**Microwave Theory and Techniques**
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**Module - 02**
**Lecture - 07**
**Waveguides - II: Parallel Plane Waveguides**

Hello. In the last lecture, we will discussed Maxwell's equations after that we; so, transverse fields in terms of longitudinal fields and then how longitudinal fields are derived using wave equations or Helmholtz equation. After that we applied these concepts of the fields in a simple wave guiding structure parallel plane waveguide and we saw how fields vary in different modes of parallel plane waveguide.

Today, we will see the cutoff frequency of parallel plane waveguides in different modes and phase constant intrinsic impedances phase velocity and group velocities of the waves in different modes, after that we will start the discussion on rectangular waveguide. So, let us start with cutoff frequency in parallel plane waveguide.

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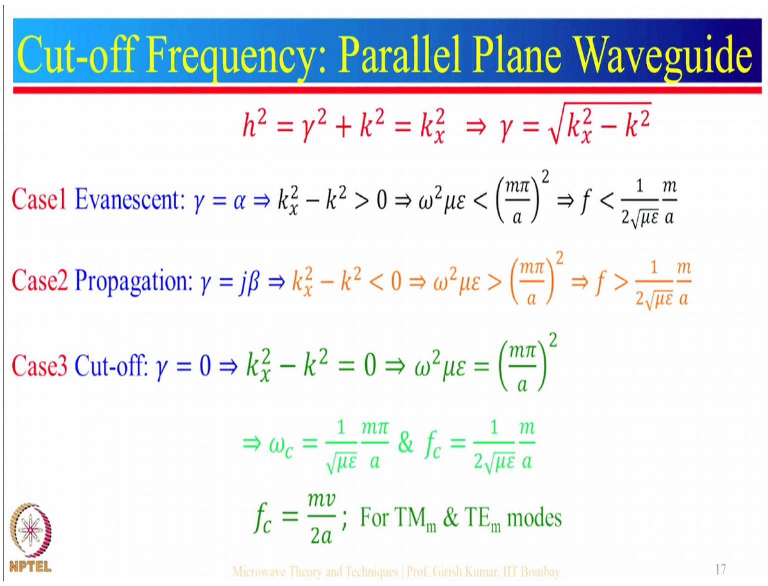


So, as we discussed earlier that h square is equal to gamma square plus k square which is equal to kx square where gamma is a propagation constant which is equal to alpha plus j beta and k is wave number which is equal to omega under root mu epsilon and k x is wave number along x direction which is equal to m pi by a.

So, from here, we can get propagation constant which will be under root kx square minus k square. There are 3 cases of the values of gamma, first case is when gamma is real, then beta will be 0 and gamma is equal to alpha, the gamma to be real this quantity inside this root should be positive. So, kx square minus k square is greater than 0, we can put the values of kx and k in this equation and we will get omega square mu epsilon is less than m pi by a whole square. From here, we can get the frequency which is less than 1 upon 2 root mu epsilon into m by a.

So, for all the frequencies less than this frequency or less than this number the propagation constant will be real number. And in this case, when propagation constant is real, then the variation of the fields along z direction will be e raised to minus alpha z which is attenuating field or we can say; the fields will die down as they will move along the z direction. So, because of this these fields are called as evanescent fields because they are attenuating field or non propagating fields. So, bellow this frequency there will not be propagation of wave as there are attenuating fields.

Now, second case is when gamma is imaginary in that case alpha will be 0 and gamma is equal to j beta and for gamma to be imaginary the term inside this root should be negative. So, kx square minus k square is less than 0, from there, we will get omega square mu epsilon is greater than m pi by a whole square and from here, we can get the frequency f which is greater than 1 upon 2 root mu epsilon into m by a.

And for all the frequencies greater than this term, the propagation constant will be imaginary number and in this case, for gamma imaginary number the variation of the fields, along z direction will be e raised to minus j beat z which is propagating wave. So, when gamma is imaginary, then propagation of waves will takes place.

