Principles of Digital Communications
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Lecture - 19
Basics of Signal Detection : ML, MAP Detectors

We have studied that an Additive White Gaussian Noise continuous channel is equivalent to N dimensional Additive White Gaussian Noise vector channel.

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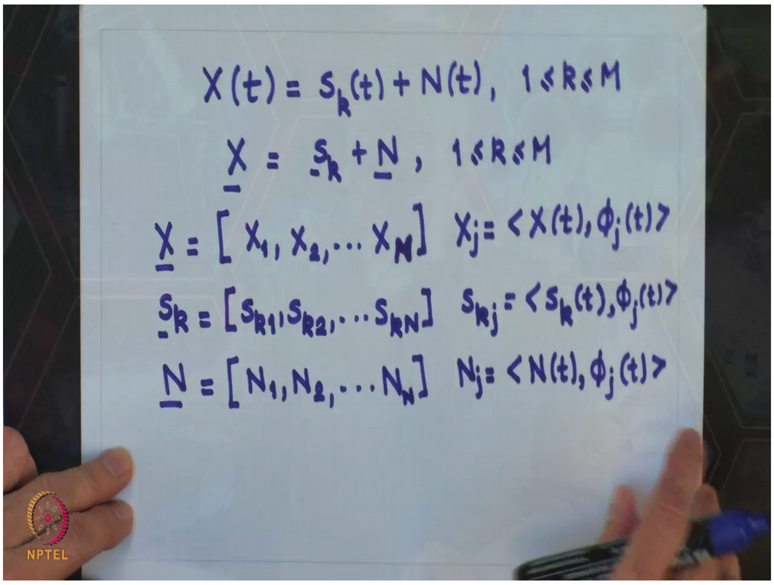

So, your continuous channel of the form shown here is equivalent to N dimensional vector channel; where, your vector X is equal to. These are the random variables. Any random variable here is a projection of X t on to phi j t. S k is S k 1, S k 2 up to S k N; where, S k j is a projection of the message signal S k t or phi j t.

And this is your noise vector. Any element out here is equal to projection of the noise over the basis signal phi j t. So, having done this a problem is now considerably simplified.

Now, this random vector N is represented by N independent Random Gaussian Variables.

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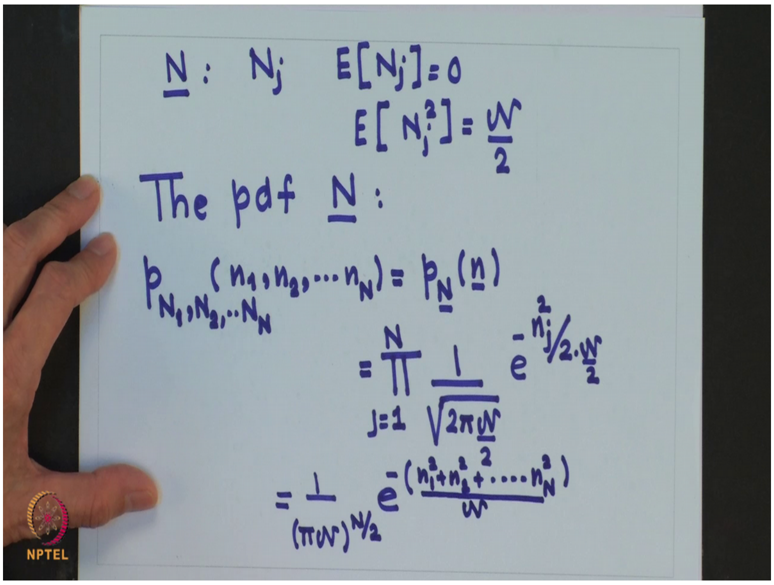

Each of this is basically with 0 mean and the variance of this is italic N by 2 which is the power spectral density of the White Gaussian Noise ok.

So, the p d f probability density function of vector N can be easily written in this case as follows. This is how we write the joint probability density function for the vector which in a simplified manner can be written as. This is equal to each of this random variable is a Gaussian p d f with the variance italic N by 2 e raised to minus N j square 2.

