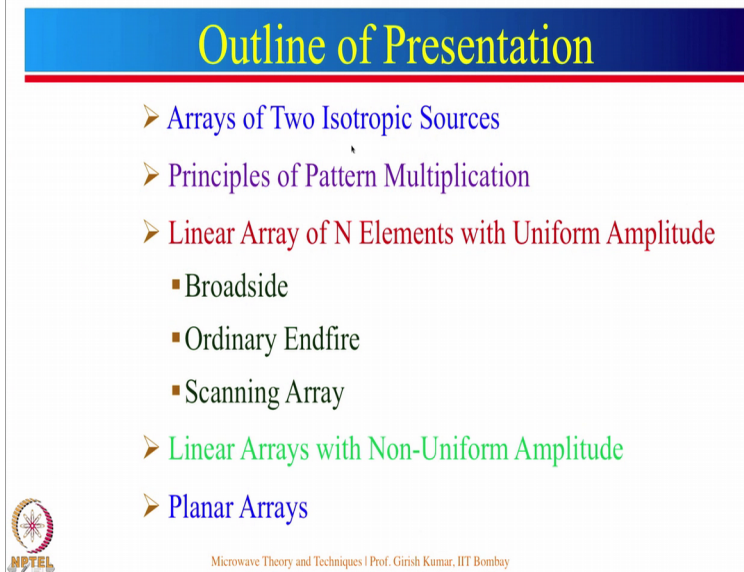


**Microwave Theory and Techniques**  
**Prof. Girish Kumar**  
**Department of Electrical Engineering**  
**Indian Institute of Technology, Bombay**

**Module - 10**  
**Lecture - 48**  
**Linear and Planar Arrays**


Hello and welcome to today's lecture on Linear and Planar Antenna Arrays. In the last lecture I have talked about dipole antenna, monopole antenna, slot antenna and loop antenna. For all those 4 antennas typical gain is of the order of 2 dB and they are omnidirectional antenna. There are many applications, where we would like to have a high gain antenna, which has a narrow beam width and also it has a directional pattern. So, today I am going to talk about linear and planar antenna arrays to increase the gain of the antenna. So, let us start with outline of presentation.

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**Outline of Presentation**

- Arrays of Two Isotropic Sources
- Principles of Pattern Multiplication
- Linear Array of N Elements with Uniform Amplitude
  - Broadside
  - Ordinary Endfire
  - Scanning Array
- Linear Arrays with Non-Uniform Amplitude
- Planar Arrays

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So, I will start with arrays of 2 isotropic sources then I will talk about principle of pattern multiplication. After, that I will cover linear arrays of N elements with uniform amplitude, we will take 3 different cases broadside radiation, pattern ordinary endfire radiation pattern and scanning array. After, that we will discuss about linear arrays with non-uniform amplitude followed by planar arrays.

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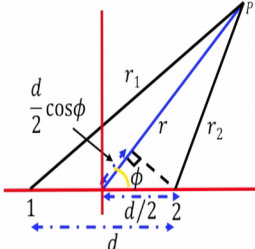
### Array of Two Isotropic Point Sources

$$E = E_0 e^{-j\beta r_1} + E_0 e^{-j\beta r_2}$$

$$\left. \begin{aligned} r_1 &\cong r + \frac{d}{2} \cos\phi \\ r_2 &\cong r - \frac{d}{2} \cos\phi \end{aligned} \right) \text{ for } r \gg d$$


$$\begin{aligned} E &= E_0 e^{-j\beta r} \left[ e^{-j\beta \frac{d}{2} \cos\phi} + e^{j\beta \frac{d}{2} \cos\phi} \right] \\ &= E_0 e^{-j\beta r} \left[ e^{-j\frac{\psi}{2}} + e^{j\frac{\psi}{2}} \right] \end{aligned}$$

$$E = 2E_0 \cos\left(\frac{\psi}{2}\right) = 2E_0 \cos\left(\frac{\pi d}{\lambda} \cos\phi\right)$$



$$\beta = k = \frac{2\pi}{\lambda}$$

$$\psi = \beta d \cos\phi = \frac{2\pi d}{\lambda} \cos\phi$$



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3

So, let us start with array of 2 isotropic point sources. So, we have a source 1 over here and there is a source 2 distance between the 2 elements is d. And these 2 elements are centered around the origin so, this distance is d by 2 this is also d by 2. Now, we want to find out the field at a point P, which is at a distance of r from the origin. And, distance from the 2 elements is r 1 and r 2.

So, let us see now, how we can find the field distribution? So, total field will be given by the term E equal to E 0, this is magnitude and e to the power minus j beta r 1 is the phase term. So, this is element number 1. So, that is distance is r 1. So, that is why the term is e to the power minus j beta r 1.

And, the term corresponding to 2 is e to the power minus j beta r 2. Now, why we have taken E 0 same, because at a very large distance amplitude. From this element and the other element will be approximately same. Of course, there is an approximation here, that r is much larger than d. And, if this is the case we can approximately write expression for r 1 and r 2. Let us see what is r 2 r 2 is this distance this distance can be obtained from this total r minus this particular length here.

So, this length is nothing, but d by 2 multiplied by cos phi. So, r 2 can be written approximately as r minus d by 2 cos phi. And, then similarly we can write r 1 which is r plus d by 2 cos phi you can think about this particular perpendicular dimension over here. So, total length will be r plus d by 2 cos phi.

Now, we can substitute the value of  $r_1$  and  $r_2$  in this particular expression. So, you can see that  $r_1$  is this term and  $E$  to the power minus  $j\beta r$  can be taken out and a rest of the terms are over here, this particular term is denoted by  $\psi$ . So, which is  $\beta d \cos \phi$ . So,  $\psi$  is  $\beta d \cos \phi$  and  $\beta$  is equal to  $2\pi/\lambda$  into  $d \cos \phi$ . I just want to mention many books write  $k$  equal to  $\beta$  equal to  $2\pi/\lambda$ . So, please be aware about the other symbol also.

So, now, this particular term over here can be written in the form of  $\cos \psi$  by 2 we know that this is nothing, but  $e$  to the power minus  $j\psi$  plus  $e$  to the power  $j\psi$  divided by 2 would be the  $\cos$  term. So, that is why 2 term comes over here. So, now, here we have substituted the value of  $\psi$ . So, this is the expression for the field at a far away point, because of the 2 elements 1 and 2 and both these elements have been fed with equal amplitude equal phase and they are isotropic point sources.

