Microwave Theory and Techniques
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Module - 02
Lecture - 08
Waveguides - III: Rectangular Waveguides

Hello. In the last 2 lectures, we have covered various concepts for parallel plane waveguide. Today, we will extend those concepts to Rectangular Waveguide, after that we will see some applications of wave guides. So, let us begin with rectangular waveguide.

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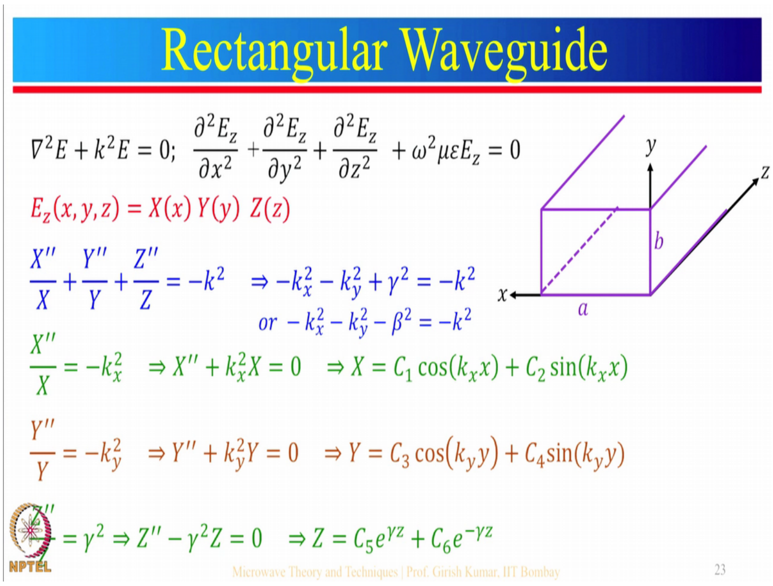

In rectangular waveguides, there are 4 finite metallic plates of which 2 metallic plates are in yz plane and those plates are separated by a distance a in x direction and the 2 metallic plates are in xz plane which are separated by a distance b in y direction.

Now, as we have discussed earlier that to find the longitudinal field we need to solve the wave equation or Helmholtz equation. So, Helmholtz equation for electric field is del square E plus k square E is equal to 0 or we can write it as dou square E z by dou x square plus dou square E z by dou y square plus dou square E z by dou z square plus omega square mu epsilon E z equal to 0.

Since in rectangular waveguide, waves are confined in 2 direction. One is in x direction, other one in y direction and the third direction is the direction of propagation. So, the field varies in all the 3 directions x, y and z. So, we can write longitudinal electric field E z as a function of X, Y and Z which is equal to function of X into function of Y into function of Z.

Now, put this is E z in this wave equation. So, we will get y into Z double derivative of x plus x into Z double derivative of y plus x into Y double derivative of z plus omega square mu epsilon x, y and z is equal to 0. Now, divide this whole equation with the x, y and z, then we will get this equation double derivative of X divided by X plus double derivative of Y divided by Y plus double derivative of Z divided by Z is equal to minus omega square mu epsilon or we can write it as k square.

Since these x, y and z functions are independent of each other and this term k square is a constant. So, the each term in this equation must be a constant. So, we can assume double derivative of X divided by X is equal to minus kx square and double derivative of Y divided by Y is equal to minus ky square and double derivative of Z divided by Z as minus beta square or we can write it as plus gamma square. So, in gamma if alpha is 0 then we will get minus beta square and the signs of these constants are taken as negative for propagating wave and if we take positive signs of this constant, then the solution of this type of equation will be a exponential function that will represent a attenuating field or a non propagating field or even a cent field.

So, for propagation of the waves; the negative signs are considered we will get these type of second order differential equation from here, we can get xy and z. So, X comes out to be C 1 cos kx into x plus C 2 sin kx into x. Similarly, Y comes out to be C 3 cos k y into y plus C 4 sin k y into y and Z comes out to be C 5 e raised to gamma z plus C 6 e raised to minus gamma z. Now, by putting these fields in this equation we will get the longitudinal field E z. So, E z will be multiplication of these 3.

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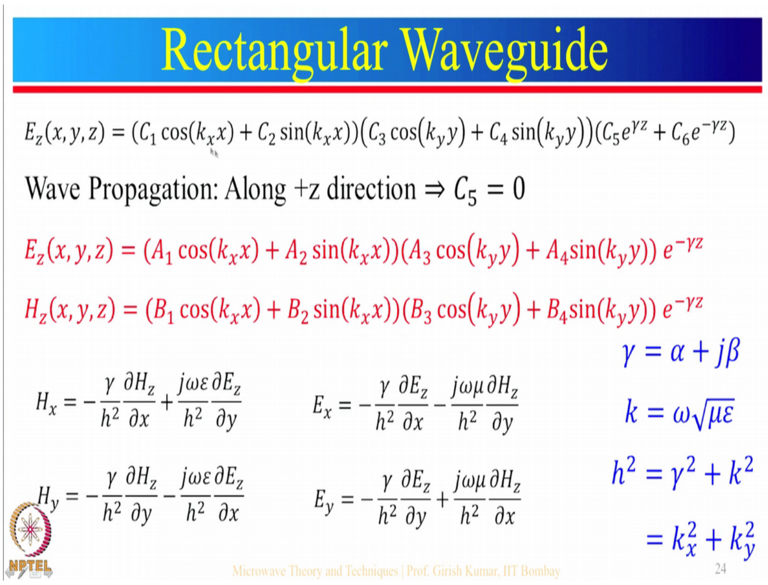

So, it is C 1 cos kx into x plus C 2 sin k x into x multiplied by C 3 into cos ky into y plus C 4 sin k y into y multiplied by C 5 e raised to gamma z plus C 6 e raised to minus gamma z. The term e raised to gamma z in this equation represent the propagating wave in negative z direction and the term e raised to minus gamma z represents the wave propagating in the positive z direction, but we have assumed that wave propagation is there in positive z direction. So, this term should be 0. So, C 5 will be 0 and the electric field E z reduces to this A 1 cos kx into x plus A 2 sin kx into x into this A 3 cos k y into y plus A 4 sin k y into y into e raised to minus gamma z.

