Principles of Digital Communications
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Lecture – 25
Uniform Quantizer

Next we study a very simple and popular quantizer and that is Uniform Quantizer.

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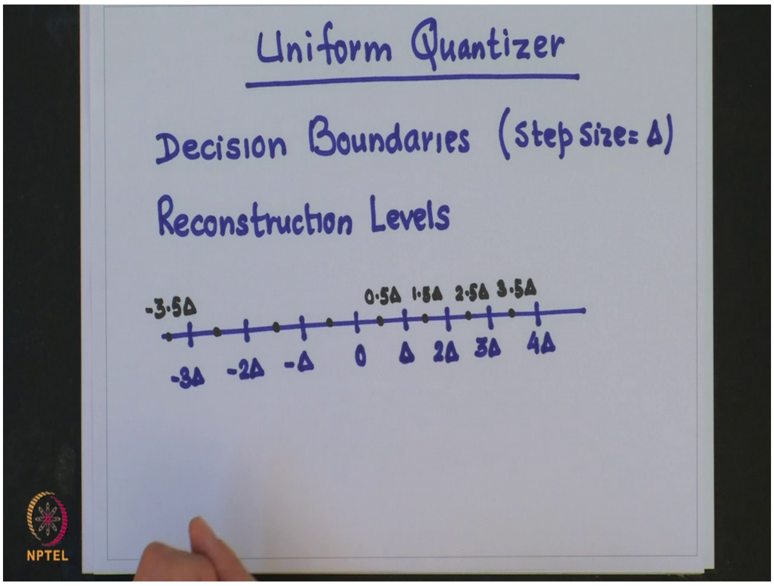

Constraining to uniform quantization makes the design easier, but performance will usually suffer. So, for a uniform quantizer the following two constraints are imposed. The first one is that decision boundaries are equally spaced, that is your step size is equal to delta which remains same for all the intervals, except possibly for the outer 2 intervals, we will see this later on. And then another condition is that the reconstruction levels are also equally spaced and these are centered between the decision boundaries.

So, pictorially this will look something like this, we have the intervals which are all equally spaced. So, let us denote this as say 0th point, the next boundary from here would be equal to delta, the second one interval would be 2 delta, 3 delta, 4 delta and similarly you will have minus delta, minus 2 delta, minus 3 delta ok.

And the next is basically we said that our reconstruction levels are equally spaced and centered between the decision boundaries. So, these are the centers. So, this will be 0.5

delta, 1.5 delta, 2.5 delta, 3.5 delta. So, similarly you can write here also minus 0.5 delta, minus 1.5 delta, minus 2.5 delta and so on and here basically this field also, if you want we can indicate as minus 3.5 delta. Now, there are two types of quantizers uniform quantizer and these are known as midrise quantizer.

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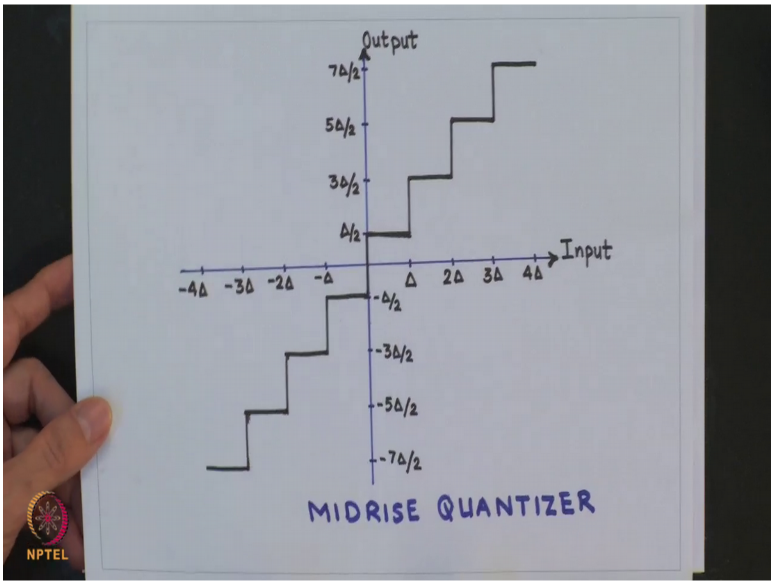

So, this is the quantizer input output mapping is shown in this figure. So, you see basically that these are the intervals and the outputs are the midpoints of this intervals, which are equally spaced, but here 0 is not one of the construction level. Now, for some applications we require 0 to be one of the reconstruction level for example, while quantizing speed signals.