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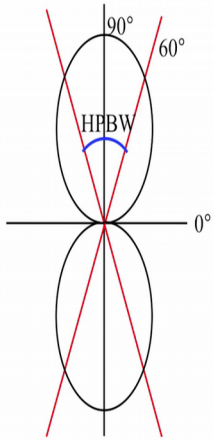
### Two Isotropic Sources of Same Amplitude and Phase


Normalized E:  $E = \cos\left(\frac{\pi d}{\lambda} \cos\phi\right)$

For  $d = \frac{\lambda}{2} \Rightarrow E = \cos\left(\frac{\pi}{2} \cos\phi\right)$

$\phi$	$0^\circ$	$60^\circ$	$90^\circ$
E	0	$1/\sqrt{2}$	1

HPBW is  $60^\circ$  in one plane and  $360^\circ$  in another plane





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4

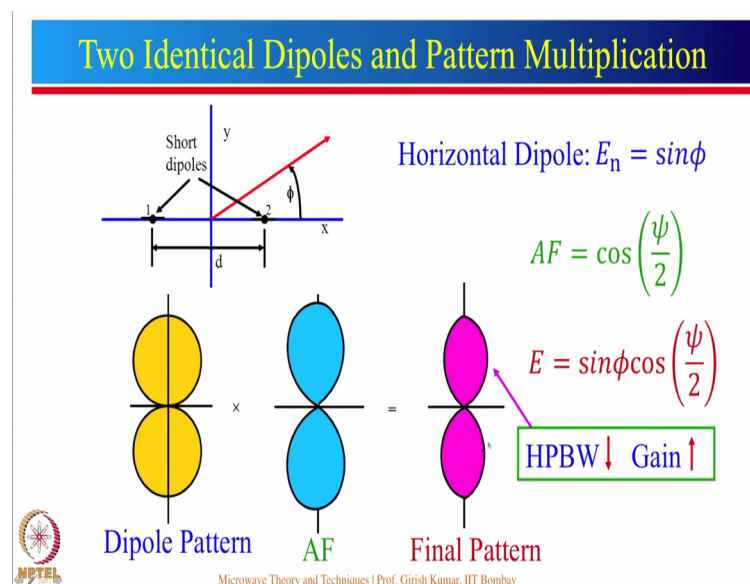
So, now let us just take a few special cases. So, 2 isotropic sources of same amplitude and phase so, we can now write normalized value of  $E$ . You can see that  $2E_0$  is not coming over here, because this is normalized field. I just want to tell you whenever we talk about antenna pattern; we always talk about normalized radiation pattern. So, let us take a case when the distance between the 2 elements is equal to  $\lambda/2$ . So, if we substitute the value of  $d$  equal to  $\lambda/2$  over here, this expression becomes  $\cos d$  is  $\lambda/2$ . So, that becomes  $\pi/2$  times  $\cos \phi$ .

Now, we need to plot the radiation pattern. I am just going to show you for 3 different cases you can build the rest on your own. So, let us take these cases now and that is when phi is equal to 0. So, phi is equal to 0 in this direction, phi is equal to 90 degree in this particular direction. So, at phi equal to 0 cos 0 will become 1. So, cos pi by 2 will become 0 at phi equal to 60 degree cos 60 degree will be 1 by 2. So, this will be pi by 4 that would be equal to 1 by square root 2 and at phi equal to 90 cos 90 will be equal to 0 so, cos 0 will be equal to 1.

So, this I have shown you for 0 60 90. Similarly, you can plot the rest of the pattern. So, from here we can say half power beamwidth is nothing, but you can say that how we define half power beamwidth? Where, the field is reduced to 1 by square root 2. So, half power beamwidth can be now determined from this particular plot and that will be 30 degree and 30 degree. So, total 60 degree.

Now, this is half power beamwidth in this particular plane, but in this particular plane field will be uniform. So, half power beamwidth in this particular plane will be 360 degree.

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So, now, instead of taking isotropic elements, let us take horizontal dipole over here and horizontal dipole over here. So, 4 horizontal dipole, we know that maximum field will be in this particular direction just recall I have mentioned to you, that you can think about a dipole as a pen.



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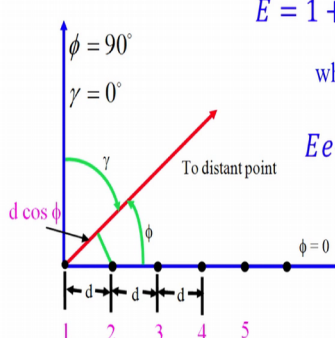
So, maximum radiation will be in this direction and minimum radiation will be in this particular direction. So, it makes a figure of 8 like this over here.

Now, this is the dipole radiation pattern. Since, these 2 elements are fed with equal amplitude and phase this is the array factor. So, we multiply these 2 we actually get a narrower pattern over here. And, you can actually obtain this in a simple way this is 1 multiplied by 1 will be equal to 1, but let us say if this is 0.8 and if this is 0.7 0.8 multiplied by 0.7 will become 0.56.

Hence, it becomes narrower pattern and if we look at half power beamwidth. So, half power beamwidth decreases hence gain will increase. So, instead of let us say gain of 2 dB for a dipole, we will now have a larger gain if we use 2 element array.

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### N Isotropic Sources of Equal Amplitude and Spacing



$$E = 1 + e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \dots + e^{j(n-1)\psi}$$


where  $\psi = \frac{2\pi d}{\lambda} \cos\phi + \delta$

$$E e^{j\psi} = e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \dots + e^{jn\psi}$$

$$E - E e^{j\psi} = 1 - e^{jn\psi}$$

$$E = \frac{1 - e^{jn\psi}}{1 - e^{j\psi}} = \frac{\sin(n\psi/2)}{\sin(\psi/2)}$$

As  $\psi \rightarrow 0$ ,  $E_{\max} = n$ ,  $E_{\text{norm}} = \frac{\sin(n\psi/2)}{n\sin(\psi/2)}$



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6

So, now instead of using 2 elements, if we use N elements so, here are N elements. So, you can start from 1 2 3 and so on. Here, we have taken the first element at origin and then other elements are at equal distance which is equal to d. So, now, we can find out the E, which is far field pattern, because of all these elements we can find the total field at a faraway distance.

So, since this particular element is at origin the term corresponding to that will be 1. This particular element is at a distance of d. So, now, we have e to the power j psi this is at 2 d. So, we have 2 psi 3 psi and so on. And, what is psi? Psi is equal to 2 pi d by lambda into cos phi plus delta. What is delta? Delta is the phase difference between these elements.

