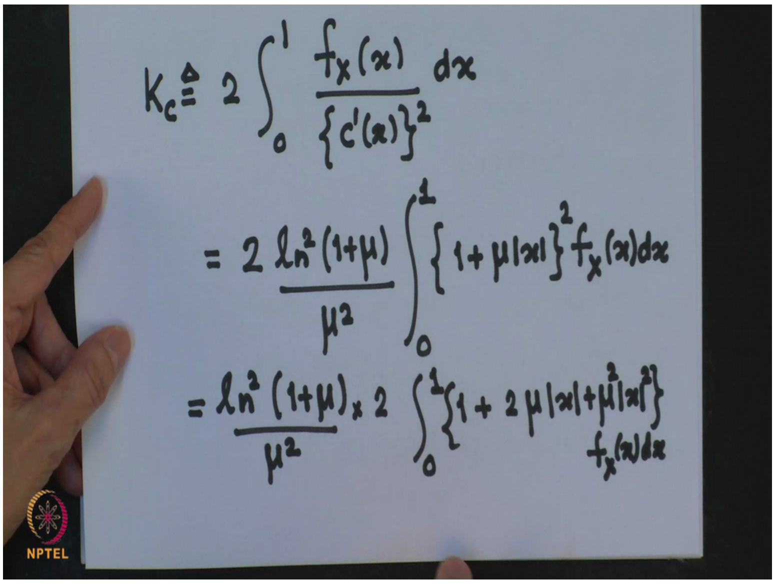

So, the value of K c, which is equal to by definition twice this quantity can be evaluated, this would be equal to twice ok.

Now, this is simple to evaluate I can write this as logs square of 1 plus mu by mu squared, twice integral 0 to 1, we can expand this we can expand this let us expand this ok. And now we can evaluate this twice the integral from 0 to 1 of f X dx is going to be 1, then we will get the mean value of mod x and we will get the mean square value of x ok.

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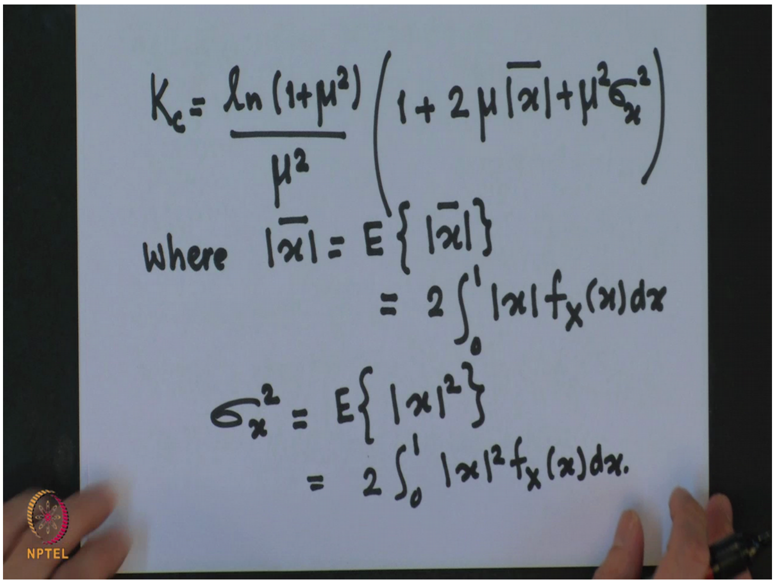

So, from here I can write this K c to be equal to log of 1 plus mu square by mu squared 1 plus 2 mu plus mu squared, where is equal to expectation of this is nothing, but twice and your signal power is expectation of and this is equal to twice 0 to 1 fine ok.

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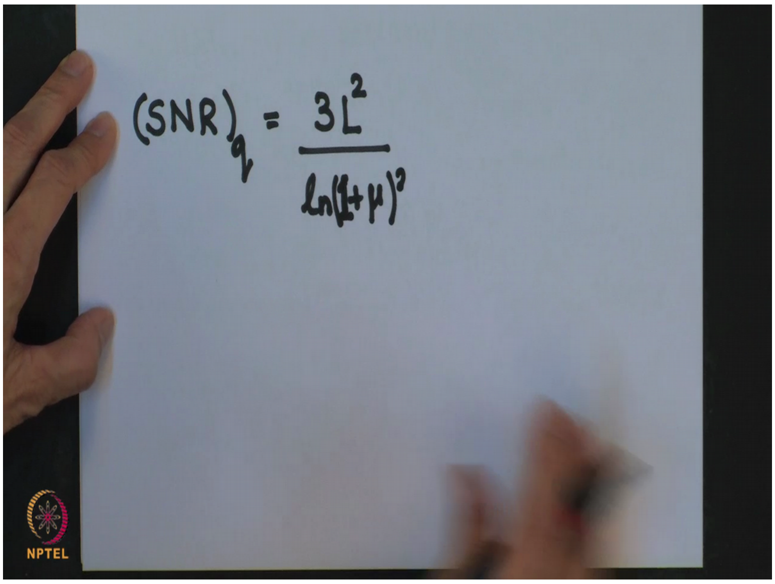

So, once we have this evaluation we can get the signal to quantization noise ratio. This is equal to 3 L squared by log of 1 plus mu.