Similarly, we can derive longitudinal magnetic field H z which will be this if we know E z and H z, then we can derive the transverse fields H x, H y, E x and E y as we have derived for parallel plane waveguide and the constants are same as before. There is only one difference which is H square is equal to kx square plus ky square because, the wave is confined in both x and y direction.

Now, let us see how these fields vary in different modes in a rectangular waveguide. So, first start with TEM mode which is transverse electric and magnetic mode in which electric and magnetic fields are transverse to the direction of propagation and in the direction of propagation, there is no electric field and magnetic field. So, H z is equal to 0 as well as E z is equal to 0.

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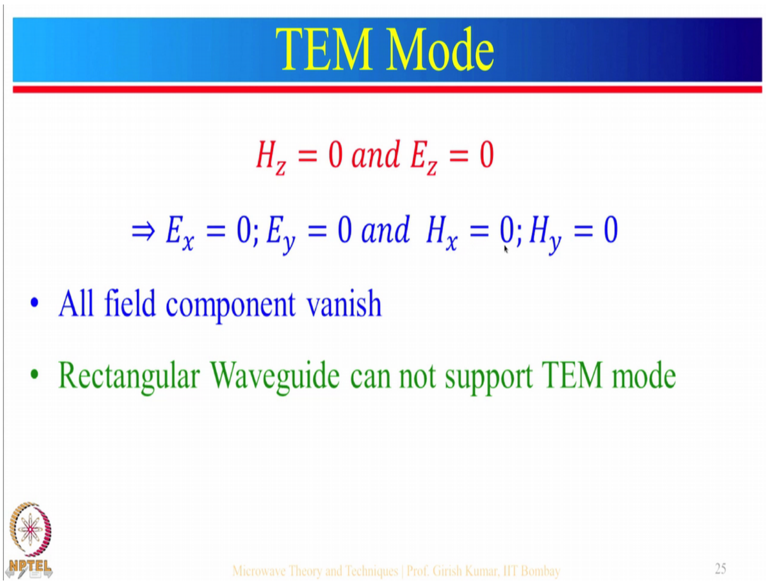

Now, if we put these values of H z and E z in the field equations, then we will get E x equal to 0 E y equal to 0 and H x equal to 0 and H y equal to 0; that means, all the field components are vanished. So, there will not be wave propagation in the waveguide in TEM mode or we can say rectangular waveguide does not support TEM mode of propagation. Now, move on to the next mode which is transverse magnetic mode in which the magnetic field is transferred to the direction of propagation or we can say magnetic field is 0 in the direction of propagation.

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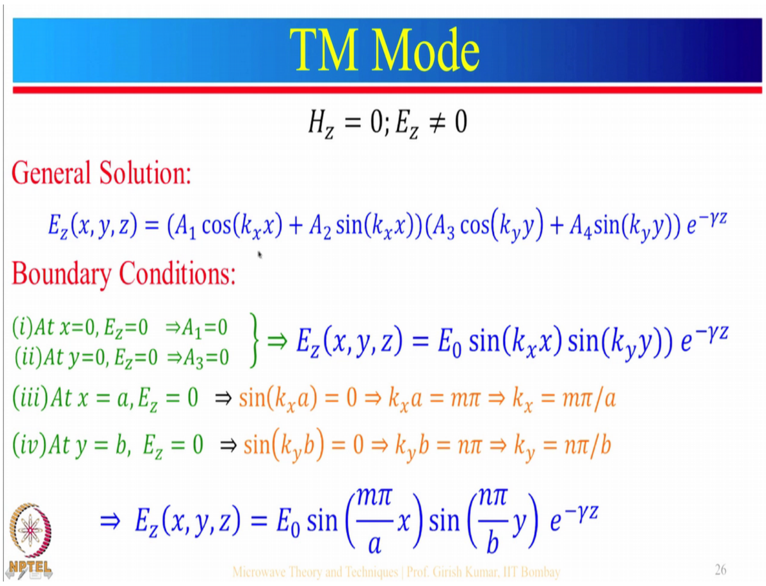

So, Hz is 0 and E z is non-zero and as we have derived earlier that the longitudinal electric field will be E z, it is equal to this. Now, we need to apply boundary conditions on this electric field and what are boundary conditions boundary conditions are that the tangential component of electric field should be 0 at the conducting boundaries and in the rectangular waveguide, we have 4 conducting boundaries. One is at x equal to 0, another one is at x equal to a and other 2 are at y equal to 0 and y equal to b; So, put x equal to 0 in this equation; so, we will get A 1 cos 0 plus A 2 sin 0 sin 0 is 0 cos 0 is 1. So, we will get A 1 into something and now equate this to 0. So, we will get A 1 is equal to 0.

Similarly, put y equal to 0 in this equation then we will get A 3 cos 0 plus A 4 sin 0 and from here, we will get A 3 is equal to 0 now put the values of A 1 and A 3 in this equation. So, E z reduces to e naught sin kx into x into sin k y into y e raised to minus gamma z. Now, apply the other 2 boundary conditions which are at x equal to a E z equal to 0 and at y equal to b E z equal to 0. So, if we put x equal to a in this equation then what will happen sin kx into a; that should be 0 which makes kx into a is equal to m pi where m is an integer. So, from there we will get the value of kx is equal to m pi by a.

Similarly, if we put y is equal to b in this equation, then we will get sin k y into b and we need to make that to 0. So, ky into b will be n pi where n is an integer from there, we will get k by is equal to n pi by b now put these values of kx and ky in this equation. So, E z will be e naught sin m pi by a into x sin n pi by b into y e raised to minus gamma z. So, this is the longitudinal electric field in TM propagation. Now, let us see; what are the propagating and non propagating modes in TM propagation.