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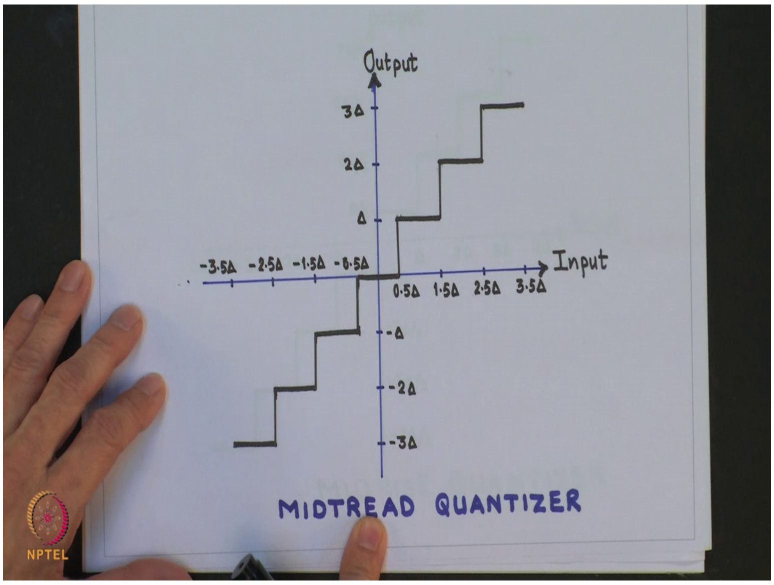

So, in that case basically we can design another form of quantizer which is shown in this figure. So, for the interval between minus 0.5 delta to plus 0.5 delta, the reconstruction level is 0 correct. So, this is known as mid tread quantizer where 0 is one of the reconstruction level; whereas, this is known as midrise quantizer where 0 is not one of the reconstruction level.

So, in this case midrise quantizer will have even number of reconstruction levels, correct. Whereas, in mid tread quantizer you will have odd number of levels correct. So, for our discussion we will presume that we are designing midrise type of quantizer. So, the analysis will become little easier, but the same analysis can be easily extended to the mid tread quatizer.

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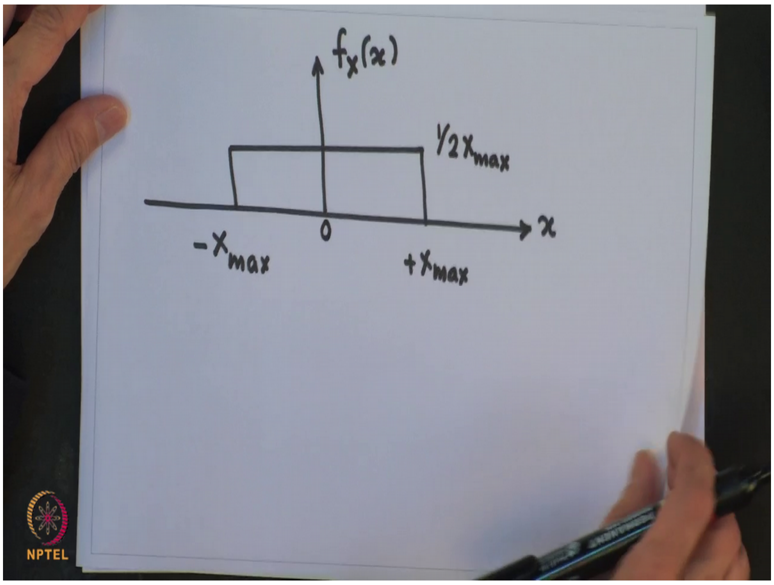

So, now let us design an l level uniform quantizer for an input that has uniform PDF in the interval between minus X max to plus X max. So, in this case we can write down the mean square quantization error as follows.