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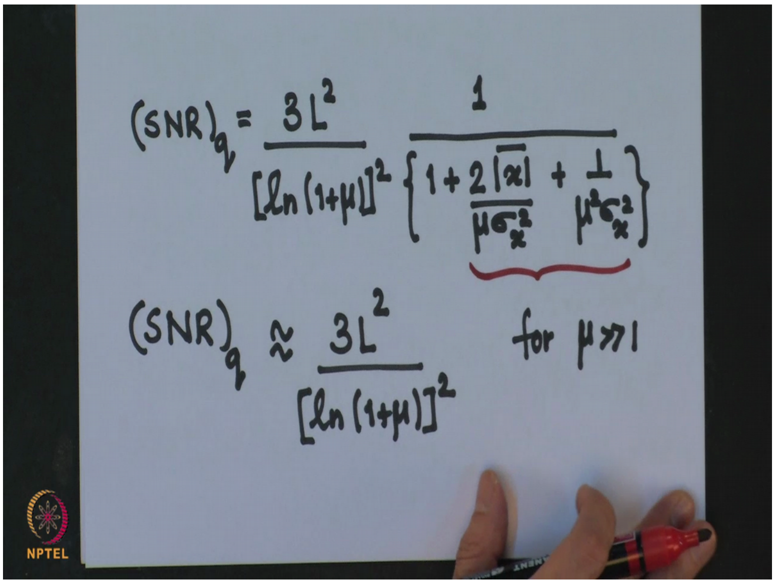

So, we can get the signal to quantization noise ratio equal to 3 L squared by log of 1 plus mu whole squared, divided by 1 plus twice, we have just plugged in the quantities which we have related in the signal to quantization noise ratios formula.

Now, from here we can see that for large value of mu this two terms can be neglected. So, signal to quantization noise ratio, would become approximately equal to 3 L squared upon log of 1 plus mu whole square correct, for large mu correct this is a constant. So, for large mu the robustness improves by deemphasizing these terms, but large mu also reduces the signal to quantization noise ratio level that can be achieved. So, if this is plotted for different value of signal power, we get the graph as follows.

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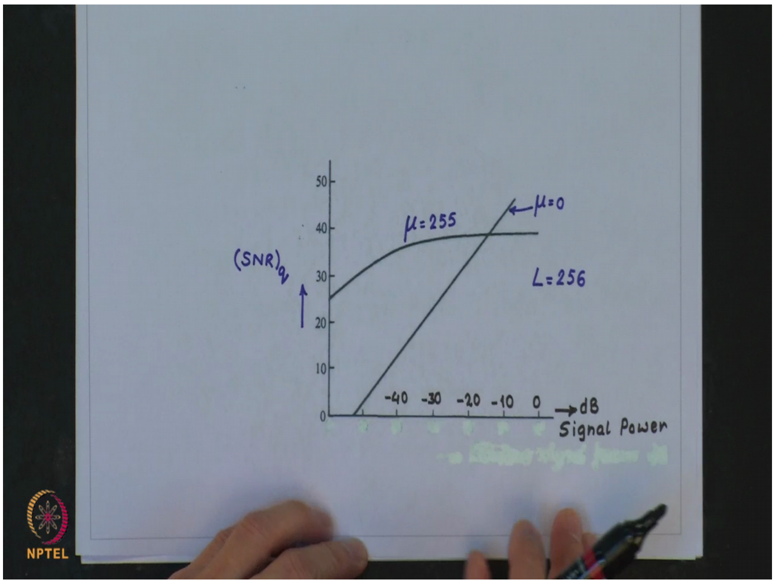

So, this graph has been plotted from mu equal to 0 and, this is for mu equal to 255. And this curve it is more or less valid both for Gaussian pdf and the Laplacian pdf. So, we see that the variation in the signal to quantization noise ratio is quite minimum, with the variation in the signal power. So, in this sense your companding provides robustness, against the mismatch due to the wrong choice of pdf, or due to the wrong choice of signal power, or we could also consider variation in the signal power.

Now, in our study so far, we have carried out the quantization of a sample independent of other samples, which is a form of scalar quantization. Now, in real signals high degree of correlation exists amongst the samples of the input signal. And this correlation can be exploited in a beneficial manner and, this is being done by one form of PCM known as differential pulse code modulation, also known as DPCM and this we will study next time.

Thank you.