Now, the third case is when gamma is 0; that means, alpha is 0 and beta is also 0 and in this case, there will be neither attenuation nor propagation of the fields and for gamma to be 0 this quantity should be 0 kx square minus k square. So, make this 0 and then we will get omega square mu epsilon is equal to m pi by a whole square. From here, we can get the cutoff frequency omega c which is equal to 1 upon root mu epsilon m pi by a and cutoff frequency fc is equal to 1 upon 2 root mu epsilon m by a. We are calling these frequencies as cutoff frequency because, bellow this frequency there is attenuating fields or non propagating fields and above this frequency, there are propagating fields. So, bellow this frequency there will not be any propagation of waves and above this frequency there will be propagation of the waves.

So, these are the cutoff frequencies or we can write this cutoff frequency as mv by 2 a also because v is one upon root mu epsilon. So, cutoff frequency for TM m mode and TE m mode is given by fc is equal to mv by 2 a where m is a mode number and it is a integer n is the waveguide is field with air then cutoff frequency will be fc is equal to mc by 2 a; where c is a speed of light in vacuum which is equal to 3 into 10 raised to 8 meter per second or 3 into 10 raised to 10 centimeter per second. So, this is the equation for cutoff frequencies for different modes.

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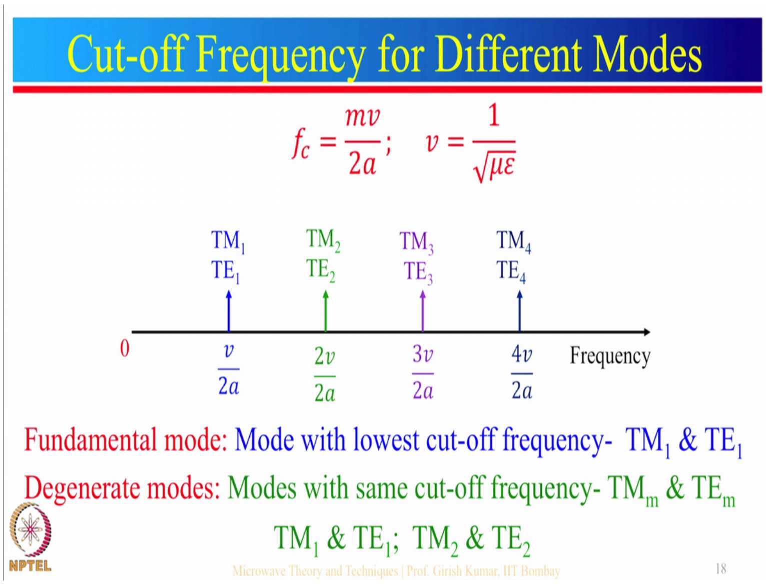


Now, let us see the graph of cutoff frequencies for different modes. So, in general the cutoff frequencies given by fc is equal to mv by 2 a where v is 1 upon root mu epsilon and if we put m equal to 1 in this equation, then we will get v by 2 a cutoff frequency for TM 1 mode and TE 1 mode. And if we put m equal to 2, then we will get cutoff frequency for TM 2 and TE 2 mode which will be 2 v divided by 2 a or we can say v by a similarly by putting the value of m, we can get the cutoff frequency for any mode.

Now, in this the lowest cutoff frequency is v by 2 a which is of mode TM 1 and TE 1 and the mode with lowest cutoff frequency is called as fundamental mode. So, in parallel plane waveguide the fundamental modes are TM 1 and TE 1.

And in this TM 1 and TE 1 have same cutoff frequency which is v by 2 a and TM 2 and TE 2 mode have same cutoff frequency which is 2 v by 2 a. Similarly, TM m mode and TE m mode have same cutoff frequency which is mv by 2 a and the modes with same cutoff frequency are called as degenerate modes.

So, the degenerate modes in parallel plane waveguides are TM m and TE m modes for example, TM 1 and TE 1 or TM 2 and TE 2 mode and so on. So, this is all about the cutoff frequency of parallel plane waveguide for different modes. Now, let us see phase constant and intrinsic impedance of the waves inside the waveguide in different modes.