So, independence implies that I can take the product of it; the pdf of the individual random variables ok. So, this can be simplified as which can be rewritten as.

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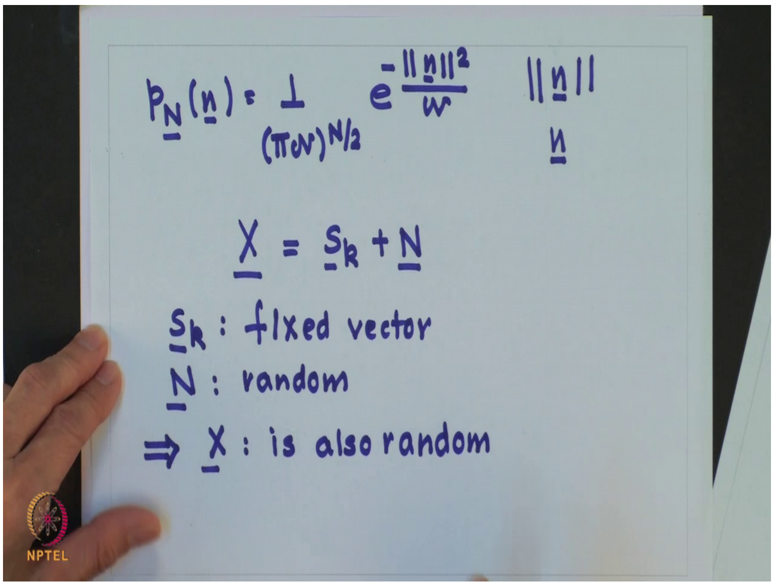

So, norm square of the noise vector. So, this shows that the pdf of the noise vector depends on the norm of the noise vector, which is the sampled length of the noise vector in the hyper space. And therefore, symmetrical if plotted in the N dimensional hyper space fine ok.

So, in this model, the irrelevant noise component has been filtered out. Because remember, this vector is basically consisting of the elements which are projection of the noise process N t on to the N dimensional space ok.

Now, this signal will vector S k is of X vector because the wave form S k t corresponding to a message m k is non-random correct. So, your vector N that is noise vector is random. So, what this implies that your vector X is also a random.

Now, because this vector N has spherical symmetry in the signal space the distribution of X is also a symmetrical distribution centered at a fixed point given by this vector S k. So, what it happens is in the hyper space is it forms some kind of a specter region it I;s not possible for me to draw N dimension.

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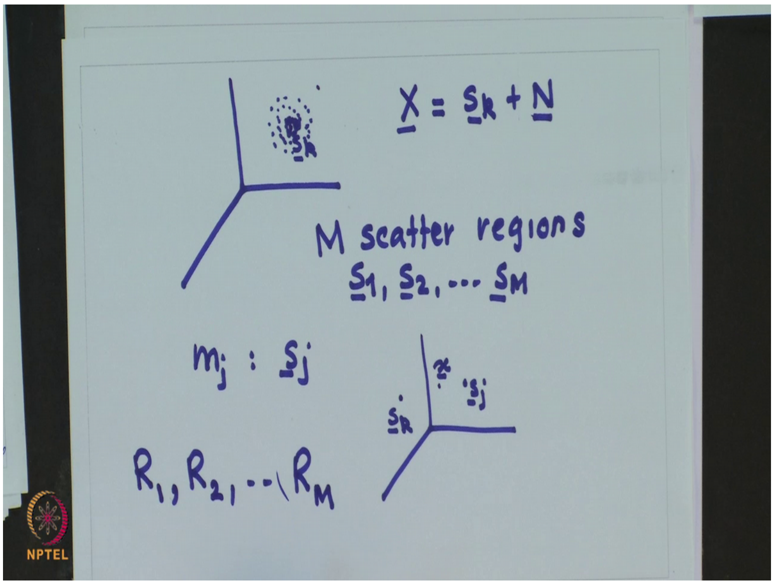

So, just for the sake of understanding, I will just draw three-dimensional plot out here. So, you have S k here corresponding vector to the message m k and because of the noise, what happens is that it forms the scatter region correct. So, like this. So, it is a point center point here. So, distribution of X which is equal to S k plus noise correct is speri spherical because of the spherical distribution corresponding to the noise vector.