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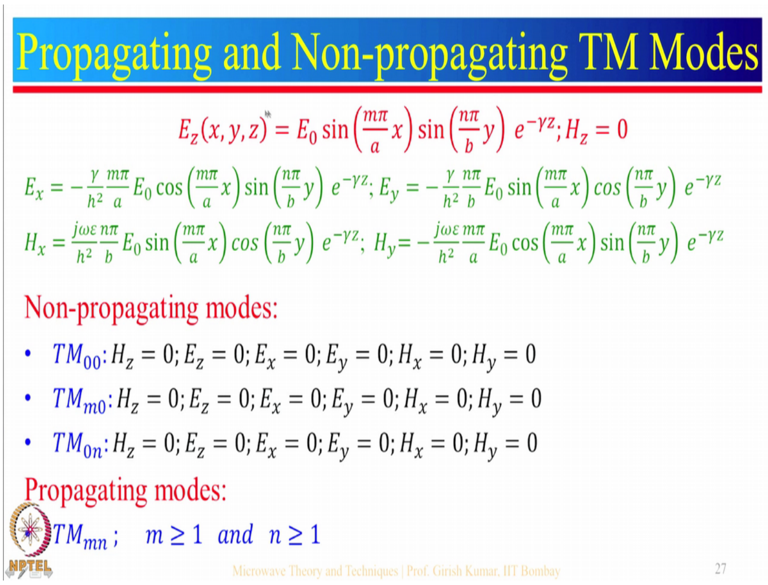

So, as we know E z and H z. So, we can derive E x E y H x and H y. So, these are the field equations for these transverse components. Now, if we put m equal to 0 and n equal to 0 in these equations, then the mode will be TM 0 0 mode. So, if we put m and n equal to 0, then this will be sin 0 into sin 0. So, E z will be 0, H z is already 0. Similarly, E x will be 0 into something 0 and E y will be 0 and like this. So, all the field components in TM 0 0 mode are 0. So, there will not be propagation of electromagnetic wave.

Similarly, if we put n equal to 0 and m can be anything, then this will be 0 if we put n equal to 0, then we will get sin 0 here. So, E x is 0 put n equal to 0 here, then 0 into something E y will be 0. Similarly, H x 0 into something, it will be 0, same way, H y is equal to sin 0 which is 0. So, this is how all the components will be 0 in TM m 0 mode. Similarly for TM 0, n mode where m is 0 and n can be anything all the field components will be 0.

So, for these 3 modes, there will not be any propagation of wave. So, these modes are called as non propagating modes for rectangular waveguides, if we put both m and n greater than or equal to 1, then these fields will not be 0. So, there will be propagation of electromagnetic wave. So, TM mn mode modes are the propagating modes in rectangular waveguide. Now, let us see how field varies in propagating modes in TM propagation.

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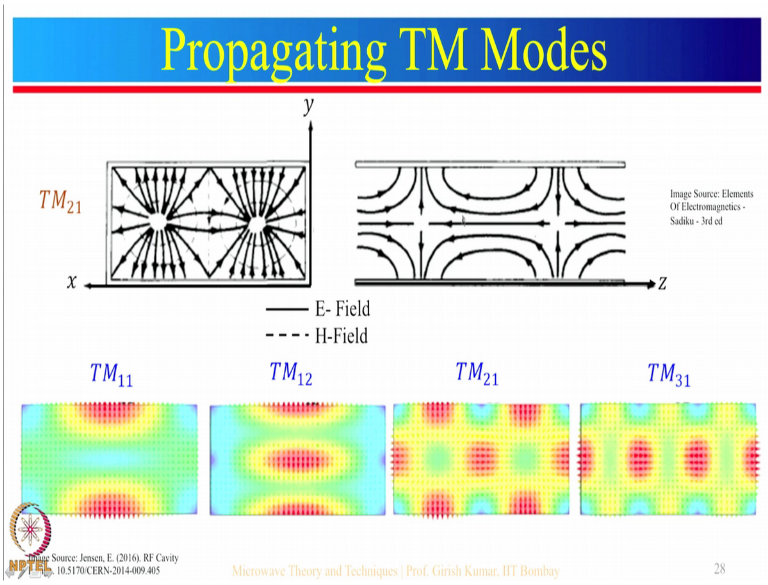

This is the example for TM 2 1 mode, this graph is for xy plane, this is the variation of fields in y z plane. So, as you can see from this, there is A 2 half sinusoidal variation of the fields in x direction and there is only one half sinusoidal variation of the fields in y direction. In general, an TM mn mode there is m half sinusoidal variation of fields along x direction and there are n half sinusoidal variations of the fields in y direction.

So, now let us see in these graphs, in the first graph along x direction, there is 0 maxima 0. This is a half sinusoidal variation along x direction and along y direction maxima 0 maxima, this is also a half sinusoidal variation. So, this mode is a TM 1 1 mode, similarly in this, there is 0 maxima 0 in x direction and in y direction, there is maxima 0 maxima 0 maxima; this is 2 half sinusoidal variation of the fields.

So, this mode is TM 1 2 mode; similarly in this in x direction there are 2 half sinusoidal variation of the fields and in y direction there is only one half sinusoidal variation of the field. So, this is TM 2 1 mode and similarly for this along x direction, there is 3 half sinusoidal variation of the fields and along y there is only one half sinusoidal of the fields. So, this corresponds to TM 3 1 mode all these 4 plots are in xy plane.

Now, let us move on to the next mode which is TE mode transverse electric mode in which the electric field is transverse through the direction of propagation and there is 0 electric field in the direction of propagation. So, E z it is equal to 0 and Hz is not equal to 0 and as we have discussed earlier that the general solution for Hz is this one.