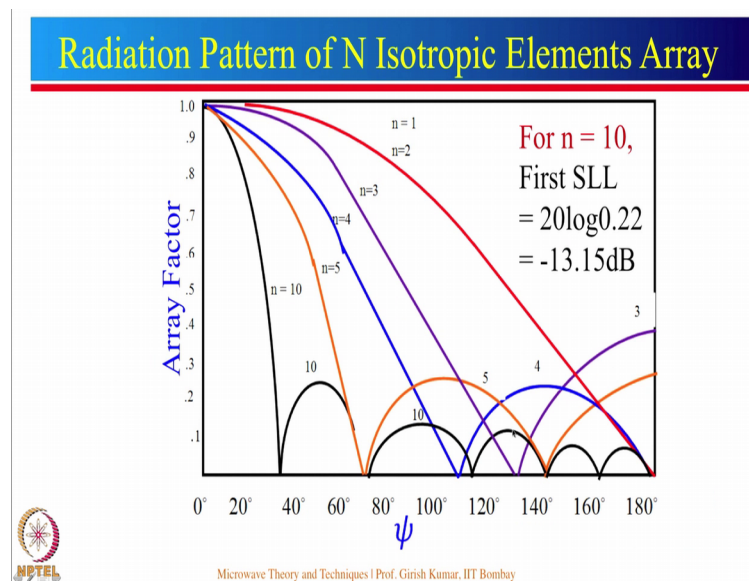
So, suppose if we feed this particular element with let us say one angle 0, then this will be 1 angle delta 1 angle 2 delta 1 angle 3 delta and so on. So, now, we have to simplify this particular equation. So, to do the simplification, let us first multiply this entire equation with the term E to the power j psi. So, then 1 will become e to the power j psi j psi will become 2 j psi and the last term will become e to the power j n psi.

Now, we take the difference between this and this if we take the difference from here to hear it will be E minus E to the power e j psi. And, from here if you see most of these terms will get cancelled, what we will be left with 1 minus e to the power j n psi. So, from here we can find the value of E which is given by this particular expression.

Now, how do we obtain this? If just think about it if we take outside  $E$  to the power  $j n \psi$  by 2 from here and from here take  $E$  to the power  $j \psi$  by 2 from the denominator. Then, this particular then this particular expression will become  $\sin n \psi$  by 2 divided by  $\sin \psi$  by 2. Now, as  $\psi$  tends towards 0. So, let us see what happens? If  $\psi$  is equal to 0 this becomes  $\sin 0$ , which is 0 divided by  $\sin 0$  this becomes 0 by 0. So, you may say it is indeterminate no, do not do that what we need to do it is we just take a limiting case when  $\psi$  is tending towards 0.

So, we know that  $\sin x$  is equal to  $x$  if  $x$  is very small. So, in this particular case if  $\psi$  is tending towards 0. So, this expression will become  $n \psi$  by 2 divided by  $\psi$  by 2 so; that means, this whole thing will be equivalent to now  $n$ . So, when  $\psi$  is equal to 0 we get maximum value of the  $e$  which is equal to  $n$ . So, normalized value of  $e$  will be you divide this expression by  $n$ . So, this is the normalized radiation pattern. Now, this radiation pattern can be calculated in this particular fashion.

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So, here is a radiation pattern of  $n$  isotropic elements array so, for  $n$  equal to 1 will have simply this kind of a pattern for  $\psi$  varying from 0 to 180 degree. So, when  $n$  is equal to 2 it varies from 1 to 0. So, for  $n$  equal to 3 you can see there is a null over here, at this particular thing is known as side lobe level. As,  $n$  increases you can see that half power beamwidth is decreasing, half power beamwidth will correspond to which point. So, this

is 1, this is 0.7 over 7 if you draw the line somewhere like this. So, this will be half of half power beamwidth, other half will come from this particular side here ok.

So, now you can also see that as number of elements increase, you can see that there are more number of side lobes are there all these are plotted by using that one simple expression. So, by using that expression, we can actually find out lot of other characteristics, let us now take some special cases of few element array.

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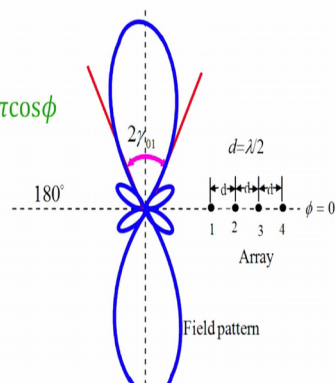
### Broadside Array (Sources In Phase)

$$\psi = \frac{2\pi d}{\lambda} \cos\phi + \delta$$


$$\delta = 0, d = \frac{\lambda}{2} \text{ and } n = 4 \Rightarrow \psi = \pi \cos\phi$$

$$E_n = \frac{\sin(n\psi/2)}{n\sin(\psi/2)} = \frac{\sin(2\psi)}{4\sin(\psi/2)}$$

$\phi$	$\psi$	E
$0^\circ$	$\pi$	0
$60^\circ$	$\pi/2$	0
$90^\circ$	0	1



Field pattern of 4 isotropic point sources with the same amplitude and phase. Spacing =  $\lambda/2$ .



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8

So, this is a broadside array in which all the sources are fed in the same phase. So, 4 example here, we have 4 elements distance between all the elements is equal to d. So, we know that psi is given by this expression, but since they are in same phase delta will be equal to 0.

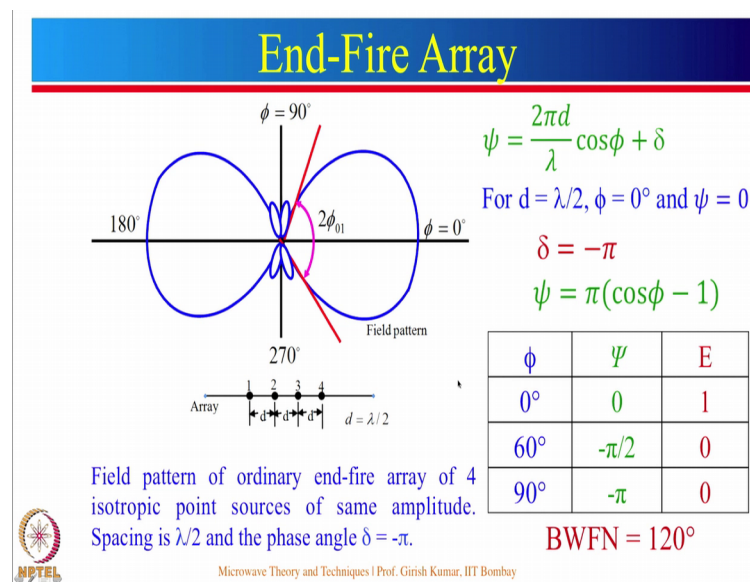
Let us take a case of d equal to lambda by 2 n equal to 4. So, if we substitute the value of delta over here d equal to lambda by 2 over here, we can simplify the expression of psi as psi equal to pi cos phi. So, what is now the normalized field? So, normalized field will be given by this expression substitute the value of n which is 4, it becomes 2 psi and this is 4 sin psi by 2.

