Principles of Digital Communications
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Lecture – 18
AWGN Vector Channel

Hello welcome back. We have studied that any complete orthonormal basis signals can be used for the expansion of a White Noise Process.

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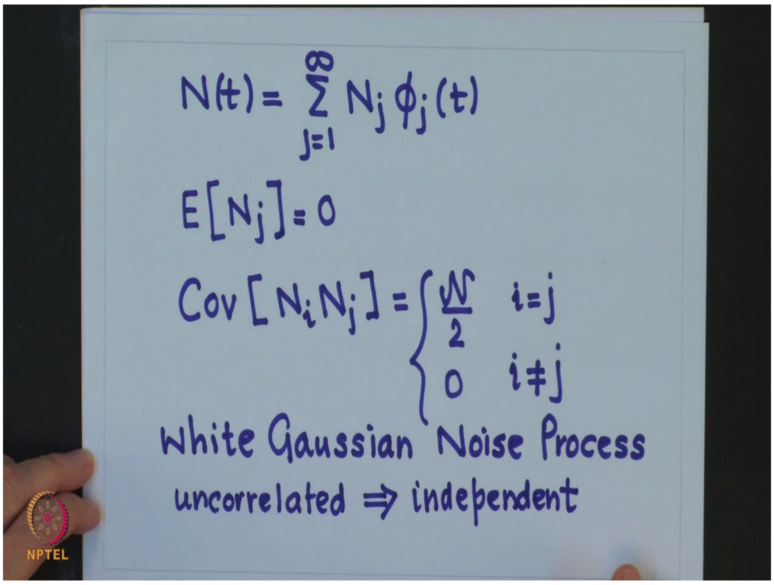

The process can be expanded as follows. It is important to note that this equality is in the mean square sense. We have also seen that for a 0 mean random process, the mean of this expansion coefficient is equal to 0 and we have also shown that the covariance of N i N j is equal to italic n by 2 which is the power spectral density of a White Noise Process for i equal to j and equal to 0 for i not equal to j. What it implies that this N i and N j are uncorrelated.

Now, if your noise process is a Gaussian. So, if you have White Gaussian Noise Process, we will assume the process to have 0 mean. In this case uncorrelated N i N j implies independent N i N j correct ok. Now, we will use this concept in our study on M-ary Communication. The model for M-ary Communication is as shown here.

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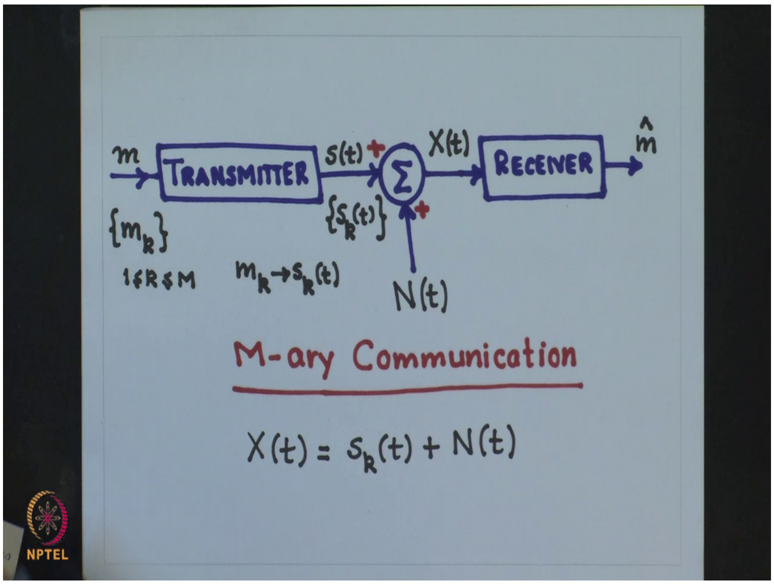

A transmitter sends a sequence of messages from a set of m messages. This messages are represented by finite energy waveforms or signal that is S k t. On the channel, we assumed that is going to be corrupted by noise which we denote by N t and we assume that this noise gets added to the signal being transmitted. So, at the receiver, what we receive is X t.