So, there will be M scatter regions for various points S 1, S 2 up to S m corresponding to m messages. So, if the message m j is transmitted, it will be in the scatter region which center at S j ok. It will form a scatter around this point S j ok. So, from the position of the received signal which is X vector, one can decide with a very small probability of error which message signal was transmitted.

Now, it is important to note that this scatter region basically extends in principal right up to infinity. But the probability of observing the received signal that is X diminishes rapidly as a point is scattered away from the center. So, you are the probability of obtaining the vector X far off from this point S k will be lower compared to the its probability of occurring at a point closer to S k.

So, this probability here would be higher than the probability say at this point correct. But there will be always some kind of overlap between 2 scatter sets. So, I have just shown you here 1. So, we can extend it to a 2 sets. So, I have one here corresponding to say S j and there could be another one corresponding to say S k correct.

So, this will have scatter region around this. This will have a scatter region around it and what would overlap correct. So, what will happen is basically that because of this overlap, it could result in non-zero error probability. So, if you find a vector X say let us say example out here. This is a vector X which I receive now, this might this looks to be closer to S j.

But it is possible that it has been generated by some other message vector which is not S j, but could be S k correct. So, the probability of such a thing happening; obviously, would be lower from S k compared to the probability points happening from this scatter region S j, if S j is transmitted correct right ok.

So, the task of the optimum receiver is basically to decide from the knowledge of this receive vector X which matches has been transmitted. So, what this implies that the receiver must break up the signal space or divided into m non-overlapping or dis-joint decision regions and this decision regions are known as R 1, R 2 up to R m. So, this corresponds to the decision region for message m 1; this corresponds to decision region for message m 2 and this corresponds to the decision religion for message mm correct.

So, if X false in the region say R k, the decision would be that the message transmitted is m k. So, the problem of designing the receiver then, reduces to choosing the boundaries of these decision regions R 1 to R m in such a way that the probability of error is minimized in this decision making process correct fine. So, with this let us try to find out an optimum receiver.

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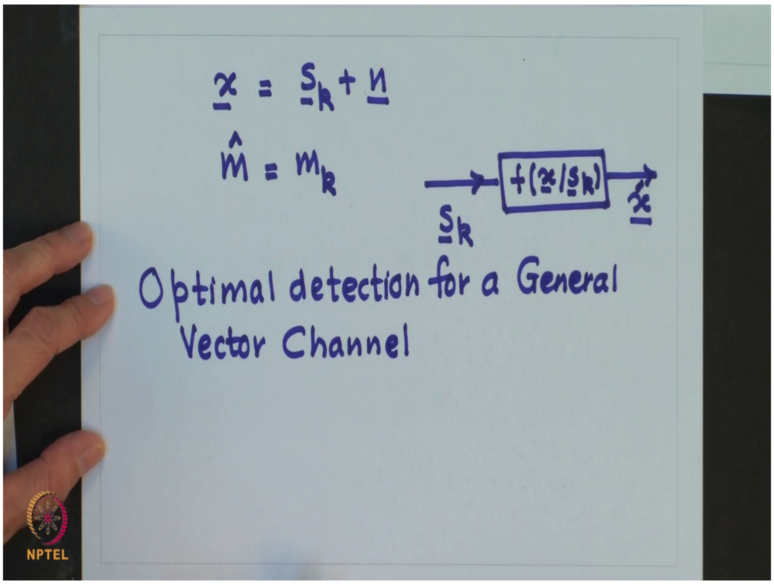

So, to recapitulate let us assume that our receive vector is equal to that is X is equal to S k plus noise vector and the receivers job is basically to decide which message has been received. So, the receiver will decide and let us say the decision is m k correct.