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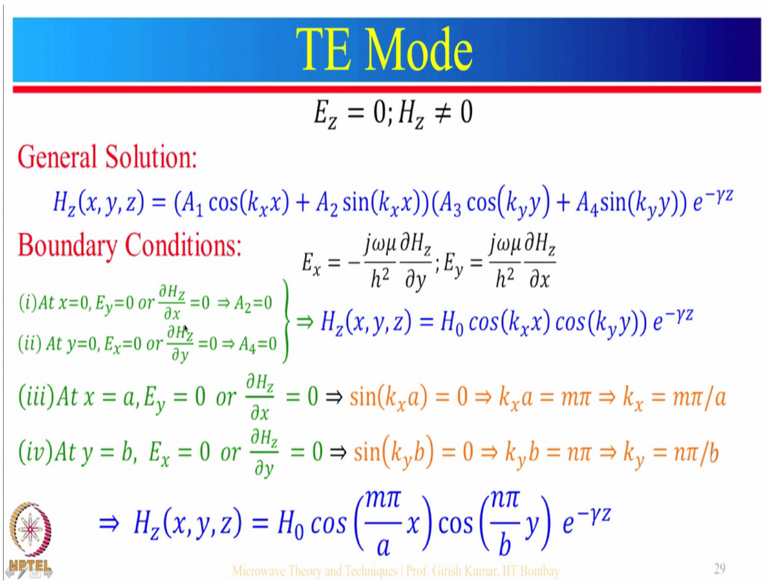

So, we need to apply a boundary condition for this and boundary conditions are the tangential component of electric field should be 0 at the conductive boundaries. There are 4 conducting boundaries x equal to 0 x equal to a y equal to 0 and y equal to b for 2 boundaries x equal to 0 and x equal to a there are 2 tangential component E z and E y E z is already 0. So, we need to make E y is equal to 0 to make E y equal to 0 first we need to find E y. So, how we will find E y since we already know E z and H z. So, we can find E y in terms of H z and E z that will be e y is equal to j omega mu by h square dou H z by dou x.

So, if we need to make E y equal to 0, then dou H z by dou x should be 0. So, put this boundary condition at x equal to 0 dou H z by dou x is equal to 0, then we will get A 2 is equal to 0 similarly for y equal to 0 and y equal to b the tangential components are E x and E z; E z is already 0. So, we need to make E x equal to 0. So, write E x in terms of H z, E x is minus j omega mu by h square dou H z by dou y if E z is 0, then dou H z by dou y will be 0.

So, if we differentiate this with respect to y, then we will get A 3 sin y plus a for cos and if we put 0 here, then we will get A 3 sin 0 plus A 4 cos 0. So, this term will be 0 and then we will get A 4 into something and then we need to make that equal to 0 then we will get A 4 equal to 0, if we put these 2 conditions here in H z, then H z will be H naught into cos kx into x into cos ky into y e raised to minus gamma z.

Now, apply the other 2 conditions, the third one is at x equal to a E y equal to 0 or we can say at x equal to a dou H z by dou x is equal to 0. So, sin kx a will be 0 from here, we will get the value of kx which is m pi by a where m is an integer. Similarly, if we apply this condition, then we will get ky is equal to n pi by b. Now, by putting kx and ky in this equation we will get H z is equal to H naught cos m pi by a into x into cos n pi by b into y e raised to minus gamma z. So, this is the longitudinal magnetic field in TE mode. Now let us see; what are the propagating and non propagating modes in TE propagation.

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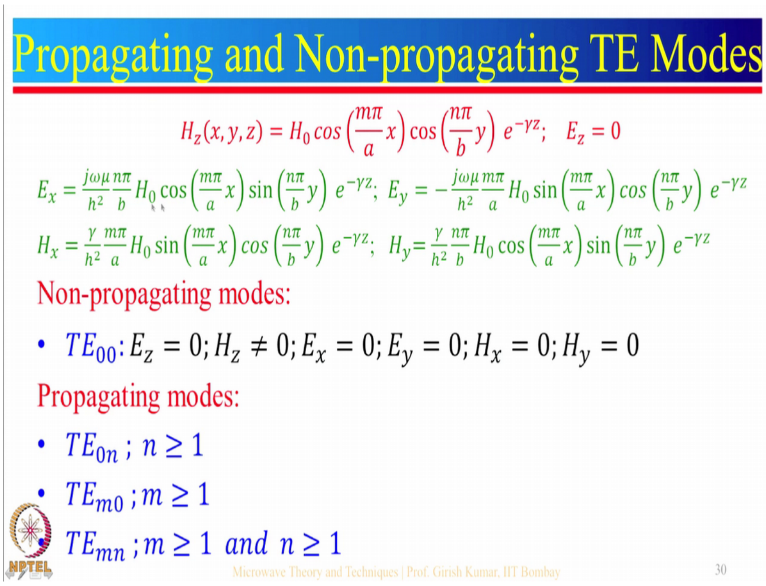

So, we know Hz and E z; we can find transverse fields E x E y H x H y in terms of these 2. Now, if we put m equal to 0 and n equal to 0 in these field equations, then what we will get E z is already 0 will get E x equal to 0 E y equal to 0 H x equal to 0 h y equal to 0 and H z is not equal to 0.

Since all, the transverse field components are 0 in this mode. So, wave will not propagate in this mode. So, TE 00 mode is a non propagating mode in TE propagation and if we put m equal to 0 and n can be anything which is greater than 1 then H z E x and H y will not be 0 and if we put n equal to 0 and m can be anything which is greater than 1, then H z, E y and H x will not be 0 and because of this, there will be propagation of the waves in these 2 modes. Similarly, if we put m is greater than or equal to 1 and greater than or equal to 1, then there will be propagation of the waves. So, these 3 modes TE 0 n TE m 0 and TEM n are the propagating modes for rectangular waveguide.

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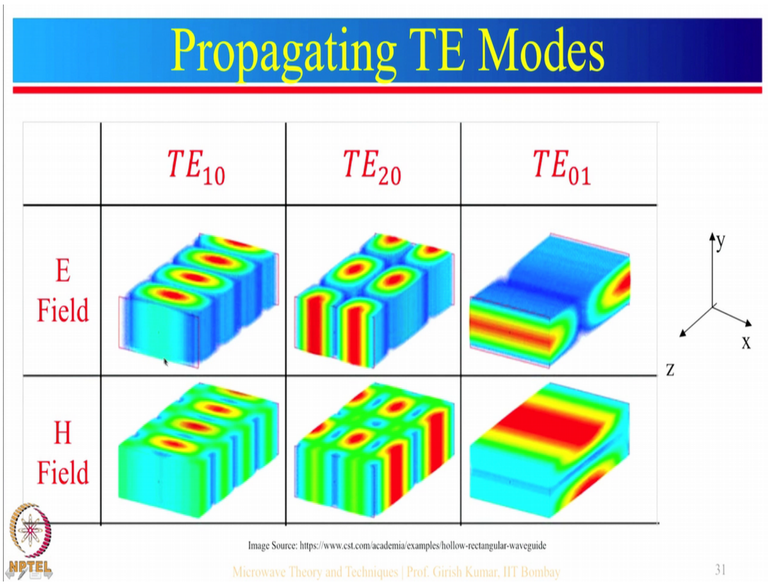

Now, let us see how fields vary in TE mode. So, as you can see from this, this is the x axis, this is y axis and this is the direction of propagation as you can see from this figure that in x direction, there is half sinusoidal variation of the field because at this point 0 field is there, here it is maximum, here it is 0. So, there is half sinusoidal variation along x direction and along y direction there is no field variation. So, this will be TE 1 0 mode.