Now, we will take again few cases of phi. So, phi 0 60 90, corresponding to this phi you can calculate the value of psi you can see that if this is 0 here. So, pi of cos 0 cos 0 will be equal to 1. So, this becomes pi. So, for phi equal to 60 cos 60 will be 1 by 2 psi

becomes  $\pi/2$  for  $\phi$  equal to  $90^\circ$   $\cos 90^\circ$  will become 0  $\psi$  becomes 0. Now, for these values of  $\psi$  we can use this expression to find the values of  $E$ . And, then we can plot these things as radiation pattern. So, you can see that this is the beam maxima in broadside direction, we have a side lobe level over here and there is a back radiation also.

In this particular case, it is easy to find out what is the beamwidth between the first null? So, you can see that  $E$  is equal to 0, which is along this then  $E$  is equal to 0 along  $\phi$  equal to  $60^\circ$  and  $E$  is equal to 1 along  $90^\circ$ . So, beamwidth between the first null will be this  $30^\circ$  and this  $30^\circ$ . So, total  $60^\circ$ .

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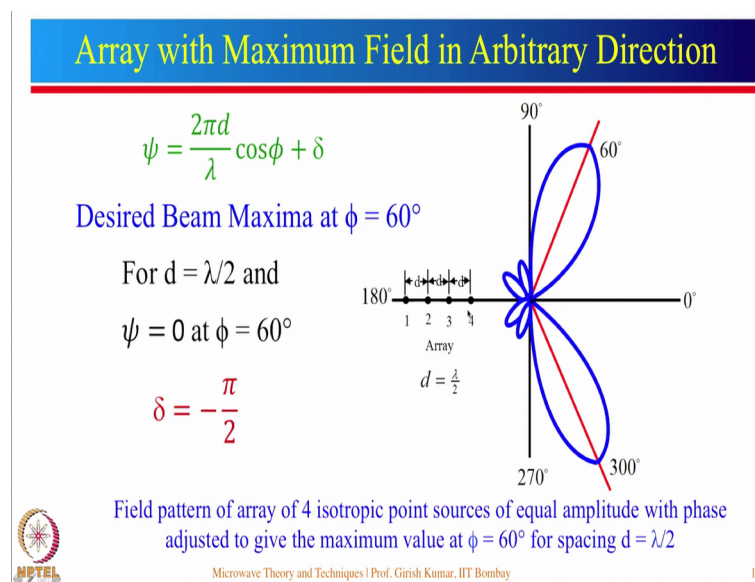
Let us take the case of end fire array. In this particular case what is that we have taken again  $d$  is equal to  $\lambda/2$ , but now we want the beam maxima to be in  $\phi$  equal to  $0^\circ$  plane. So, what you do you put  $\psi$  equal to 0? Because,  $\psi$  equal to 0 corresponds to the maximum value of the radiation.

So, for  $\psi$  equal to 0 and desired  $\phi$  equal to  $0^\circ$   $\delta$  comes out to be  $-\pi$  and for this value of  $\delta$  and  $d$  you can calculate the value of  $\psi$ , and again do the same thing plot the radiation patterns. So, you can see that now the radiation pattern has beam maxima in this particular direction. I just want to mention over here, this was the desired radiation pattern this is totally undesired radiation pattern. In fact, this is known as grating lobes we will see after few slides, what is grating lobe?

So, just I want to mention here this is actually a undesired radiation pattern end fire array should basically be radiating only in 1 direction. So, just to tell you to design a proper end fire array we should never ever take d is equal to lambda by 2. In fact, generally d is taken as lambda by 4 for end fire array.

When, I talk about Yagi Uda Antenna array I will take this particular case and I will show you that when you take d around lambda by 4, you get a endfire radiation pattern. So, please remember this is not a desired distance; you should always take d less than lambda by 2 for endfire array.

(Refer Slide Time: 16:29)



Now, let us see array with maximum field in arbitrary direction. So, let us say we want desired beam to be along phi equal to 60 degree. So, how do we find the value of delta well? Let us say for d equal to lambda by 2 what we need to do we put the condition psi equal to 0 at phi equal to 60 degree.

So, by substituting these values in this particular equation, you can find the value of delta which comes out to be minus pi by 2. So, if you have phase difference of 1, then minus pi by 2 then minus pi minus 3 pi by 2 you will get this kind of a radiation pattern. So, simply by changing the phase difference between different elements, you can scan the beam from broadside to any direction to all the way to the endfire direction. So, this is the concept of the phased array antenna.



Now, let us see how we can find the null direction? So, it is very easy to find the null direction for the array, we know that this is the array factor.

(Refer Slide Time: 17:38)

### Null Directions for Arrays of N Isotropic Point Sources

$$E_{norm} = \frac{\sin(n\psi/2)}{n\sin(\psi/2)}$$


For Finding Direction of Nulls:

$$\sin\left(\frac{n\psi}{2}\right) = 0 \rightarrow \frac{n\psi}{2} = \pm k\pi \quad \text{where, } k = 1, 2, 3, \dots$$

$$\psi = \pm \frac{2k\pi}{n}$$

For Broadside Array,  $\delta = 0$

$$\psi = \frac{2\pi d}{\lambda} \cos\phi_0 = \pm \frac{2k\pi}{n} \rightarrow \phi_0 = \pm \cos^{-1}\left(\frac{k\lambda}{nd}\right)$$


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11

All you do it is you make this particular term equal to 0, when this will be equal to 0, whenever inside this is multiple of k pi where k can be 1 2 3. So, we know that for 180 degree or multiple sin of that term will be equal to 0. So, by substituting this value of n psi by 2 equal to plus minus k pi, we can find the expression for psi. Now, for broadside array delta is equal to 0. So, you substitute the value of psi over here equated, we can find the direction of null by using this particular expression.