So, we could model this something like this. For the receiver, I have the signal vector S k. The receiver observes the vector X and we assume that conditional pdf between the two is specified. So, this is the conditional pdf of receiving vector X given, you have transmitted the vector S k. So, what is the optimum receiver for this? Ok.

So, we are trying to find out what is known as Optimal detection for a general vector channel which has been modeled as discussed now ok.

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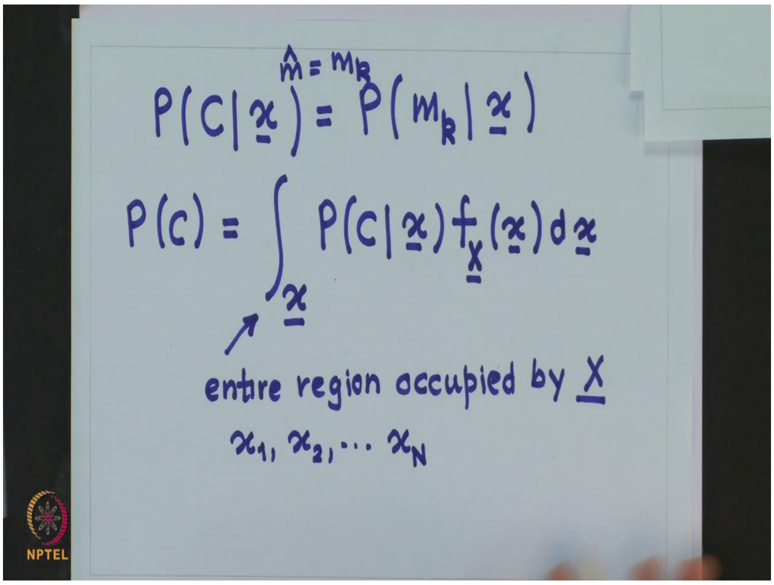

So, let us assume that the conditional probability of making the correct decision given that X is received is equal to this. So, this is given by what is the probability that message m k was transmitted given that you have received the vector X correct. Remember the receivers decision is this is a decision m hat is equal to m k and this is correct. So, this relationship is valid.

So, the unconditional probability of correct decision which we denote by P C would be given by the integral of this quantity. Probability of correct decision for the given vector X correct and this has to be integrated and fold integration over the pdf distribution of vector X ok.

So, this basically is entire region occupied by the random vector X correct. So, this will be N fold integration with respect to the variables X 1, X 2 up to X n over the signal waveform duration fine.

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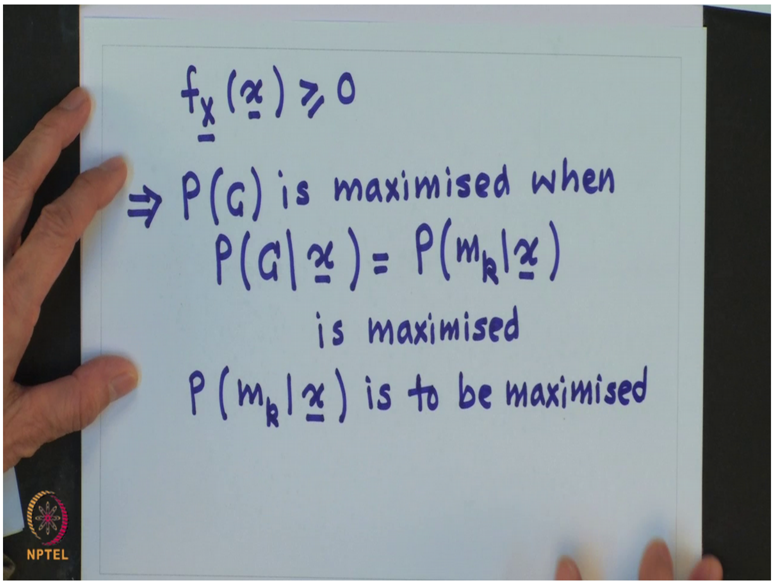

Note that this pdf is always greater than or equal to 0. So, what this implies is this that probability of correct detection is maximized when conditional pdf this is nothing but equal to is maximized, this is always positive.