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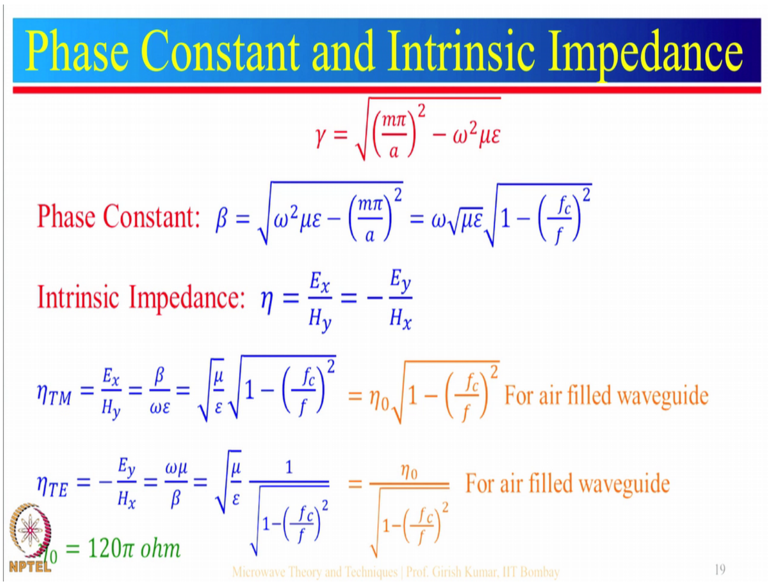


So, as we discussed earlier that the propagation constant of the wave is equal to under root m pi by a whole square minus omega square mu epsilon and for a propagating wave gamma is imaginary. So, alpha is equal to 0 and gamma will be j beta. So, equate j beta to this. So, j beta is equal to under root m pi by a whole square minus omega square mu epsilon.

From here we can get beta which is a phase constant beta is equal to under root omega square mu epsilon minus m pi by a whole square take omega square mu epsilon outside of this root and we will get omega root mu epsilon into under root 1 minus fc by f whole square. So, this is the phase constant beta.

Now, let us see intrinsic impedance intrinsic impedance is given by ratio of electric field and magnetic field that is either E x divided by H y or minus E y divided by H x in transverse magnetic mode as we have seen that E y and H x are 0 and E x and H y are nonzero and we have derived E x and H y in terms of longitudinal field is an sock put those field equations in this, then we will get eta TM intrinsic impedance and TM mode is equal to beta divided by omega epsilon. So, beta we have derived which is this one. So, divide this by omega epsilon, then we will get under root mu divided by epsilon into under root 1 minus fc by f whole square.

So, this is the intrinsic impedance for TM propagation now if the waveguide is filled with air, then the term under root mu by epsilon can be replaced by a constant eta naught which is impedance of wave in free space and the value of eta naught is one twenty pi ohms or 3 seventy seven ohms. So, eta TM will be eta naught into under root 1 minus fc by f whole square for air filled waveguide.

Now, similarly we can find intrinsic impedance for TE mode. So, in transverse electric mode as we have seen that E x and H y are 0 and E y and H x are nonzero and we have a derived this in terms of longitudinal field H z. So, put the values of the fields and we will get eta TE is equal to omega mu by beta and by putting the value of beta here, we will get under root mu by epsilon into 1 upon under root 1 minus fc by f whole square and for air filled waveguide it will be eta naught divided by under root 1 minus fc by f whole square this is for air filled waveguide. So, these are the equations of intrinsic impedances in TM mode and TE mode. Now, let us see how these impedances vary as frequency changes.

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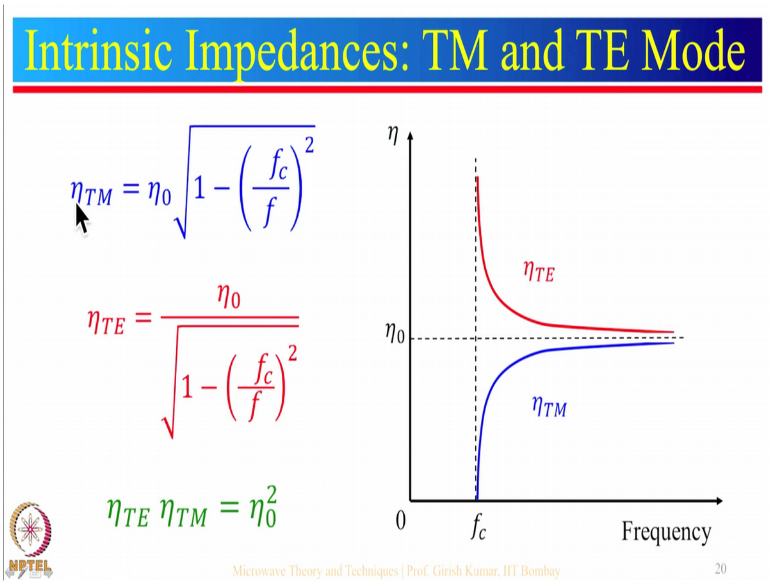