(Refer Slide Time: 18:21)

### Directions of Max SLL for N Elements Arrays

$$\sin \frac{n\psi}{2} = \pm 1 \rightarrow \frac{n\psi}{2} = \pm \frac{(2k+1)\pi}{2} \quad \text{where } k=1,2,3,\dots$$


$$\psi = \pm \frac{(2k+1)\pi}{n}$$

Magnitude of SLL:  $AF = \left| \frac{\sin \frac{n\psi}{2}}{n \sin \frac{\psi}{2}} \right| = \left| \frac{1}{n \sin \left( \frac{(2k+1)\pi}{2n} \right)} \right|$

For very large n:

$$AF = \left| \frac{1}{n \times \left( \frac{(2k+1)\pi}{2n} \right)} \right| = \frac{2}{(2k+1)\pi} = 0.212 \text{ for } k=1 \text{ (First SLL)}$$

SLL in dB =  $20 \log 0.212 = -13.5 \text{ dB}$


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12

Now, how to find the direction of maximum side lobe level for elements? It is simple to find out maximum side lobe level, what you need to do it is make numerator equal to plus minus 1. And, this will be plus minus 1 when n psi by 2 is nothing, but odd multiple of pi by 2 which is 90 degree.

So, now, we can simplify this particular thing we get expression for psi, you can substitute the value of psi which is equal to 2 pi d by lambda cos phi. Now, here phi will be the direction of maximum SLL. So, by solving this particular equation you can find the direction of maximum side lobe level.

Now, what is the magnitude of side lobe level? Well, again we can use this particular expression now. So, array factor is given by this term and for SLL numerator should be equal to 1. So, which is equal to 1, and what about this term here? Well, this is n sin and this is the expression for psi we substitute that over here. So, here we will just make a special case.

So, for very large n this term will become small. So, we can say sin x is approximately equal to x. So, that can be now substituted over here. Simplify, it this value comes out to be 0.212 for k equal to 1 which is first side lobe level. For k equal to 2 you can find the value of second side lobe level and similarly you can find the value of other side lobe level.

So, you can see that side lobe level in dB comes out to be 20 log of 0.212, which is equal to minus 13.5 dB.

(Refer Slide Time: 20:14)

### Half-Power Beamwidth (HPBW) of Array


For calculating HPBW, find  $\Psi$ , where radiated power is reduced to half of its maximum value:  $\Rightarrow AF = \frac{\left| \sin \frac{n\psi}{2} \right|}{n \sin \frac{\psi}{2}} = \frac{1}{\sqrt{2}}$

For large n, HPBW is small:  $\Rightarrow AF \cong \frac{\left| \sin \frac{n\psi}{2} \right|}{n \frac{\psi}{2}} = \frac{1}{\sqrt{2}} \Rightarrow$  Solution:  $n\Psi/2 = 1.3915$

For Broadside:  $\psi = \frac{2\pi d}{\lambda} \cos\phi = 2.783/n$

$\cos\phi = 1.3915 / (\pi nd/\lambda) = 0.443/L_\lambda$  (radian)

$HPBW \simeq 2 \times (90 - \phi) = 50.7^\circ / L_\lambda$


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13

Now, similarly we can find out half power beamwidth of the array, what you need to do we know that expression for array factor is given by this. So, this should be equal to 1 by square root 2. Why, we are equating to 1 by square root 2? The reason for that is normalized value of this is equal to 1 for maximum value. So, for half power, it will be 1 by square root 2. Again, we are making a little simplification here for large n, half power beamwidth will be small. Because, larger the array half power beamwidth will be small. So, we can make an approximation that is sin psi by 2 can be written as psi by 2, this is a sinc function, which is equal to 1 by square root 2 and you can solve this particular thing.

So, solution of that comes out to be n psi by 2 is equal to this. In fact, what you can do you substitute this value and calculate this using calculator, you will get this value of 1 by square root 2 please remember this is in radians. So, again for broadside, we can say psi is given by this expression. And, that is now equal to from here you can obtain this 2 goes over here 2.783 divided by n. So, from here we can find the value of phi and then half power beamwidth is given by this particular expression.

(Refer Slide Time: 21:39)

**Grating Lobes for Arrays of N Isotropic Point Sources**

To Avoid Grating Lobes:


$$\psi = \frac{2\pi d}{\lambda} (\cos\phi - \cos\phi_m) \leq 2\pi$$

where,  $\phi_m$  is direction of max. radiation

$$\frac{d}{\lambda} \leq \frac{1}{\cos\phi - \cos\phi_m} \rightarrow \frac{d}{\lambda} \leq \frac{1}{1 + |\cos\phi_m|}$$

For Broadside Array:  $\frac{d}{\lambda} < 1 \rightarrow d < \lambda$

For Endfire Array:  $d < \frac{\lambda}{2}$



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14

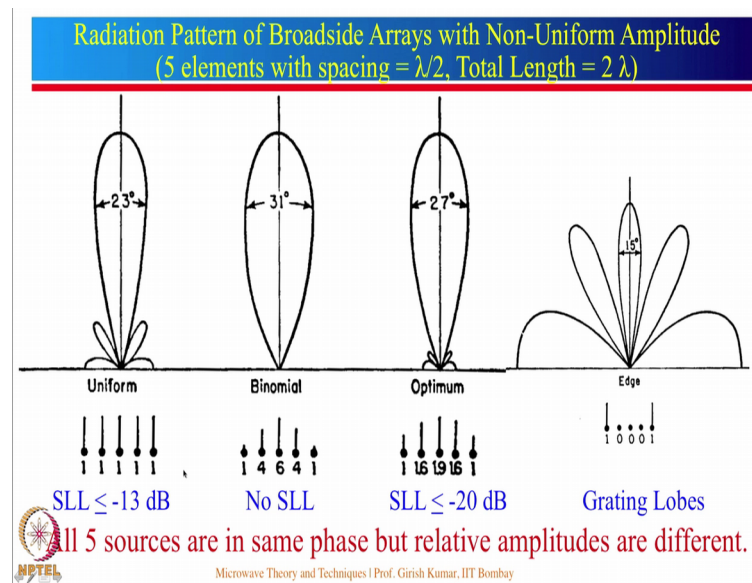
Now, I am going to talk about what is grating lobe and how we can avoid these grating lobes? Ok. So, to avoid grating lobes, we have to put the condition that  $\psi$  should be always less than or equal to  $2\pi$ . We had seen that beam maxima comes for  $\psi$  equal to 0, but again beam maxima comes when  $\psi$  becomes equal to  $2\pi$ . So, if we maintain this condition that  $\psi$  should be always less than  $2\pi$ , then we can avoid grating lobe.

So, this particular expression now can be simplified you can see from here  $d$  by  $\lambda$  should be now less than or equal to  $2\pi$   $2\pi$  gets cancelled 1 divided by  $\cos\phi$  minus  $\cos\phi_m$ , where  $\phi_m$  is direction of maximum radiation.