Similarly, for this in the x direction, there are 2 half sinusoidal variations of the field and in the y direction there is no variation of the field. So, this is TE 2 0 mode. Similarly, for this in x direction there is no variation of the field and in y direction, there is one half sinusoidal variation of the field. So, this is TE 0 1 mode. So, this is how we can plot the field variation of any TE mn mode or TM mn mode inside a rectangular waveguide. Now, let us see cutoff frequencies for rectangular waveguide in different modes.

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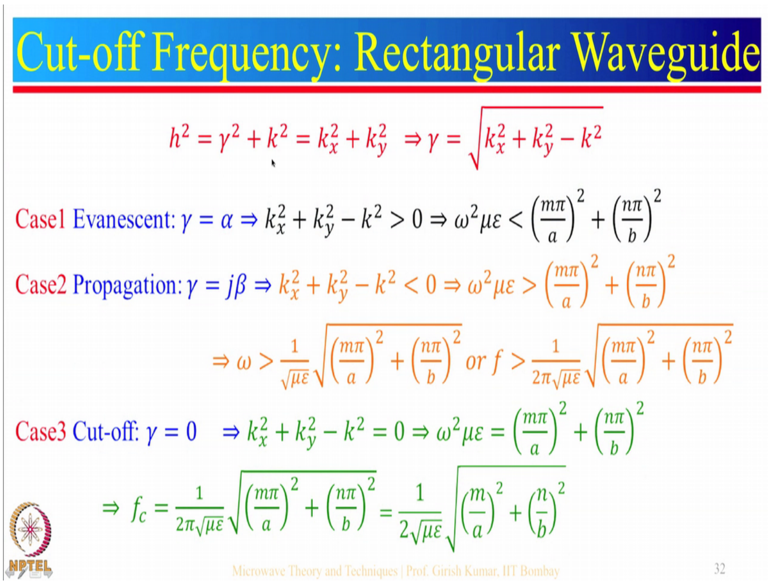

So, as we have earlier that gamma square plus k square is equal to kx square plus ky square for rectangular waveguide. So, from here, we can get propagation constant gamma which is equal to under root kx square plus ky square minus k square from this, there are 3 conditions of gamma. First one is if gamma is real, then beta will be 0 and gamma is equal to alpha. In this case, this quantity should be positive. So, kx square plus ky square minus k square should be greater than 0 from here, we can get omega square mu epsilon is less than this and in this mode, the variation of the field along z direction will be e raised to minus alpha z and that is a attenuating field, we call it as evanescent field.

Now the second case is when gamma is imaginary; that means, alpha is 0 and gamma is equal to j beta and from here this quantity should be negative. So, kx square plus ky square minus k square is less than 0, from here, we can get omega square mu epsilon is greater than this one and from here, we can get frequency should be greater than 1 upon 2 pi root mu epsilon into under root m pi by a whole square plus n pi by b whole square. And for this case, the variation of the fields along z direction will be e raised to minus j beta z which is a propagating wave. So, for all the frequencies greater than this term, there will be propagation of the electromagnetic wave.

The third case is when gamma is equal to 0; that means, alpha is equal to 0 beta is also equal to 0. So, there is neither attenuation nor propagation of the wave. So, from here make this quantity to 0, then we will get fc is equal to this which we can simplify as 1 upon 2 root mu epsilon under root m by a whole square plus n by b whole square or we can write it as v divided by 2 under root m by a whole square plus n by b whole square. And if the waveguide is filled with the air, then the cutoff frequency is given by c by 2 under root m by a whole square plus n by b whole square this is the cutoff frequency of. So, this is how we calculate the cutoff frequency for different modes.

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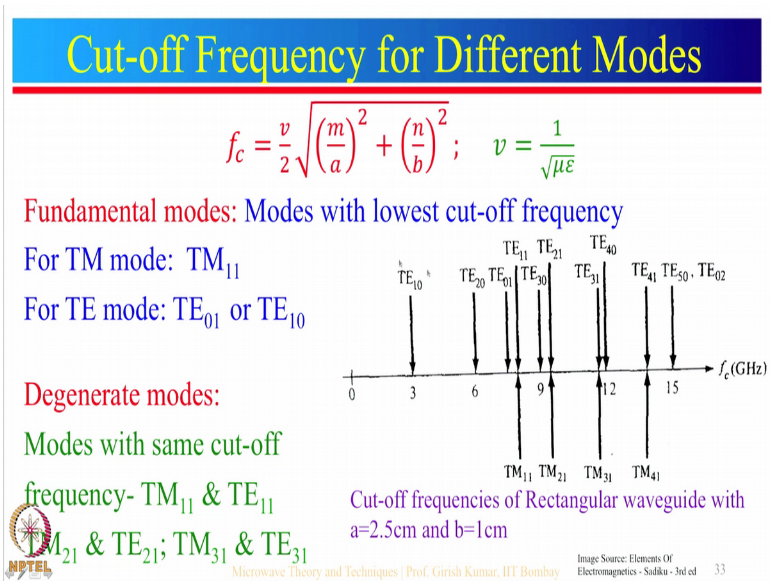

Now, let us see the graph of cutoff frequencies this plot represents the cutoff frequency of a rectangular waveguide with a is equal to 2.5 centimeter and b is equal to 1 centimeter. So, for TE 1 0 mode, the cutoff frequency for this waveguide comes out to be 3 gigahertz; for TE 2 0 mode, it is 6 gigahertz. Similarly, for other mode, we can calculate in this the lowest cutoff frequency is of TE 1 0 mode.