So, this has to be maximized this quantity has to be maximized inside correct ok. If this happens the error probability is minimized correct. So, my problem reduces basically is probability of m k given X is to be maximized.

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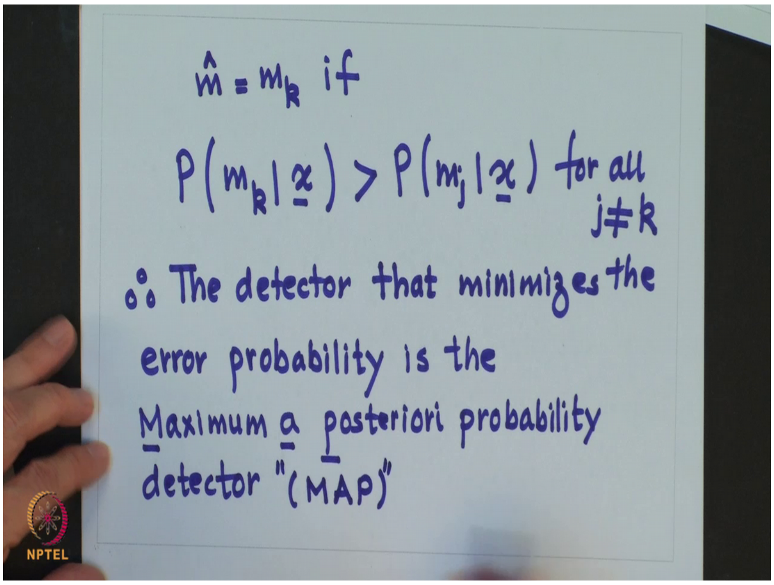

So, now the receiver will decide that m hat equal to m k; if probability of m k given the received vector X is greater than probability of m j given X for all j not equal to k correct. This detector is known as repeat, this detector will minimize the probability of error is known as the Maximum a posteriori probability detector. So, in short it is known as MAP detector.

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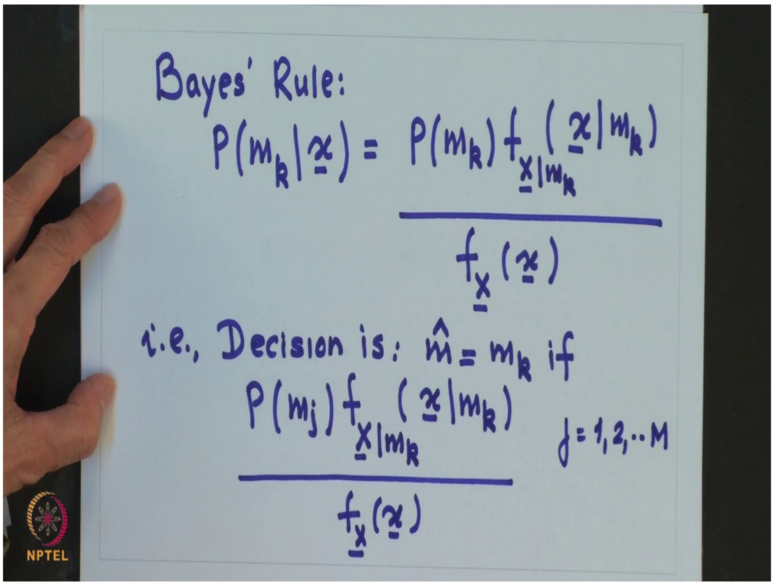

Now, this MAP detector can be put in a different form by using Bayes rule which states the probability of m k given vector X is equivalent to writing ok. That is we can say that decision is m is equal to m hat equal to m k, if this is maximize, for j equal to 1 to M. So, for j equal to k, this is maximum. Then, I will decide in favor of m k correct.

So, remember that when you are evaluating this for the given X, this is common for all j.