So, now we need to find what is the worst case condition? We always do the design for worst case condition. So, that gives us  $d$  by  $\lambda$  should be always less than this particular term over here. So, for broadside array this particular thing reduces to  $d$  by  $\lambda$  less than 1; that means,  $d$  must be less than  $\lambda$  and for endfire array  $d$  should be less than  $\lambda$  by 2. So, recall I did mention to you for endfire array that example I have taken  $d$  equal to  $\lambda$  by 2 and that is why there was grating lobe, had we taken  $d$  less than  $\lambda$  by 2 there would not have been a grating lobe.

Now, let just look at the problem of uniform feed. What is the problem with the uniform feed? The problem with the uniform feed is here the example is taken for 5 elements all the elements are fed with uniform amplitude and phase.

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For this particular case the problem is that side lobe levels are less than minus 13 dB or so in fact, in fact even for a very large uniform array side lobe level can be at best 13.5 dB. There are many applications where we would like side lobe level to be less than 20 dB or less than 30 dB. So, you can see that for 5 elements fed with equal amplitude and equal phase half power beamwidth is of the order of 23 degree, but side lobe levels are of the order of 13 dB.

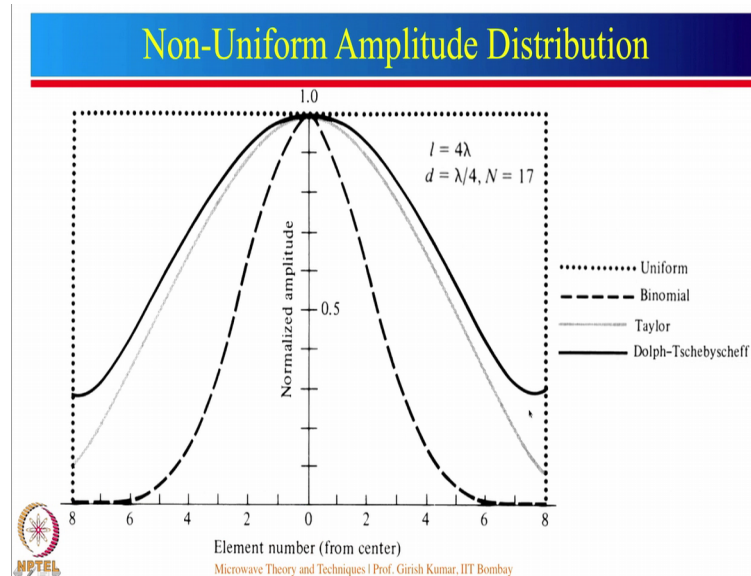
Now, if instead of taking this distribution, if you take binomial distribution, which will be amplitude 1 4 6 4 1, in this particular case there is no side lobe level. However, there is a problem with this particular case; half power beamwidth is 31 degree, which is much larger than 23 degree. Hence, gain of this particular array will be smaller than this particular array.

Now, this is the another distribution which is 1 1.6 1.9 1.6 1. In this particular case side lobe level is of the order of minus 20 dB, you can see this is smaller than this over here, in this particular case half power beamwidth is about 27 degree. So, gain of this will be smaller than this, but gain of this will be larger than this ok.

Now, just to show you the case of grating lobe, if only 2 elements are fed these are the 2 extreme elements. Now, what is the distance between this element and this see we had taken spacing as lambda by 2. So, total length is 2 lambda. So, because of 2 lambda

length, you can see now there are 2 grating lobes which have occurred over here. So, this particular condition should be avoided.

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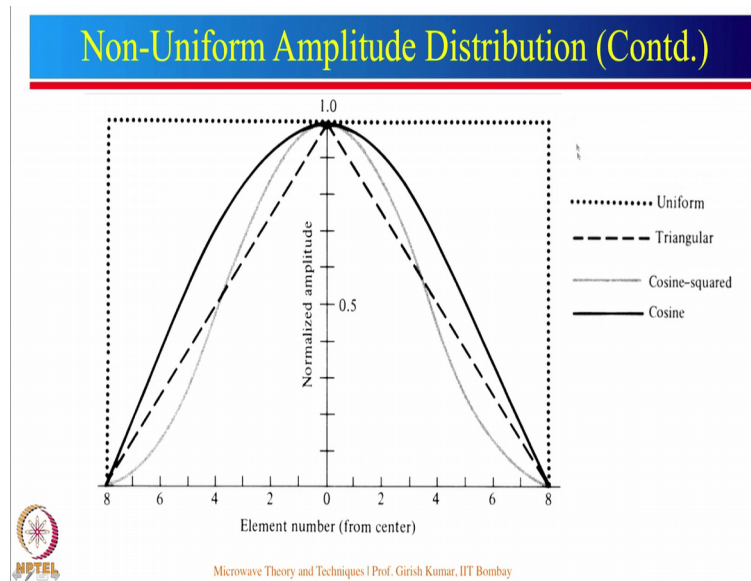


So, how we can get this non uniform amplitude distribution? So, this is the uniform distribution case shown over here is for 17 elements all the elements have equal spacing of lambda by 4. So, total length of this particular array will be from here to here. So, total length of the array will be equal to 4 lambda.

So, this is the distribution uniform all elements are fed with equal amplitude. For binomial distribution you can see this will be the distribution, and you can actually notice that the last few elements are fed with very very small amplitude. So, In fact, this almost looks like equivalent to a 13 element array instead of 17 element array that is why gain of binomial distribution is relatively small. So, this is the Taylor distribution this is the Dolph-Tschebyscheff distribution.

However, I am going to show you somewhat similar to this distribution.

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Which is actually uniform, triangular, cosine squared, cosine. I will tell you little later why I have shown you this particular thing, but let us plot these thing. So, uniform as before it is over here triangular will be the distribution is like this cosine distribution corresponds to this particular plot here. And, cosine squared is somewhat similar to this. So, now, let me show you the correlation you can see that this field distribution is almost similar to that of binomial distribution except that some amplitude is there even at this particular element; the reason why this is more popular because analytical expressions are available for this particular distribution.