So, the mode with lowest cutoff frequency is called as fundamental mode. So, the fundamental modes in rectangular waveguide is TE 1 0 mode, and if we want to know the fundamental mode for TM propagation, then the lowest frequency in TM propagation is of TM 1 1 mode. So, TM 1 1 mode is the fundamental mode for TM propagation. Now, the TE 1 1 and TM 1 1 have the same cutoff frequency. Similarly, TE 2 1 and TM 2 1 have same cutoff frequency. So, these modes are called as degenerate modes.

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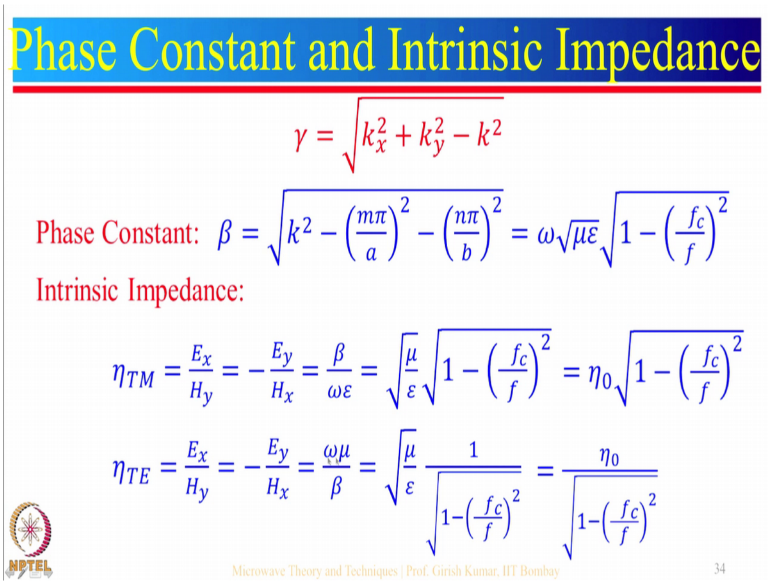

Now, let us move on to phase constant and intrinsic impedance. So, as we know gamma is equal to under root kx square plus ky square minus k square in a rectangular waveguide and for propagating wave gamma is imaginary. So, alpha is 0 and gamma is j beta. So, by equating this to j beta, we will get the value of beta which is equal to under root k square minus m pi by a square minus n pi by b square. And after simplifying this, we will get omega root mu epsilon under root 1 minus fc by f whole square. So, this is the phase constant in rectangular waveguide.

Now, as we have discussed earlier in parallel plane waveguide, the intrinsic impedance for TM mode is given by under root mu by epsilon into one minus fc by f whole square and the intrinsic impedance for TE mode is given by under root mu by epsilon 1 upon under root 1 minus fc by f whole square and for a air filled waveguide, it is given by this for TM mode intrinsic impedance for TE mode is given by this.

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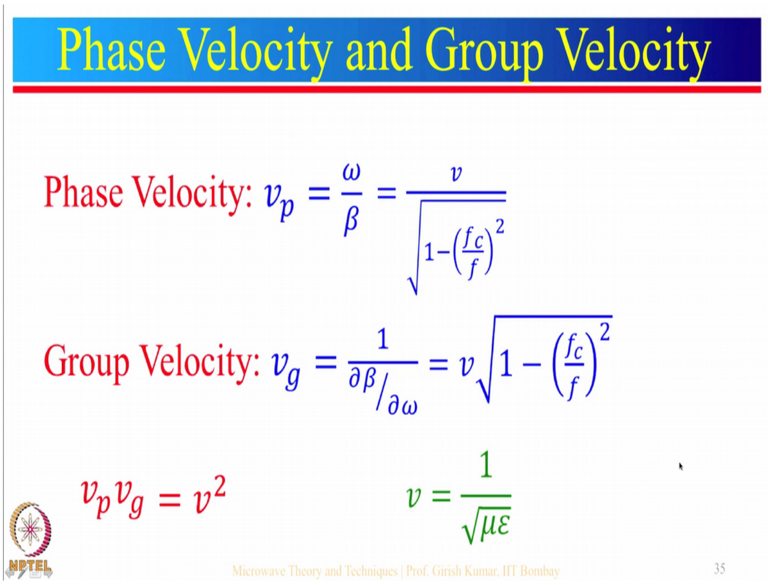

Now, let us see the phase velocity and group velocity the phase velocity as we have discussed earlier that it is given by omega by beta. So, if we put the value of beta here, then we will get the phase velocity vp is equal to v divided by under root 1 minus fc by f whole square similarly we can calculate the group velocity which is equal to 1 upon dou beta by dou omega by solving this, we will get v into under root 1 minus fc by f whole square.

So, as we have seen earlier for parallel plane waveguide that phase velocity decreases from infinity to speed of light as frequency changes from cutoff frequency to infinity, whereas, group velocity increases from 0 to speed of light as frequency changes from cutoff frequency to infinity. But the product of these 2 is always a constant which is equal to v square or c square for air filled waveguide this is all about the phase velocity and group velocity. Now, let us take an example and see how different calculations are done in rectangular waveguide.