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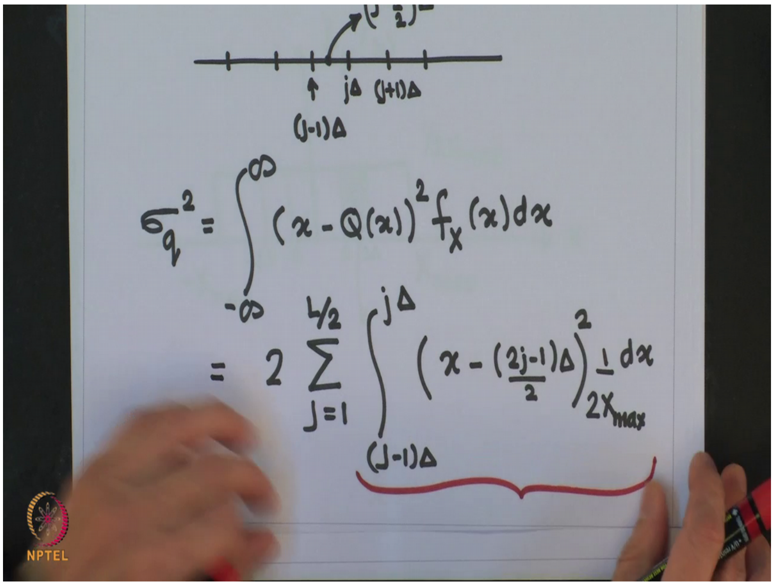

So, this will be the intervals we will have if I call this boundary point has j delta, then this would be j minus 1 delta and in this case the reconstruction value would be equal to, so, let us take this otherwise j plus 1 delta correct. So, here this would be equal to j minus half delta would be the reconstruction value here correct ok.

So, given this we can easily calculate the mean square quantization error as follows, this would be equal to we are assuming midrise type of a quantizer. So, the number of levels will be even, L will be even. The boundaries would be given by these two values for a j minus 1 interval. The reconstructed value would be this so, that is nothing, but 2 j minus 1; this would be the error the PDF of this would be given by this. So, this is 1 by 2 X max and dx correct and we have l intervals. So, this is the distortion which we get.

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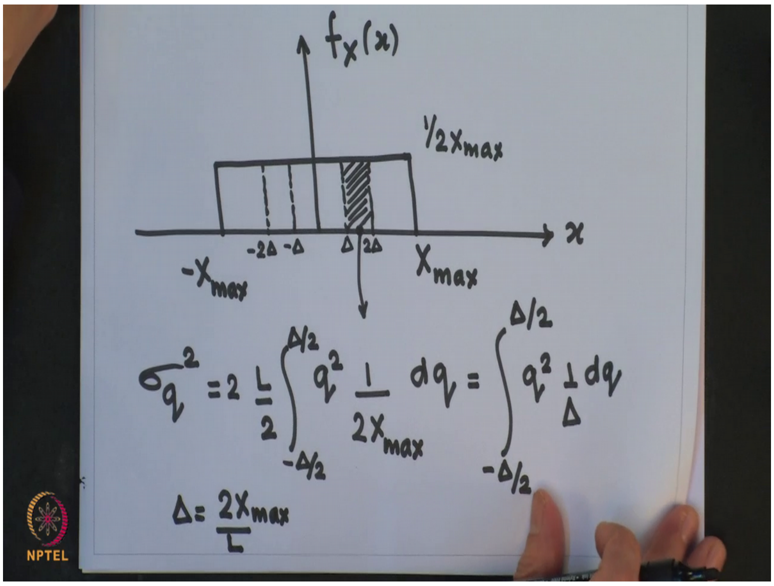

Now, it is important to know that when we carry out this quantization what will happen is depicted in this figure, this is X max minus X max and you have so, you have this delta 2 delta so on here, minus delta, minus 2 delta. So, in a particular interval this is the reconstruction value will have.

So, now it is easy to realize that this integral is identical for all the intervals. So, this is the error which you get and, that is going to be the same for every interval. So, in that case I can write my so, I can write mean square quantization noise as q denotes the quantization error in a particular interval.