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### Current Distribution for Line-Sources and Linear Array

Distribution	Uniform	Triangular	Cosine	Cosine-Squared
Distribution $I_n$ (analytical)	$I_0$	$I_1 \left(1 - \frac{2}{l} z'  \right)$	$I_2 \cos \left(\frac{\pi}{l} z' \right)$	$I_3 \cos^2 \left(\frac{\pi}{l} z' \right)$
Distribution (graphical)				
Space factor (SF) $u = \left(\frac{\pi l}{\lambda}\right) \cos \theta$	$I_0 \frac{\sin(u)}{u}$	$I_1 \frac{1}{2} \left[ \frac{\sin \left(\frac{u}{2}\right)}{\frac{u}{2}} \right]^2$	$I_2 \frac{\pi}{2} \frac{\cos(u)}{(\pi/2)^2 - u^2}$	$I_3 \frac{1}{2} \frac{\sin(u)}{u} \left[ \frac{\pi^2}{\pi^2 - u^2} \right]$
Space factor  SF				

Microwave Theory and Techniques | Prof. Girish Kumar, IIT Bombay
18

So, just to show you uniform, triangular cosine, cosine squared. So, this is the uniform distribution, this is the triangular distribution, cosine distribution, and this is cosine squared distribution. And, for this particular case here, space factor is shown. In fact, space factor is similar to array factor with the condition that the element spacing is very very small and number of elements are large. So, you can see that analytical expressions are available for this distribution.

So, what are the properties of this distribution let us look at these.

(Refer Slide Time: 27:59)

Radiation Characteristics for Line-Sources and Linear Array				
Distribution	Uniform	Triangular	Cosine	Cosine-Squared
Half-power beamwidth (degrees) $l \gg \lambda$	$\frac{50.6}{(l/\lambda)}$	$\frac{73.4}{(l/\lambda)}$	$\frac{68.8}{(l/\lambda)}$	$\frac{83.2}{(l/\lambda)}$
First-null beamwidth (degrees) $l \gg \lambda$	$\frac{114.6}{(l/\lambda)}$	$\frac{229.2}{(l/\lambda)}$	$\frac{171.9}{(l/\lambda)}$	$\frac{229.2}{(l/\lambda)}$
First sidelobe max. (to main max.) (dB)	-13.2	-26.4	-23.2	-31.5
Directivity factor ( $l$ large)	$2 \left( \frac{l}{\lambda} \right)$	$0.75 \left[ 2 \left( \frac{l}{\lambda} \right) \right]$	$0.810 \left[ 2 \left( \frac{l}{\lambda} \right) \right]$	$0.667 \left[ 2 \left( \frac{l}{\lambda} \right) \right]$



So, these are the expressions for half power beamwidth. So, you can see that this is the expression for half power beamwidth for uniform, this is for triangular, this is for cosine, this is for cosine squared. So, you can see that here beamwidth is much larger compared to this particular case over here. Hence the gain of this is also relatively small. Let just look at the advantages, what are the advantages of cosine squared?

So, 4 uniform we get sidelobe level of minus 13.2. If, we take triangular distribution it is minus 26.4, if we take cosine distribution minus 23.2 and for cosine squared we get minus 31.5 dB. So, by choosing this distribution, you can actually reduce the sidelobe level.

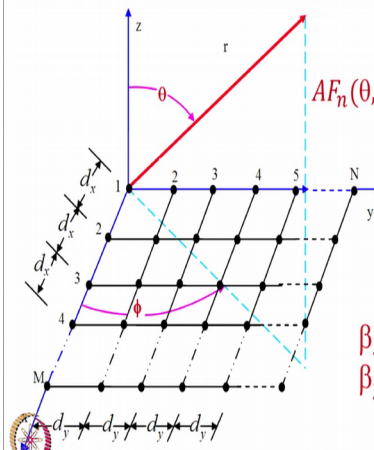
However, 1 has to pay the penalty for that and that penalty is paid in terms of reduced directivity. So, directivity expression for uniform field is given by 2 times  $l$  by  $\lambda$ ,



where  $l$  is the length of the array. So, directivity of triangular waveform is given by this particular expression, you can see that it is reduced by a factor of 0.75 for cosine it is reduced by a factor of 0.8 1, and for cosine squared it is reduced by a factor of 0.667. So, you can actually see the directivity of this is about two third of the directivity of uniform amplitude array.

(Refer Slide Time: 29:35)

Rectangular Planar Array



$$AF_n(\theta, \phi) = \left\{ \frac{1}{M} \frac{\sin\left(\frac{M\psi_x}{2}\right)}{\sin\left(\frac{\psi_x}{2}\right)} \right\} \left\{ \frac{1}{N} \frac{\sin\left(\frac{N\psi_y}{2}\right)}{\sin\left(\frac{\psi_y}{2}\right)} \right\}$$


where

$$\psi_x = kd_x \sin\theta \cos\phi + \beta_x$$

$$\psi_y = kd_y \sin\theta \sin\phi + \beta_y$$

$$\beta_x = -kd_x \sin\theta_0 \cos\phi_0 \text{ for } \psi_x = 0$$

$$\beta_y = -kd_y \sin\theta_0 \sin\phi_0 \text{ for } \psi_y = 0$$


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20

Now, we will shift to the next configuration, which is rectangular planar array. So, here we have  $n$  elements along this particular axis. And, there are  $m$  elements along  $x$  axis. So, distance between the elements along the  $x$  axis denoted by  $d_x$  and  $d_y$ . And, along this particular direction distance between the elements is  $d_y$  all these elements have equal distance. So, now, let us see how we can find out the overall radiation pattern for this rectangular planar array?

So, first what we do we combine the radiation pattern of 1 linear array. So, just think about this 1 linear array. So, we know how to find out the radiation pattern of a linear array you can see that there are  $M$  elements. So, this is the normalized radiation pattern for all these elements over here along  $x$  axis, notice there is a small change instead of  $\psi$  there is a  $\psi_x$   $\psi_x$  is given by this particular expression. I tell you in a short while what does that mean?

Now, think about all these elements, now we have found out the array factor of all of these elements. Now, think about the principle of pattern multiplication, which we

discussed earlier. So, all these elements now can be represented by single array factor. All of these elements now can be represented by single array factor. So, you can now think about there are only  $N$  elements and we know what is the radiation pattern of each element now? So, all we need to do it is we multiply this particular array factor with an array factor of the elements along the  $y$  axis.

So, you can see that there are  $N$  elements. So, this is the array factor for this particular case over here. So, total array factor for this rectangular planar array can be obtained by multiplying the array factor of this linear array with this linear array. So, this is the overall array factor for this planar array. So, let us see now, what are the  $\psi_x$  and  $\psi_y$  values. You can see here that I have written  $\psi_x$  is equal to  $k d_x \sin \theta \cos \phi + \beta_x$  but  $\beta_x$  is nothing, but phase difference between the elements along  $x$  axis.