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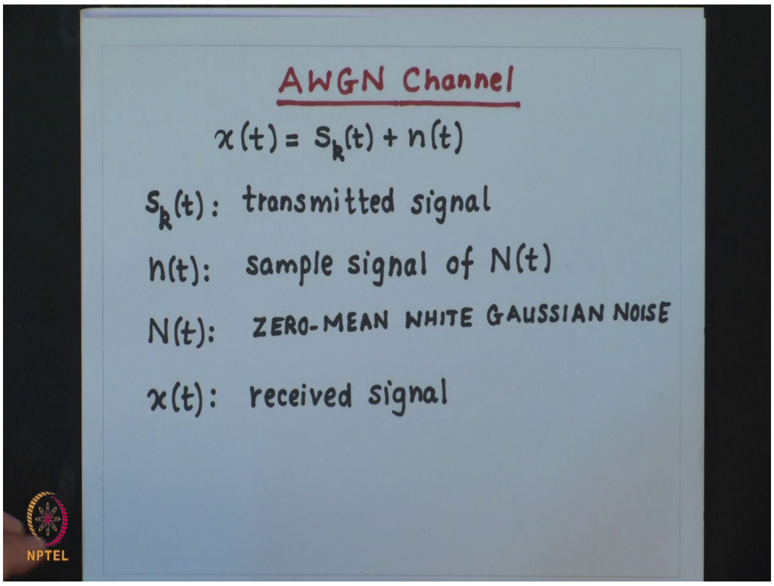

So, the model which we use is this that is Additive White Gaussian Noise Channel. So, S k t is your transmitted signal corresponding to message m k; n t is a sample signal of the Noise Process N t and this noise process N t is assumed to be Zero-Mean White Gaussian Noise and x t is the received signal.

Now, the receiver observes the received signal and based on this observation makes the optimal decision about which message was transmitted. By an optimal decision, we mean a decision rule which minimizes the probability of error; that is the decision rule that minimize the probability of disagreement between the transmitted message and the and the detected message ok.

Now, this model of Additive White Gaussian Noise Channel may seem to be very limiting, but it study is beneficial. First, noise is a major type of corruption introduced by many channels. Therefore, isolating it from other impairments and studying its effects results in better understanding of its effect on all communication system.

The Additive White Gaussian Noise Channel although very simple is a good model for studying deep space communication, which historically was one of the first challenges which a communication engineer encountered. And thirdly, this attractive property of Gaussian pdf makes the analysis of Additive White Gaussian Noise Channel mathematically tractable correct. So, that is the reason for using AWGN Channel in our analysis.

Let us assume that we have used the Gram Schmidt Orthogonalization Procedure to derive the orthonormal basis signal.

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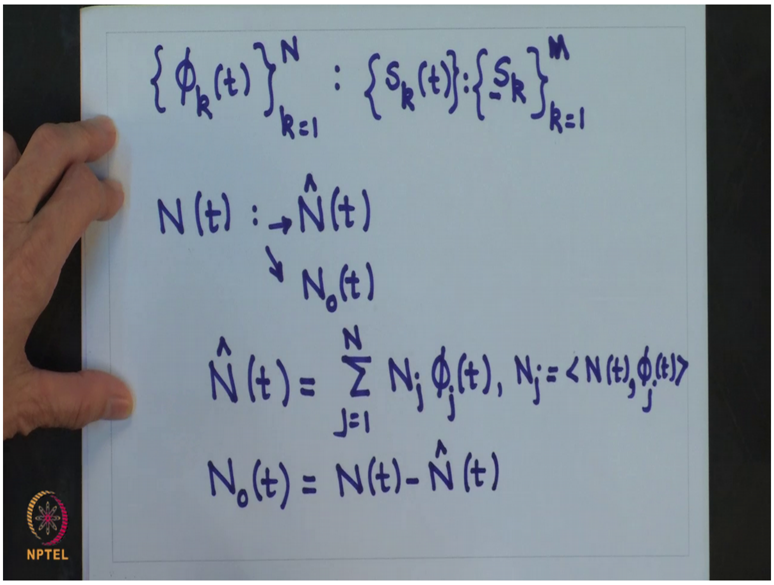

Phi k t for representations of the signals and using this set, the vector representation of the signal is given by S k fine. Now, we know that any Orthonormal basis can be used for expansion of a Zero-Mean White Gaussian Process. So, we can use this Orthonormal basis with appropriate extension for representing the Noise Process in our study.