So, for calculation I can even consider minus delta by 2 to plus delta by 2, it is not going to make a difference. So, if I do this, then this would be the quantization noise in that interval between minus delta by 2, 2 plus delta by 2. So, this will be the quantization noise in any of the interval and there are L by 2 on this side. So, multiplied by L by 2 and there is L by 2 on this side also so, multiply this by 2 correct.

So, this is equal to now remember, we have a uniform quantizer. So, your delta is going to be equal to 2 X max divided by the number of levels L. So, I substitute here this is delta.

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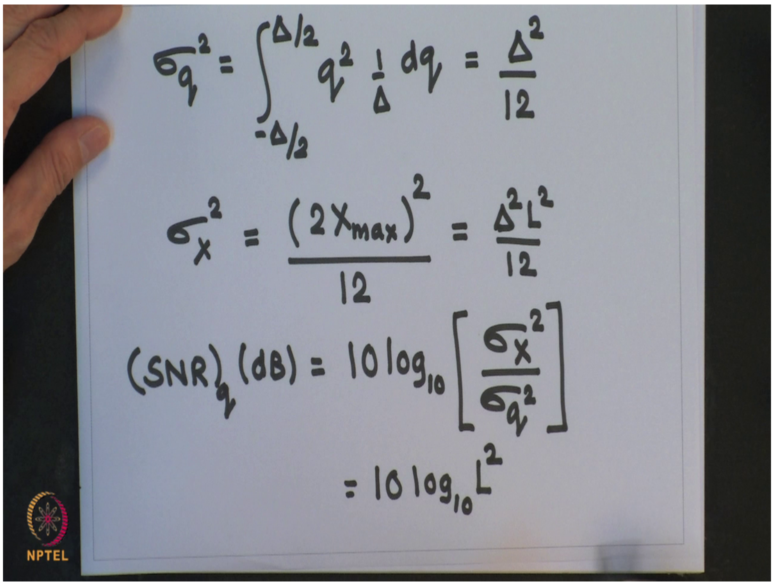

So, now, the result is from this I get equals delta square by 12 correct. So, this is the quantization noise. Now, to get the signal to noise ratio from quantization, we need to find the variance of power of the signal. Now, since the signal is uniformly distributed, we know from the probability theory, that variance of the signal would be equal to this quantity, we know that X max will be equal to delta L by 2, so, if I substitute this I get delta squared L square by 12.

And then the Signal to Noise Ratio when I say noise it is a quantization noise. So, this denotes this, this in dB is equal to 10 log to the base 10 of signal power divided by the noise power and if I just substitute here I get 10 log to the base 10 of L square. Now, if we assume that we are doing binary encoding your, L will be equal to 2 raise to n.

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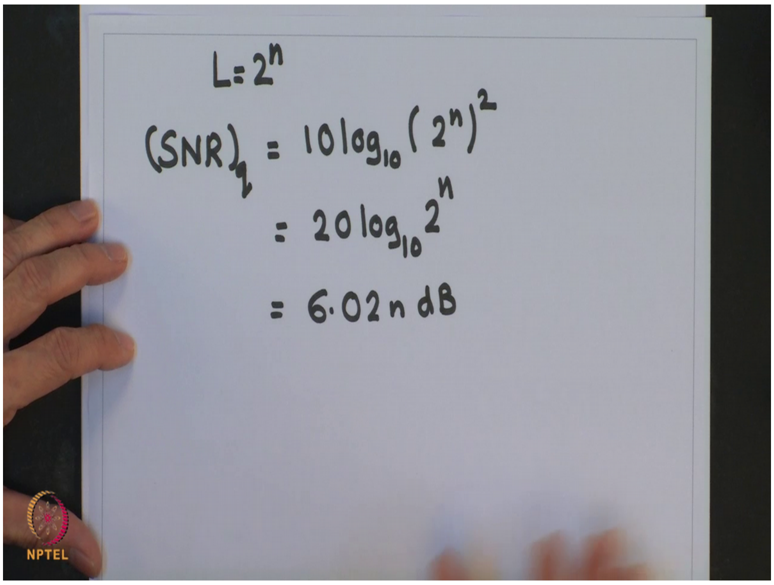

So, if I substitute L is equal to 2 raised to n, I get signal to quantization noise ratio to be equal to which is equal to 20 log. So, it is important to know that for every additional bit, I get about 6 dB per bit improvement in signal to noise ratio. So, we can also plot the distortion versus rate curve for uniform quantizer of a uniform random variable as follows.