So, let us see now why these 2 terms are there? So,  $k$  is same as  $\beta$ , which is equal to  $2\pi/\lambda$ .  $d_x$  is the distance along the  $x$  axis. Now, why we have these term here? So, let us see now we are trying to find the radiation pattern at a point  $p$  which is at a distance  $r$ . So, what we do we take the projection of this particular point along  $x-y$  plane? So, we take the projection over here and then draw a line like this here. So, now, corresponding to this particular  $x$  axis this angle is  $\phi$  so; that means, this whole thing has to be multiplied by  $\cos \phi$  and that is why the term  $\cos \phi$  comes into picture.

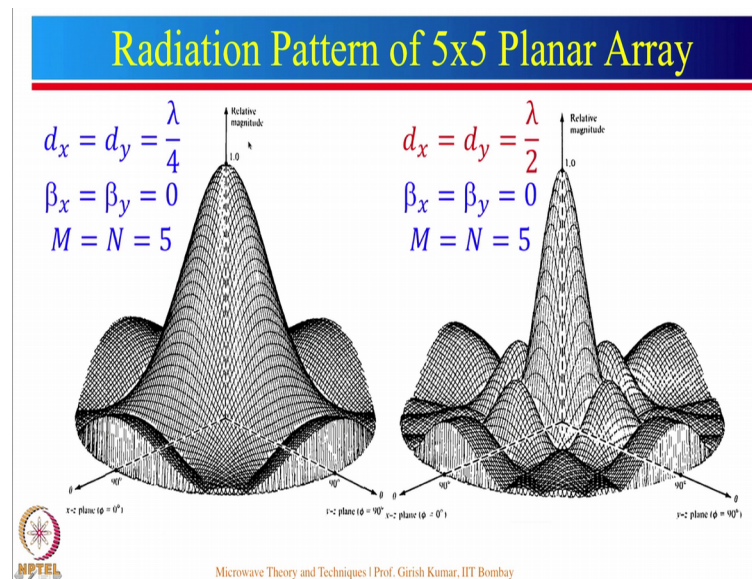
Now, this has to be taken along this particular direction. So, this will be now this angle is  $\theta$ . So, this will be  $90 - \theta$ . So, this will be now multiplied by  $\sin \theta$ . So, that is why  $\psi_x$  has  $\sin \theta \cos \phi$ . Let us see what is happening for  $\psi_y$ ? Again, now we take the projection of this particular thing on the  $x-y$  plane. Now in this particular case we have to move along  $y$  axis. So, this will be now  $90 - \phi$ . So, that will be equivalent to  $\sin \phi$  and then again this has to be taken along this that is to be multiplied by  $\sin \theta$ . And,  $\beta_y$  is the phase difference between all the elements in the  $y$  direction.

So, now let us say we want to find out what should be the value of  $\beta_x$  and  $\beta_y$  for the desired direction of beam maxima along  $\theta_0$  and  $\phi_0$ . So, what you do you put  $\psi_x$  equal to 0  $\psi_y$  equal to 0. So, by putting this we can find the value of  $\beta_x$  and  $\beta_y$ . So, whatever is the desired direction? Suppose we want beam maxima to be let us say at  $\theta_0$  equal to 30 degree  $\phi_0$  equal to 45 degree. So, you substitute this value.

So, let us say we want the desired beam maxima to be along theta 0 equal to 30 degree and phi 0 equal to 45 degree substitute these values and you can find the value of beta x and beta y.

So, just to tell you so, what will happen in that particular case? So, this will be 0 this will be beta x this will be 2 beta x this will be 3 beta x and so on. This will be 0 beta y 2 beta y 3 beta y and so on. What about this here this will be beta y plus beta x corresponding to this here over here it will be 2 beta x plus beta y. So, this is how you actually have to calculate the phase difference for all these element.

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So, let us see the radiation pattern of 5 by 5 planar array, I have shown here 2 different cases of the distances d x is equal to d y equal to lambda by 4 over here. And, in this particular case the only difference is that d x is equal to d y equal to lambda by 2. So, in this particular case we wanted a broadside radiation pattern. So, for broadside radiation pattern beta x equal to beta y equal to 0. And, we have taken a square array of 5 by 5 elements. So, M is equal to N equal to 5.

So, you can see the radiation pattern for d equal to lambda by 4 and for d equal to lambda by 2, you can see that this has a much narrower beam the reason for that is the distance is larger. So, total aperture area will be more and if the total aperture area is more gain will be more and hence half power beamwidth will be small.

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## Directivity of Rectangular Planar Array

$$D = \pi D_x D_y \cos \theta_0$$

For Broadside Array ( $\theta_0 = 0$ ):

$$\begin{aligned} D &= \pi D_x D_y \\ &= \pi (2L_x / \lambda) (2L_y / \lambda) \\ &= 4\pi L_x L_y / \lambda^2 = 4\pi A / \lambda^2 \end{aligned}$$

where A = Area of the Array



So, now let us see how to find the directivity of the rectangular planar array? So, to find the directivity of the planar array all you need to do it is use this particular expression, where  $D_x$  is the directivity along the x axis for the linear array  $D_y$  is the directivity of the elements along the y array.

Now, what is this  $\cos \theta_0$ ?  $\cos \theta_0$  comes into picture if the beam is not in the broadside, let us say if the beam maxima is at let us say  $\theta_0$  equal to 30 degree. Then you have to substitute  $\cos 30$  degree, that would mean directivity is relatively smaller by a factor of  $\cos \theta_0$ , but for broadside array  $\theta_0$  is equal to 0. So, we can find the value of directivity as this particular expression.

Now, let us try to put the expression of directivity for broadside array over here. So, the expression for  $D_x$  I had shown you in the table. So,  $D_x$  is  $2L_x / \lambda$   $D_y$  is  $2L_y / \lambda$ . So, if we now simplify it comes out to be  $4\pi L_x L_y / \lambda^2$ . And, what is the area of the array  $L_x$  multiplied by  $L_y$ , you can see that this is the familiar expression directivity is given by  $4\pi A / \lambda^2$ .

So, just to summarize today we discussed about linear array, we talked about uniform amplitude followed by non-uniform amplitude array, we also talked about broadside array, endfire array, and the beam maxima in any desired direction. This is the principle of phased array antenna, that you change the phases between the different elements. And

just by changing the phase difference between the different element, you can scan the beam from broadside to all the way to the endfire direction.

So, by changing the phase you can scan the beam in this particular direction or you can also scan the beam in this particular direction. So, that is the principle of phased array antenna. Then, we briefly talked about rectangular planar array and we saw how to find the radiation pattern of the rectangular planar array, by simply using the pattern multiplication concept, where we multiply the directivity of 1 linear array with the another linear array. So, in the next lecture I will talk about microstrip antennas till then bye.