Now, the noise process cannot be completely expanded in terms of the basis correct. So, we decompose the noise process N t into two components correct. So, the one component which we denote by N hat t is the is part of the noise process that can be expanded in terms of the basis phi k t; k equal to 1 to N.

So, basically this will be the projection of the noise N t onto the space spanned by this basis signal and the other part, we will call it as N 0 t. This is the part that cannot be expressed in terms of the basis signal.

So, with this definition we have N t, N hat t sorry is equal to the projection of the noise onto the basis signals here right. So, where your N j is equal to projection of N t over phi j t and your N 0 t is equal to N t minus N hat t.

Now, we know that this basis signal is complete in the sense that we can take any signal S k t and represent it in terms of this basis signal.

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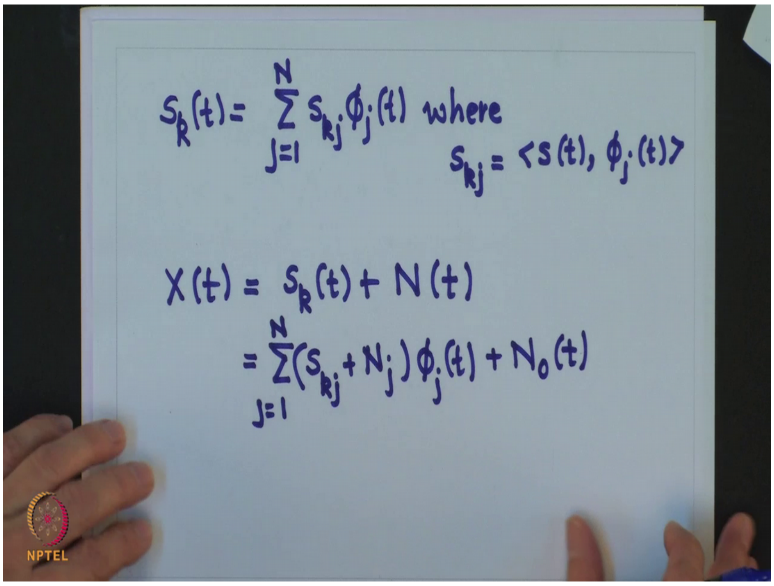

So, it means that we can write S k t as S k j phi j t j is equal to 1 to N; where, S k j is the protection of S t over phi j t right. Now, we have used the Additive White Gaussian Noise Channel model right. So, this is the receive signal.

So, we can write in terms of process as X t is equal to. This is the receive process, this is a transmitted signal and this is the Additive White Gaussian Noise Process. So, this we can rewrite it using this relationships and this relationship as fine.

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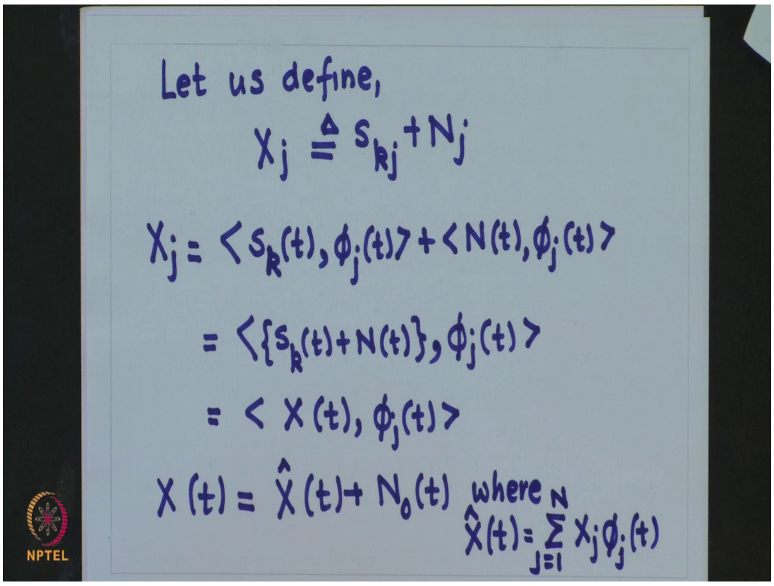