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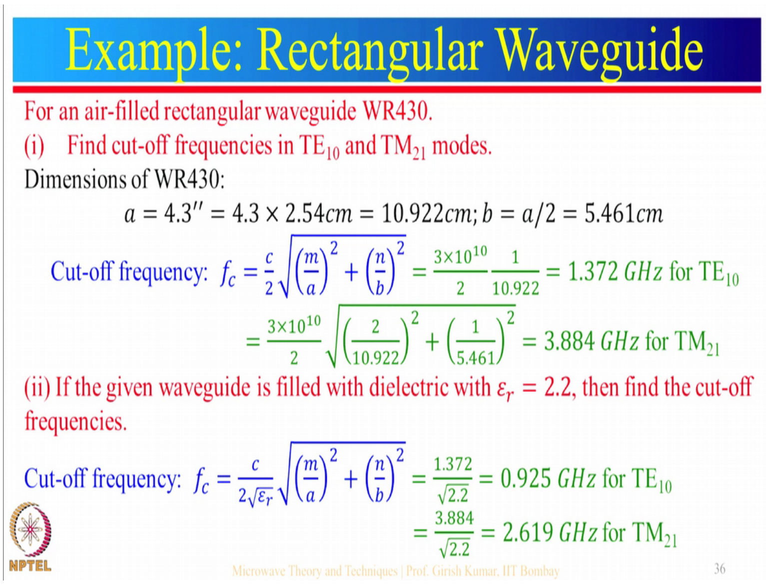

So, let us say we have an air filled rectangular wave guide named WR430, then we have to find cutoff frequencies in TE 1 0 mode and TM 2 1 mode. Now first thing which we need to know is; what are the dimensions of WR430 wave guide. So, the convention of these type of waveguides is that the dimension a is equal to number 430 divided by hundred in inches. So, a is equal to 4.3 inch. Now if we have WR230, then a will be 230 divided by hundred which is 2.3. So, a is equal to 2.3 inches.

Similarly, if we have WR90, then a will be 0.09 inches. Now, we need to convert this a from inches to centimeter. So, to convert from inches to centimeter we need to multiply with 2.54. So, a will be 4.3 into 2.54 centimeter which comes out to be 10.922 centimeter and in general, in these type of waveguides b is taken as half of a. So, b will be 5.461 centimeter and the formula for cutoff frequency as we have discussed is fc is equal to c by 2 into under root m by a whole square plus n by b whole square for air filled waveguide.

Now, put n equal to 0 and m equal to 1 for TE 1 0 mode, then we will get c by 2 into 1 by a. So, 3 into 10 raised to 10 divided by 2 into 1 by 10.922 by simplifying this one, we will get 1.372 gigahertz for TE 1 0 mode. So, this is cutoff frequency for TE 1 0 mode. In the same way, we can calculate the cut off frequency for TM 2 1 mode by putting m is equal to 2 and n is equal to 1 and a is equal to 10.922 and b is equal to 5.461 and after simplifying this we will get 3.884 gigahertz for TM 2 1 mode.

Now, the next question is if the given waveguide is filled with dielectric with epsilon are 2.2 instead of air, then what will be the cutoff frequencies for these modes, the cutoff frequency for different dielectric mediums will be c divided by 2 under root epsilon r into under root m by a whole square plus n by b whole square we have already calculated these terms. So, we need to divide cutoff frequencies we have calculated earlier by under root of epsilon r only. So, 1.372 divided by under root 2.2 will be the cutoff frequency for TE 1 0 mode which comes out to be 0.925 gigahertz.

Similarly, for TM 2 1 mode it will be 3.884 by under root 2.2 which comes out to be 2.619 gigahertz. So, this is how cutoff frequencies in the rectangular wave where are calculated. Now, let us move on to the applications of wave guides. There are various applications of wave guides, these wave guides can be used to transport large powers from one point to another in any system some applications of rectangular waveguides includes E-plane Tee H-plane Tee magic Tee attenuators couplers. So, let us see these one by one.

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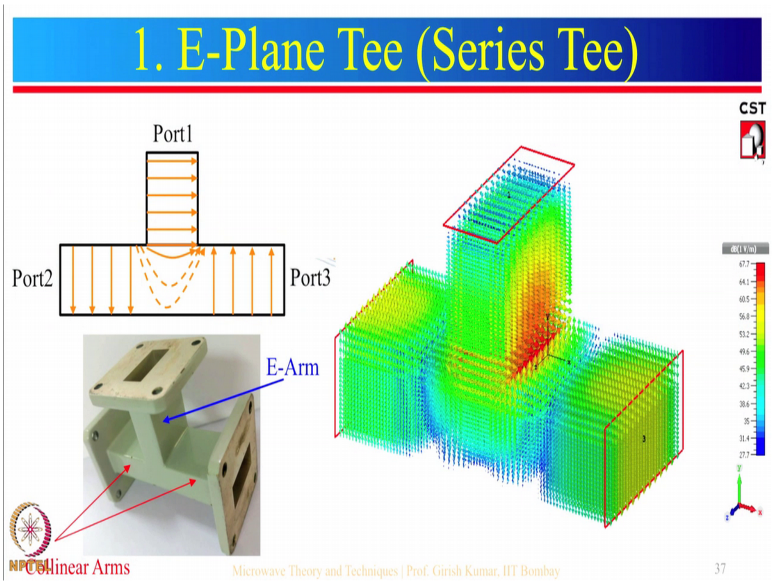

So, first we will have E-plane Tee and E-plane Tee. This is the port one which is called as E-arm this is port 2 and this is port 3. These 2 are called as collinear arms, if input is given to port number 1, then the power will be divided in these 2 ports, but the outputs at these ports will be 180 degree out of phase as you can see from this figure if the direction of electric field is like this. So, it will propagate like this in E-arm and after this, it will be divided something like this.

So, the direction of electric field in this port 3 will be in this direction and the direction of electric field is in this direction; that means, these 2 are 180 degree out of phase you can see from this also this is port 1, this is port 2, this is port 3. And we are providing input from this port and the output is getting divided at this point and the direction of the arrows and these 2 ports are opposite to each other; that means, they are 180 degree out of phase I have this E-plane Tee with me also. So, in this is the port 1, this is port 2, this is port 3. So, if we provide input at this the power will be divided in this and this, but the outputs will be in 180 degree out of phase.