So, for air filled waveguide eta TM is this one and eta TE is this one. Now if we plot these equations with the respect to frequency, then the plot will be something like this. So, eta TM varies from 0 to eta naught as frequency increases from cutoff frequency to infinity this eta TE decreases from infinity to eta naught as frequency changes from cutoff frequency to infinity. But, the product of these 2 is always a constant which is equal to eta naught square for air filled waveguide in general the product of impedances of TM and TE modes is equal to mu by epsilon. So, these are the impedance variations. Now, let us move on to phase velocity and group velocity of waves in a waveguide.

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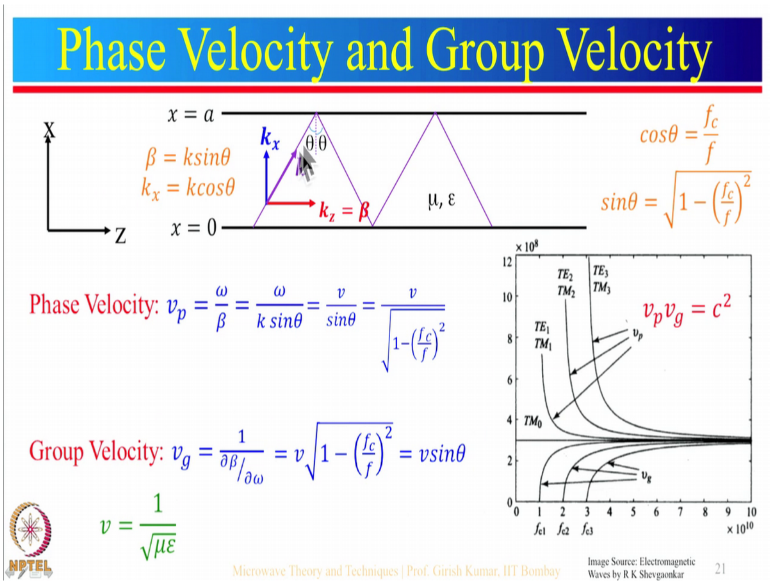

These are the 2 plates in parallel plane waveguide which are separated by a distance a in X direction and these plates are extended to infinity in Y direction and this is Z direction which is the direction of propagation. In general, the wave is propagated in any waveguide because of the total internal reflection and; let us say if wave is launched inside, this waveguide at an angle theta, then by that second law of reflection which states that the angle of reflection is equal to angle of incidence. So, r angle of reflection theta r will be same as theta i angle of incidence which is equal to theta. So, if we have launch the wave at an angle theta then it will be reflected from the walls of the waveguide at an angle theta.

Now, again this reflected wave will go to other wall at an angle theta and that will be reflected at an angle theta. So, this is how by total internal reflections from the walls of the waveguide the wave is propagated in this zigzag path and because of this zigzag path, there are 3 components of wave number, this k is a wave number along the path of the electromagnetic wave where k is equal to omega root mu epsilon and the k z is the wave number along z direction k x is wave number along x direction

Since the resultant reflected waves will travel along z direction only. So, k z is same as beta and we can take component of this k and then we will get beta is equal to k sin theta and k x is equal to k cos theta where cos theta is fc by f and sin theta is under root 1 minus fc by f whole square.

Now, if we put the value k and sin theta, we can get the value of beta which comes omega root mu epsilon into one under root one minus fc by f whole square. So, this is the same value of beta which we have derived earlier and similarly we can derive the value of k x also. So, is omega root mu epsilon and cos theta is fc by f. So, put the value of fc and then we will get k x is equal to m pi by a which is same as we got earlier and same as this wave number the velocity of the electromagnetic wave will have 3 components.