Now, let us define X j as S k j plus N j. So, if I do this, I can write X j is equal to S k j is nothing but the projection of S k t over phi j t and n j is nothing but the projection of N t over phi j t. So, this I can rewrite it as S k t plus N t projection of this signal over phi j t correct. And this is equal to basically the noise sorry the received process.

So, the projection of the received signal over phi j t correct; So, from here what I get is my X t equal to X hat t plus N 0 t ; where, X hat t is the projection of my X t over phi j ts correct. So, this is equal to right and your X j is the projection or X t over phi j t ok.

So, now my received signal is composed of two part; one part is basically the projection of the received signal over the N dimensional space and this is N 0 t. The question I am asking is now, is it possible for me to neglect N 0 t and just considered X hat t in my decision-making process.

Now, it is important to note that we will that N t is independent of our signal S k t.

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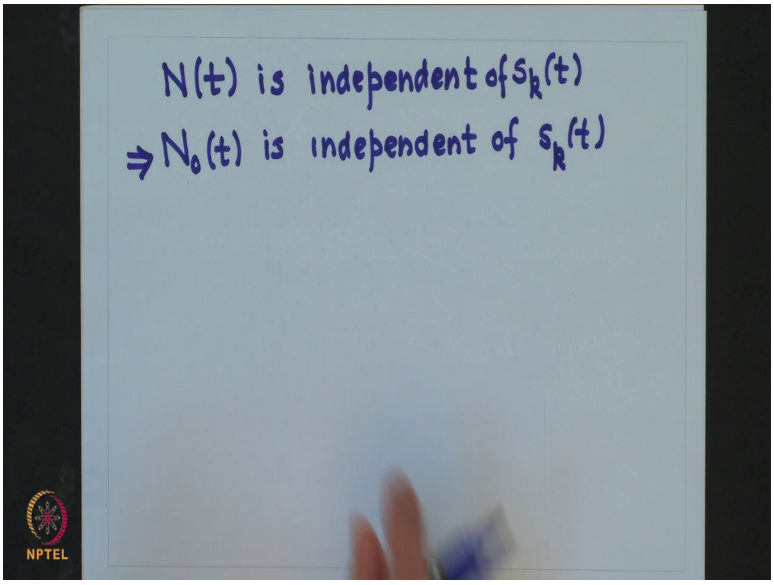

Now, N 0 t is part of a component of N t. So, if this implies that this is also independent of S k t. So, what this implies that as far as S k t is concerned there is no there is no relationship between N 0 t and S k t correct. So, no information about the transmission transmitted signal S k t is contained in N 0 t.

So, discarding such a component from the received signal X t will not cause any loss of information regarding the signal waveform S k t correct. So, fine. The next question is but, this is not enough we must also make sure that the discarded component that is N 0 t is in no way related to the noise component N hat t correct.

So, this X hat t will contain both; signal component and the noise component projected on the N dimensional space. So, it is important for us to see that N 0 t should be independent of N hat t ok. So, let us try to prove this formally.