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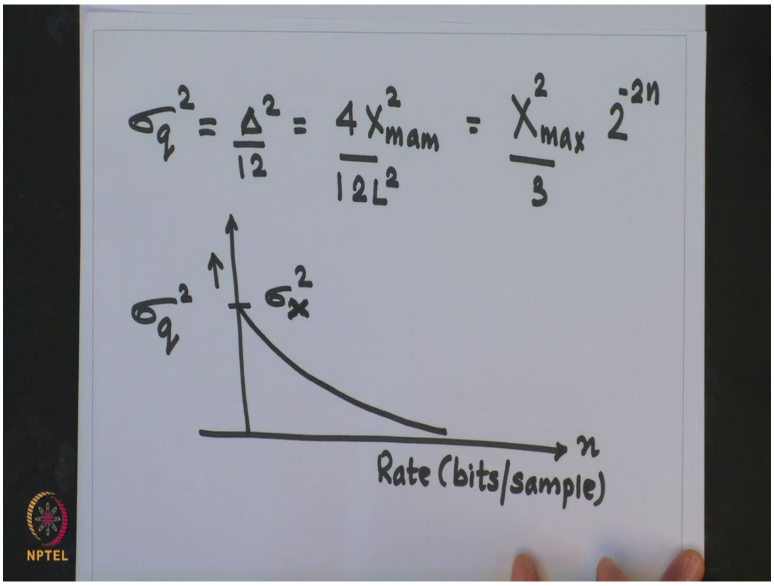

This is equal to we just showed this is equal to this so, this is nothing, but 4. So, in this n denotes the rate, this denotes the distortion. So, if we plot it rate that is bits per sample which we use for quantization and this is on access the distortion, then this will be something like this. This value for this will be at this point when n is equal to 0, will be the variance of the signal itself ok.

So, we have studied uniform quantizer quantization for the case of a source which has uniform PDF, but quite often the sources which we deal in practical scenario do not have a uniform PDF; however, we still desire the simplicity of a uniform quantizer. So, in this case even if the sources are bounded simply dividing the input range by the number of quantizer levels does not produce a very good design. To understand this let us take one example.

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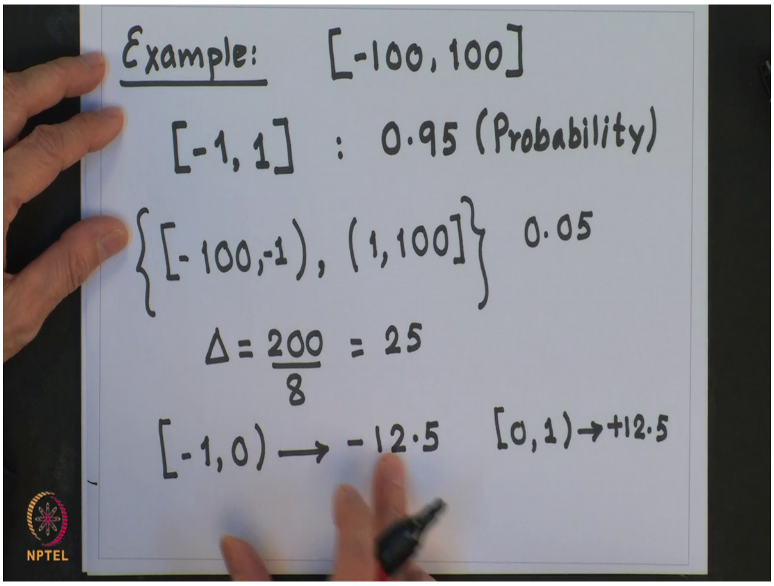

Assume input range between minus 100 to plus 100 and also assume that input falls in the interval between minus 1 to plus 1 with probability given by 0.95. And it falls in the interval between minus 0 to open interval 1 and so in this between minus 100 to less than minus 1 and 1 to 100 the probability will be obviously, equal to 0.05 and we are required to design an 8th level uniform quatizer.