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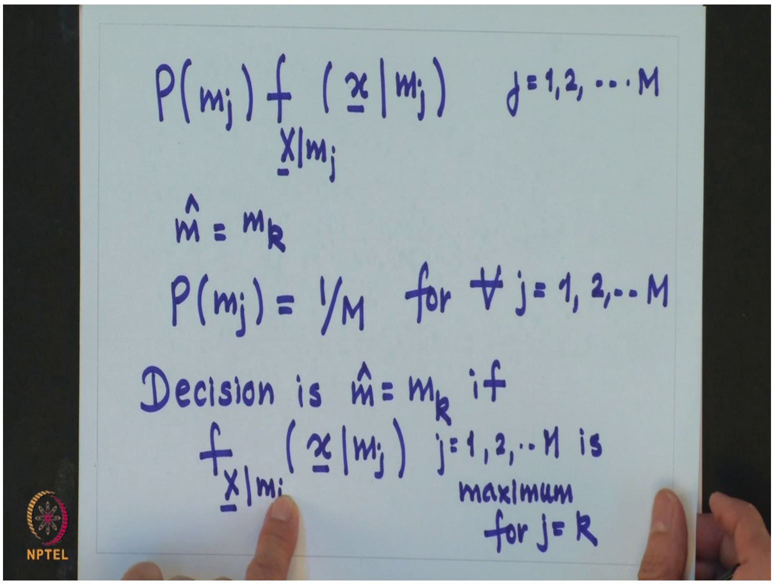

So, your MAP detector basically reduces to evaluating this quantity. So, this is your MAP detector. So, for j equal to k it is maximum, I will decide m hat is equal to m k.

Now, when all the signals are transmitted with equal probability; then, your decision gets simplified; that means, what I want to convey is that if this is equal to 1 by m for all j equal to 1 to M; then, your Decision is m hat is equal to m k, if this conditional pdf is maximized, because log function is monotonically increasing function with for positive arguments. It is convenient to work with the log of this function correct.

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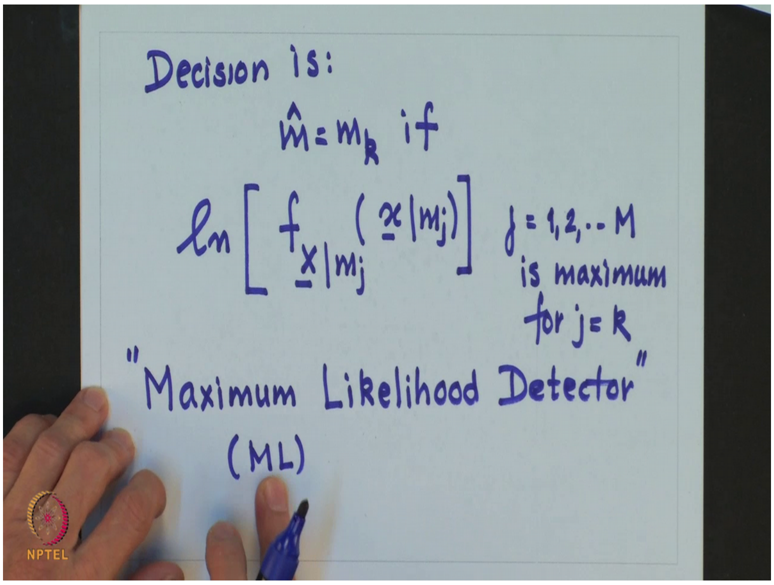

So, if you do this; then, your Decision is: m hat is equal to m k, if log of this conditional pdf is maximized for j equal to k. In such a case, this is known as Maximum Likelihood Detector or this is also known as ML detector. It is important to know that this ML detector is not an optimal detector unless the messages are equiprobable. So, the ML detector; however, is a very popular detector since in many cases having exact information about the message probability is difficult.

Now, having studied the optimal detection for a general vector channel, we will see basically what happens for a specific case of Additive White Gaussian Noise vector channel and this will take up in the next class.

Thank you.