One is v which is equal to 1 upon root mu epsilon, another one is phase velocity and the third one is group velocity, the phase velocity is the velocity with which the locus of constant phase surfaces propagate inside the waveguide and it is given by vp is equal to omega by beta we know beta which is equal to k sin theta and omega by k is equal to v. So, vp will be v divided by a sin theta or sin theta is equal to under root one minus fc by f whole square. So, we can get phase velocity is equal to v divided by under root one minus fc by f whole square.

We can get phase velocity by putting beta directly here also. So, beta is omega root mu epsilon under root 1 minus fc by f whole square. So, we will get the same value of phase velocity. So, if the waveguide is filled with air then this phase velocity will be c divided by sin theta or c divided by under root one minus fc by f whole square.

Since sin theta is always less than equal to 1; that means, phase velocity is always greater than equal to c or phase velocity is always greater than equal to a speed of light, but Einstein's relativity theory says that message or information cannot travel faster than speed of light and for waveguides energy cannot travel faster than velocity of light.

So, does this vp greater than equal to c violates the Einstein's relativity theory the answer to this question is that no it does not violate the Einstein's relativity theory because energy does not travel inside the waveguide with the phase velocity, whereas, it travel in the waveguide with group velocity and the group velocity is the velocity with which the resultant reflected waves travel inside the waveguide and it is given by vg is equal to 1 upon dou beta by dou omega.

Now, we know the value of beta which is equal to omega root mu epsilon into under root one minus fc by f whole square. So, if we differentiate it with respect to omega then we will get group velocity is equal to v into under root one minus fc by f whole square or we can write it as v sin theta and if the waveguide is filled with air, then the phase velocity will be c into sin theta and since sin theta is less than equal to 1. So, the group velocity is always less than or equal to a speed of light and because of this there will not be violation of Einstein's relativity theory because the energy in the waveguide will travel with this velocity group velocity.

So, these are the equations of phase velocity and group velocity. Now let us see; how these 2 vary as frequency changes. So, if we plot phase velocity with frequency, then the plots will looks something like this. So, this is the plot for phase velocity of TE 1 and TM 1 mode, the second one is for TE 2 and TM 2 mode, third one is for TE 3 and TM 3 mode. Similarly, if we plot group velocity then the plots will be something like this. So, first one is for TE 1 and TM 1 mode, second one is for TM 2 and TE 1 mode third one for TE 3 and TM 3 mode.

So, as you can see from these curves, the phase velocity decreases from infinity to a speed of light which is 3 into 10 raised to 8 meter per second as frequency changes from cutoff frequency to infinity. Whereas, this group velocity increases from 0 to a speed of light as frequency changes from cutoff frequency to infinity these fc 1, fc 2 and fc 3 are cutoff frequencies for mode TM 1 TE 1; this one is for TM 2 and TE 2 and this fc 3 is for TE 3 and TM 3.

So, this is the variation of the velocities with frequency, but the product of these 2 velocities is always a constant which is equal to c square for air filled waveguide and in general, the multiplication of phase velocity and group velocity is always a constant which is equal to mu into epsilon and one more thing. Since, this phase velocity and group velocity change with the frequency it means that different frequency components travel in the waveguides with different velocities. And because, of this the time taken by different frequency components will be different to reach from input to output and at the output different frequency components will reach at different time instants and this effect is called as dispersion effect.

So, in waveguide, there will be wave dispersion which is not really desirable. So, this is all about the phase velocity and group velocity. Now let us take an example of a parallel plane waveguide and see how calculations are done in parallel plane waveguide.