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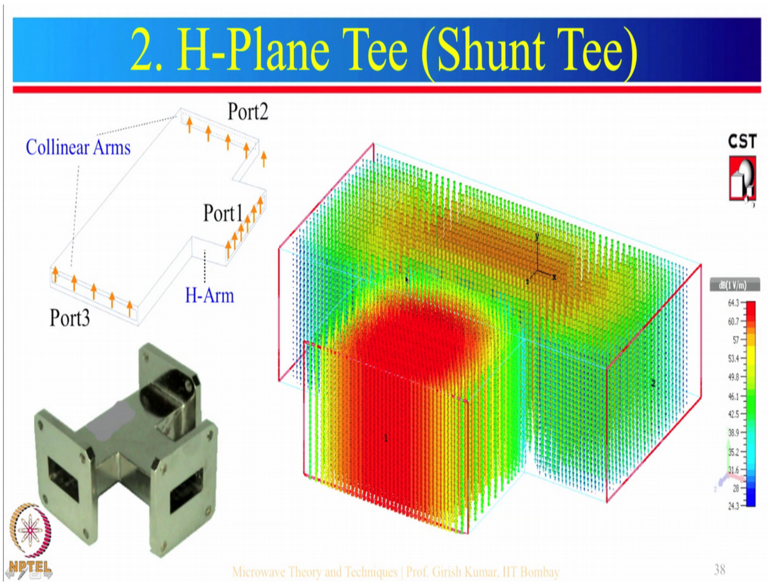

Similarly, the other application is H-plane Tee, in H-plane Tee the port 1 is called as H-arm, port 2 and 3 are called as collinear arms, if we give input to port 1, then power will be divided equally and the output powers will be in the same phase. So, as you can see from this figure also in this; this is the port 1, this is port 2 and this is port 3. So, we are giving input at this port and the power is getting divided at this point and it is going in the same phase the 2 output are in the same phase.

So, depending upon our requirement of the power division; so, if we want equal power division and output should be in same phase, then we can use H-plane Tee and if we want equal power division, but output should be at 180 degree out of phase, then we can use E-plane Tee.

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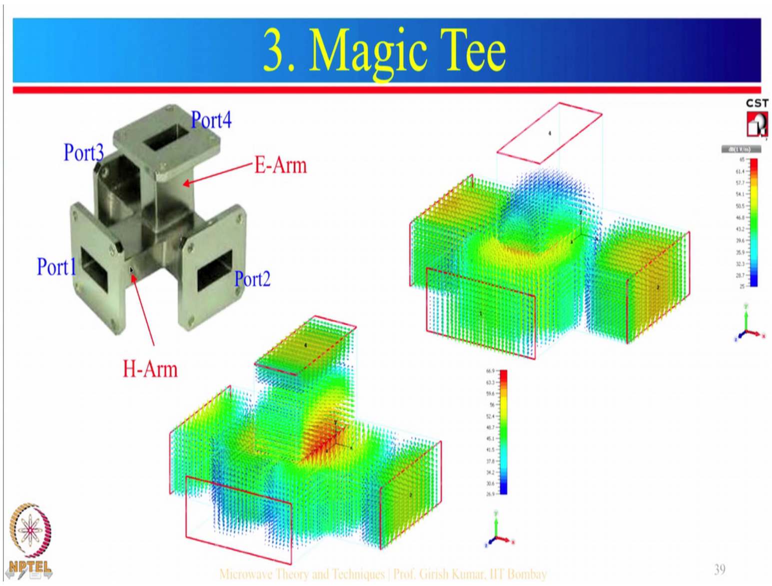

Now, there is one more Tee which is magic Tee in this port one is H-arm and port 4 is E-arm and 2 and 3 are collinear ports. So, in this if we give input to port 1, then power will be divided equally in 2 and 3 and the outputs will be in same phase and the port 4 will be isolated port as you can see from this figure in this, this is the port 1, this is port 2 and 3 and this is port 4. And if we give input to this port, then output is getting divided and these 2 ports equally with same phase as you can see from the direction of the arrows and the port 4 is isolated port.

Similarly, if we give input to port 4, then power will be divided in 2 and 3 equally, but in opposite phase and port 1 will be isolated as seen from this and one more thing, if we give input at port 1 and port 4 simultaneously, then the sum of these inputs will be provided at one of these ports 2 and 3 and the difference will be provided at the other port. Now, the next application is a coupler. So, in coupler, we have 4 ports input port through port coupled port and isolated port.

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So, in this is the input port, this is the through port this is the coupled port and this is the isolated port, this isolated port in this coupler is terminated with a mesh load this coupler is a 10 dB coupler. So, if we give input 0 dB here, then we will get minus 10 dB here at the coupled port.

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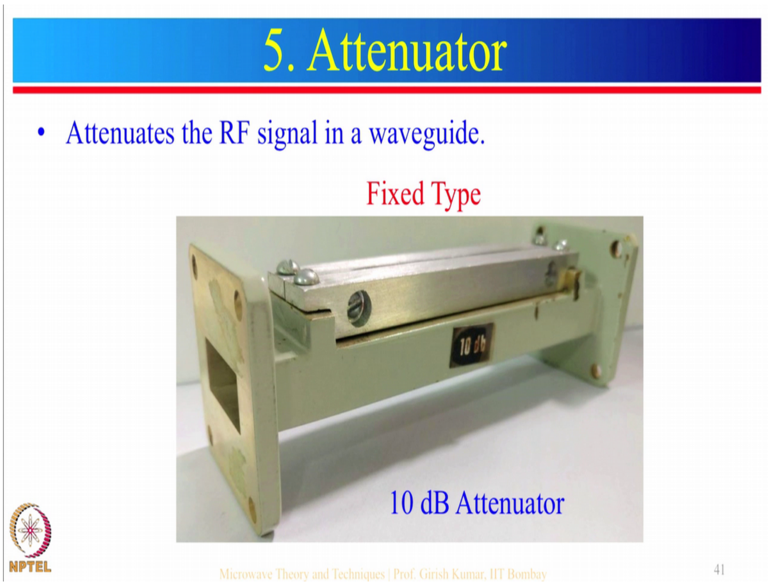

Now, the next application is attenuator. So, this is the attenuator.

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In this, this is the waveguide, we will give input here and get output from here and there is a absorbing material which will absorb the power flowing through the waveguide, this is 10 dB attenuator. So, if we give 0 dB input, here we will get minus 10 dB output here. So, this is a fixed type of attenuator, in general, the wave guides are used to transport a high power from one point to another in a system.

Thank you.