Now, if you follow the procedure which we discussed earlier and if we take minus X max to be minus 100 and plus X max to be 100 and adopt the procedure which we did for the uniform PDF, then delta I would get it as 200 by 8, which is equal to 25.

So, now when you have in the input in the interval between minus 1 and 0, then this will be quantized to the value minus 12.5, because the spacing interval spacing step size is

25. So, the midpoint of the interval is 12.5. So, on this side between minus 1 to 0 it will fall in that first interval on the left hand side. And similarly for 0 to 1 values these are going to be quantized to plus 12.5.

So, the maximum quantization error that can incurred when it lies in any of this intervals, it will be 12.5. However, at least 95 percent of the time the minimum error that will be incurred is 11.5 correct.

So, the difference between these 2 or difference between this and this; obviously, this is not a very good design. So, what would be the better approach? The better approach would be to use smaller step size, for those region where the probability of occurrence of the signal is high correct.

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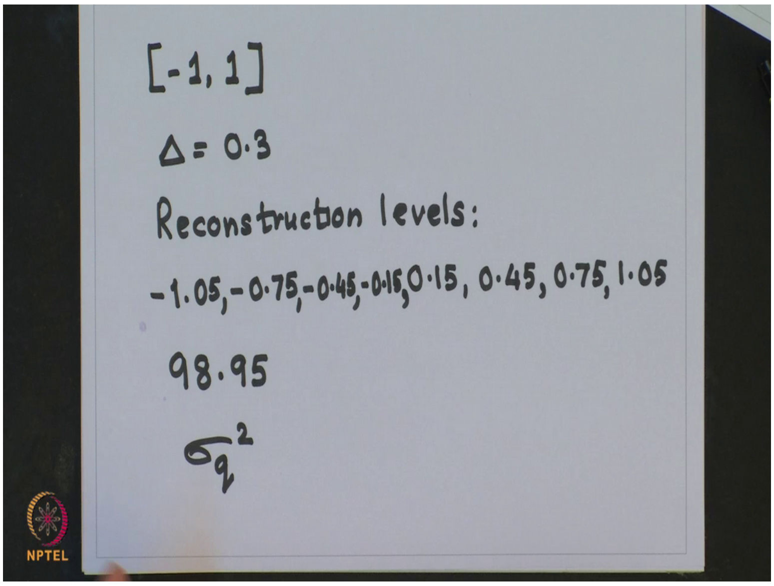

So, better in presentation of the values in the range between minus 1 to plus 1 interval is desired, even if it is meant a larger maximum error. So, let us say we use a step size of 0.3. So, if you use a step size of 0.3 and if you again design a uniform quantizer, our reconstruction levels would be as follows. So, you will have from 0 to 0.3 first, the 0 will be 1 boundary the next will be 0.3, next would be 0.6 and like that. So, the reconstruction level will be the midpoint of that. So, you will have on the positive side 0.15, you will have the next reconstruction level space by 0.3. So, 0.45 you will have 0.75 and you will have 1.05 and on the negative side you will have, similarly minus 1.05, minus 0.75, minus 0.45 and minus 0.15.

So, in this case the maximum quantization error will be; obviously, equal to 98.95. The maximum error will get quantization error will be 98.95 and, this will happen when we have the input value to be say equal to 100 and, it is going to get quantized to 1.05 or you have a value minus 100 and it will get quantized to the reconstruction level which is minus 1.05. But; however, 95 percent of the quantization error will be less than 0.5 correct because, most of the time our signal occurs in this interval. Therefore, in this case the mean square quantization error will be substantially less.

So, conclusion is that if source PDF is not uniform, it is not a good idea to obtain the step size by simply dividing the input range, by the required number of L levels. Approach is totally impractical when a source is model which say, unbounded PDF correct, say example Gaussian PDF. So, in that case we must include the PDF or the source in the design process and we will study this in the next class.

Thank you.