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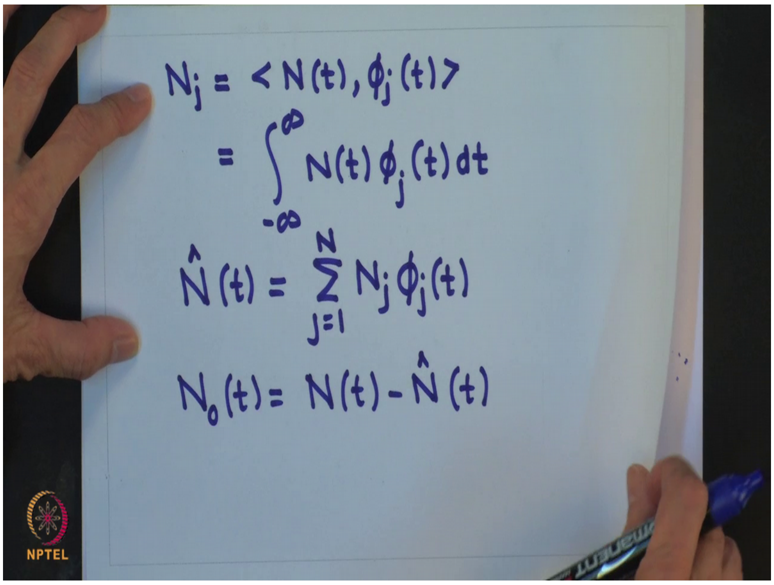

Now, note that our N j is nothing but projection of N t over phi j t which is nothing but this integral evaluation right. Now, your N t is a Gaussian process. So, this basically is a linear combination of Gaussian process.

So, what this implies that N j will be also a Gaussian random variable. And now, your N hat t which is equal to N j phi j t; N j is a Gaussian random variable and linear combinations of Gaussian random variables here right. So, what will happen is that this N hat t is also a Gaussian process.

So, your N 0 t which is equal to N t minus N hat t will be also a Gaussian process because this is a Gaussian process, this is a Gaussian process and this N 0 t is a linear combination of two Gaussian process correct fine ok.

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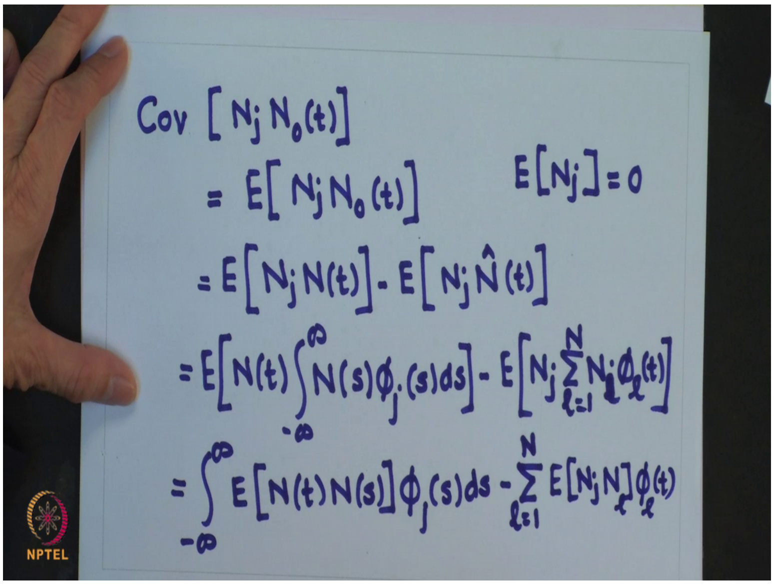

So, now remembering this, let us try to evaluate for any given time t, what is the covariance between N j and N 0 t for a fixed value of t right. So, this is equal to expectation of N 0 N 0 t. Please note that expectation of random variable N 0 is equal to 0; we have proved it.

So, your covariance will reduce to this expression and this we can evaluate as follows. I write N 0 t as a difference between N t and N hat t. So, if I do this right. This I can rewrite it as N j is evaluated by the projection of the noise process on the basis phi j minus N hat t is expansion in terms of basis signal. This I can rewrite it as change of integral and expectation. I will take expectation inside fine.