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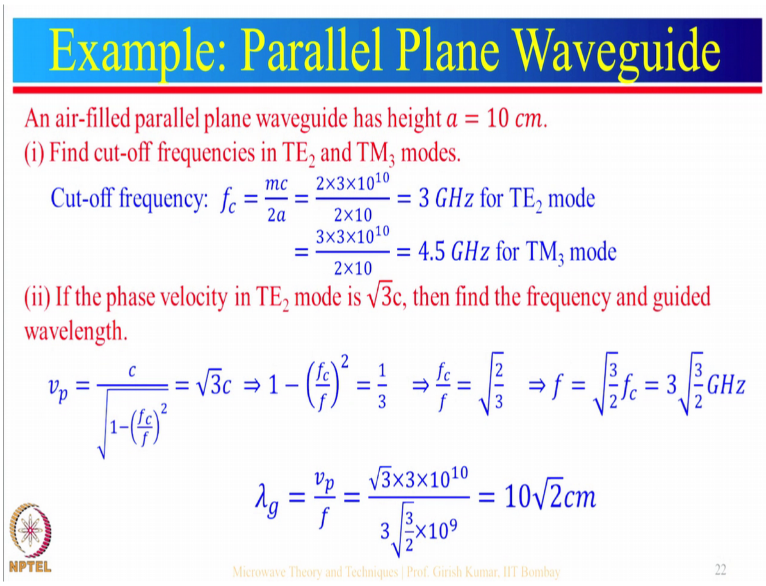

So, let us say, we have in air filled parallel plane waveguide in which the 2 plates are separated by a distance a is equal to 10 centimeter. Now we need to find cutoff frequencies in TE 2 and TM 3 modes. So, general formula for cutoff frequency is fc is equal to mc by 2 a. So, if we put m equal to 2, then we will get cutoff frequency for TE 2 mode which will be 2 into a speed of light which is 3 into 10 raised to 10 centimeter per second and 2 into a is 10 centimeter. So, by simplifying this, we will get 3 gigahertz for TE 2 mode.

Similarly, we can get cutoff frequency for TM 3 mode. So, put m equal to 3, then we will get 3 into c is 3 into 10 raised to 10 centimeter per second divided by 2 into 10 centimeter. So, we will get 4.5 gigahertz for TM 3 mode. Now, the next question is if the phase velocity in TE 2 mode is root 3 c, then what will be the frequency and guided wavelength. So, the formula for phase velocity is vp is equal to c divided by under root 1 minus fc by f whole square. Now equate this 2 given phase velocity which is root 3 c. Now cancel this c and c; a square these 2 sides.

So, we will get 1 minus fc by f whole square is equal to 1 by 3 and 1 minus 1 by 3 is 2 by 3. So, we will get fc by f whole square is equal to 2 by 3. So, fc by f will be under root 2 by 3 from here the frequency will be f is equal to under root 3 by 2 times fc and we have already calculated cutoff frequency for TE 2 mode which is 3 gigahertz. So, put this value here. So, frequency comes out to be 3 into root 3 by 2 gigahertz.

Now, to find guided wavelength. So, guided wavelength is given by lambda g is equal to phase velocity divided by frequency phase velocity is root 3 times speed of light. So, root 3 times 3 into 10 raised to 10 centimeter per second divided by frequency is 3 into root 3 by 2 gigahertz. So, 3 into root 3 by 2 into gigahertz is 10 raised to 9 hertz. So, after simplifying this we will get lambda g is equal to 10 root 2 centimeter.

So, this is how the problems in parallel plane waveguides are solved. So, till now, we have discussed all about parallel plane waveguide in which we discussed different modes TE m mode TE mode TM modes and how fields vary in these modes. And what is the cutoff frequency for these modes and after that we discussed phase constant intrinsic impedance and phase velocity and group velocity inside the waveguide

We have developed some concept for parallel plane waveguide, we extend these concept to rectangular waveguide because, there is only one difference in the structure of parallel plane waveguide. And rectangular waveguide which is in parallel plane waveguide the wave is confined in only one direction which is in x direction and z is the propagation direction in y direction the wave is not confined because of that all the fields are constant along y direction and they vary along x and z directions only.

Whereas, in rectangular waveguides, the wave is confined in 2 directions which are x and y and wave is propagated along z direction. So, all the fields will be vary in all the 3 directions x, y and z directions. So, we will extend the concepts of parallel plane waveguide in rectangular waveguide and we will see what happens to the transverses fields and what happens to the longitudinal fields and how fields are different and TEM mode TM mode and TE modes and we will see the cutoff frequency intrinsic impedances phase velocity and group velocity we will discuss all these thing in the next lecture..

So, in the next lecture we will start from rectangular waveguide and after that; we will discuss some applications of the waveguides.

Thank you.