Now, we know that N t is a White Gaussian Noise with power spectral density given by italic N by 2. So, I can this is nothing but the autocorrelation of the noise process because the power spectral density is flat White Noise.

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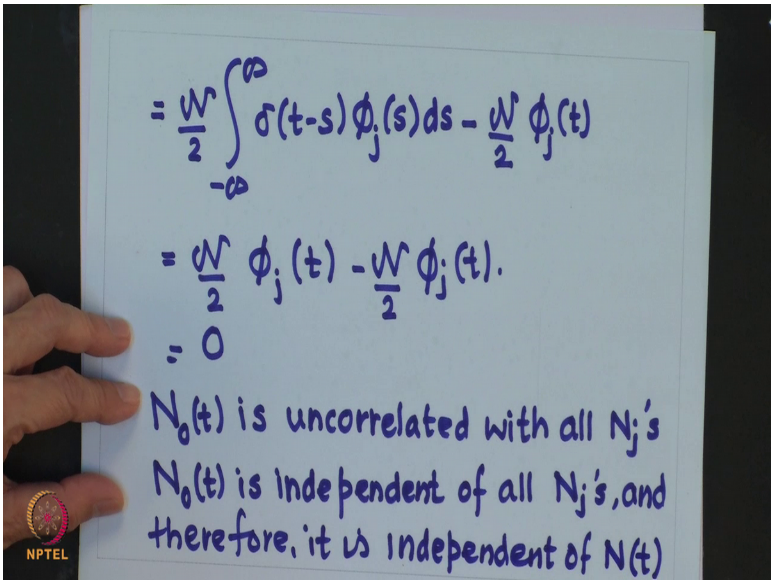

This I can write as an impulse equal to ; this quantity out here remember, this is we have shown that this will be equal to italic N by 2, for l equal to j; otherwise it is equal to 0. So, that is why I get this quantity right and here, this will turn out to be integration will be right. So, this is equal to 0.

So, what this result implies that your N 0 t is uncorrelated with all N j's correct. And since, both are jointly Gaussian N 0 t is independent of all N j's. And therefore, and it is independent of N t hat.

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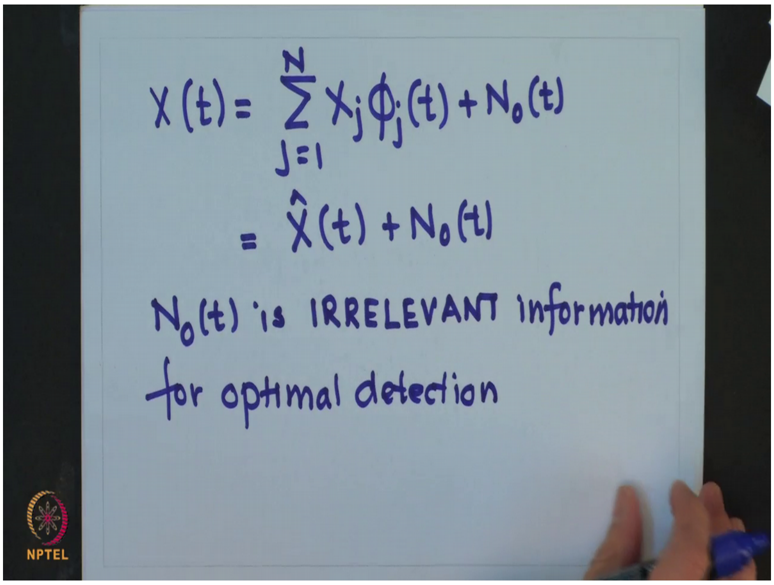

In the equation, X t equal to X j phi j t, the first component out here is the only component that carries the transmitted signal and the second component is independent of the first component.

So, the second component cannot provide any information about the transmitted signal and therefore, has no effect in the detection process and can be ignore without sacrificing the optimality of the detector correct.

So, in other words, N 0 t is Irrelevant information for optimal detection.

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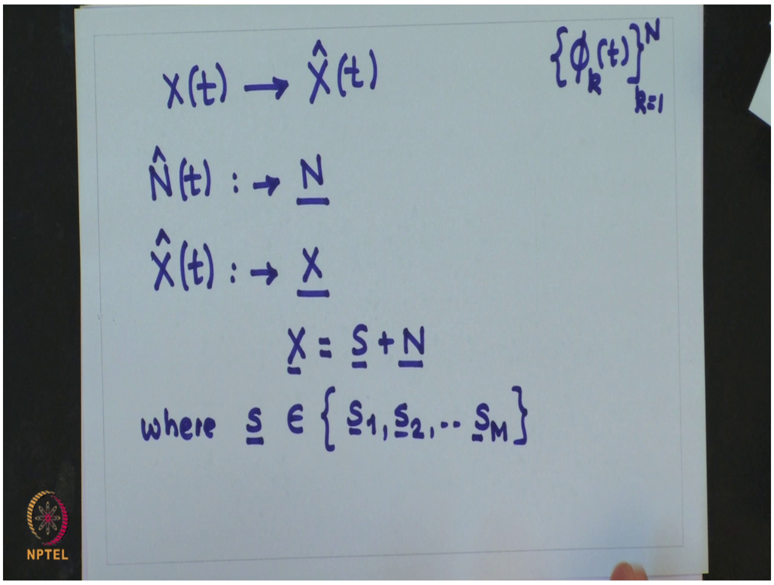

So, now, we can summarize the receive signal X t is reduced to X hat t which contains the desired signal waveform and the projection of the channel noise on the N dimensional signal space correct, spanned by phi k t; k is equal to 1 to N.

Now, let the vectors representing N hat t which is the projection of the noise onto this basis signal be denoted by and the vector representing the projection X hat t by vector X correct. So, what we get is this relationship; where, your vector X belongs to anyone of this vector S 1, S 2 up to S m.

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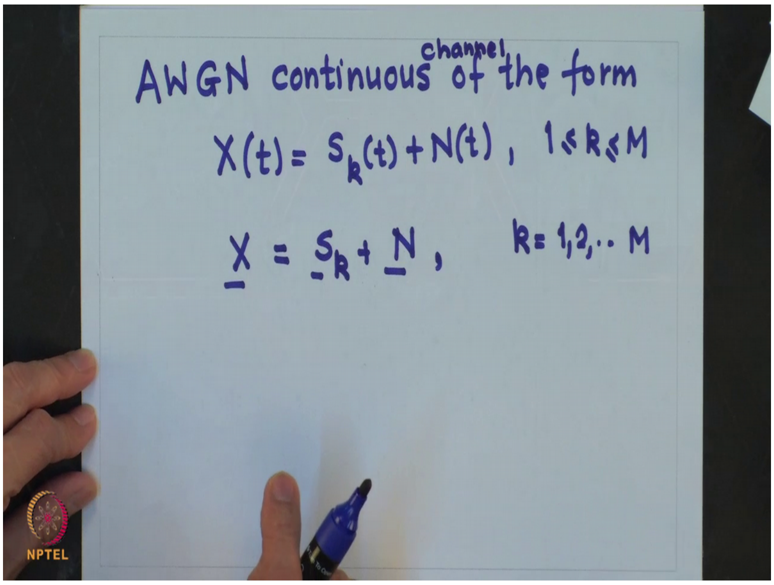

So, from this, it is clear that for the design of the optimal detector the Additive White Gaussian Noise continuous channel also known as a Waveform channel of the form is equivalent to the N dimensional vector channel which is given by this relationship.

So, quickly summarizing, given this model of Additive White Gaussian Noise continuous channel, we can convert it to what is known as Additive White Gaussian Noise vector channel this denotes the projection of X t over the N dimensional space spanned by the basis signal which is complete for representation of S k t signals.

This is your transmitted signal vector and this is a Additive White Gaussian Noise vector. This is the projection of the noise process N t onto the same basis signal used for representing S k t. We will try. Based on this, we will try to find out the optimum detector and we will continue this in the next class.

Thank you.