# Microwave Theory and Techniques

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## Module - 07
## Lecture - 35
## Low Noise Amplifiers - I: Noise Sources and Noise Figure

Hello, in the last few lectures we have been talking about microwave amplifiers. To design a microwave amplifier using either a transistor or even an IC amplifier you must first see what are the s parameters at the desired frequency, what are the biasing conditions. After that you find out what is the value of delta and k so that you know whether the amplifier is stable or not at that particular frequency. If it is not stable, in that particular case, what do you do, you draw input and output stability circles and wherever the stability circles cut the Smith chart, you avoid that particular portion and choose the gamma s value and gamma l values in the different part of the Smith chart which does not intersect the stability circle.

After you check the stability part, then what we did then after that you calculate what is the g t u max, so that you know whether the desired amplifier can be designed for a given gain. So, once you know that desired gain is less than the g t u max, then start with g t u; then you choose the value of g s and g l such a way that they will give the value of g t, after that you draw the gain cycles for both gamma s and gamma l. Choose the values of gamma s and gamma l on the constant gain circles, which are relatively closer to the centre point which is 50 ohm; so that impedance matching network becomes easier and simplified.

Today, we are going to talk about low noise amplifiers. So, low noise amplifiers are one of the most important amplifier at the input of the receiver.For example, the signal received by the receiver through an antenna is very very small and it has travel through the atmosphere, so it actually has lot of noise built into that. So, we do not want an amplifier which also adds noise to it. However, you cannot avoid the noise, what you can do you design an amplifier with minimum noise figure. So let us see; what are the sources of the noise and then how we can design a low noise amplifier.

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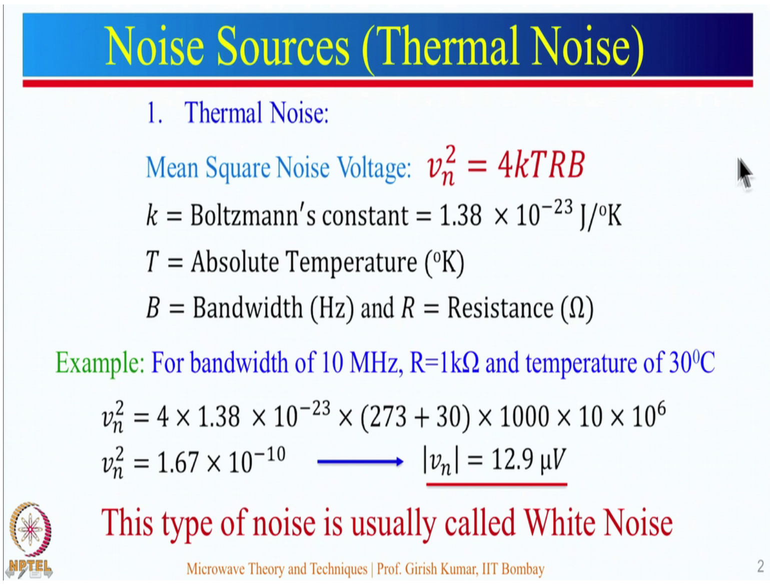


So, we are going to look at different noise sources. So, let us start with the first one which is a thermal noise ok. So, thermal noise is basically because of the resistors in the circuit. So, how thermal noise comes in to picture well when a current flows through the resistor, there will be power dissipation in the resistor and that will be given by i square R. So, this power dissipation leads to the heat and that is what is thermal effect, and that is what is known as thermal noise.

The mean square noise voltage v n square is given by 4 k T R B. What is k? K is Boltzmann's constant; the value is 1.38 into 10 to the power minus 23 joules per degree K, where K is temperature in Kelvin. What is T? T is absolute temperature in Kelvin, this is actually equal to 273 degree plus temperature in centigrade. B is bandwidth, so bandwidth over which that particular receiver is operating and R is the resistance value.

Let us take an example to calculate v n square for given parameters of bandwidth which is equal to 10 megahertz, 10 megahertz is reasonable bandwidth to assume for any cellular band. For example, we know that g s m 900 volts from 890 to 960; of course, that is divided into two separate bands 890 to 950 and 935 to 960 megahertz. Of course, one band is not going to be 10 megahertz that is generally going to be of the order of 1 megahertz, but for let us say Wi-Fi communication frequency values for Wi-Fi communication is from 2.4 to 2.483 gigahertz. Now that means, bandwidth of 83 megahertz, but that is not really correct part of the bandwidth is only used for transmission part, of the bandwidth is only use for reception and that bandwidth is of the order of 20 megahertz. So, 10 megahertz we can say is between that cellular band and the Wi-Fi band.

So, for a bandwidth of 10 megahertz and let us say resistance is equal to 1 kilo ohm and room temperature is let us say 30 degree centigrade. So, what will be the v n square well let us look at this expression, So 4 into k 1.38 into 10 to the power minus 23 into T into Kelvin that will be 273 plus 30 multiplied by resistor which is 1000 multiplied by bandwidth which is 10 into 10 to the power 6 you do the calculation. So, noise voltage comes out to be 12.9 micro volt. Now, this type of noise is also known as white noise; the reason for that is resistors are generally frequency insensitive. So, white noise is basically is a noise, which can span from dc to terahertz and more frequency range. So, the thermal noise generated by resistor is also known as white noise.

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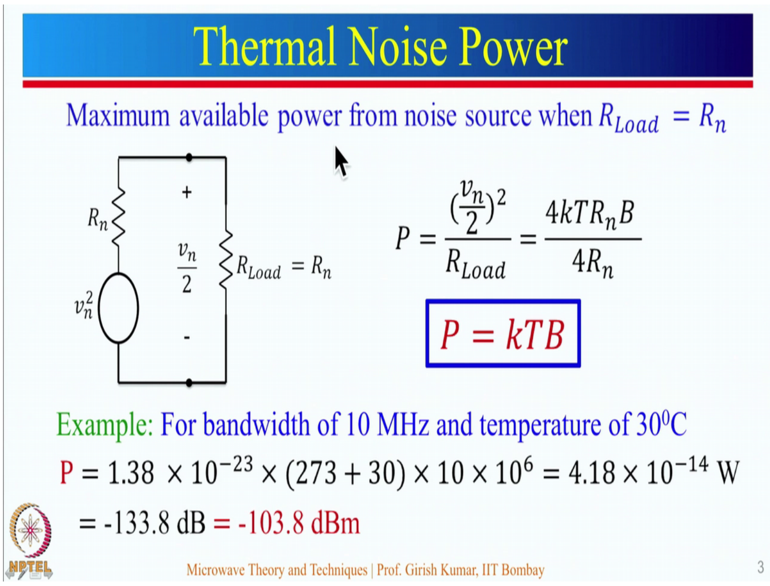


So, let us say see how much is the noise power associated with the noise resistor. So, maximum available power from noise source will be when R load is equal to R n. Now, we are not trying to optimize the maximum noise, the reason why we choose R load equal to R n or may be the source resistance. So that maximum signal is also, so that maximum signal is transmitted to the load ok. So, let us say this is the noise source which has an associated resistance R n. So, for maximum signal power transmission which will also lead to maximum noise power, R load should be equal to R n. So, we can say that this voltage will be equal to v n by 2. I just want to mention, that I have not deliberately written here v n; the reason for that is v n sign is not predictable. So, it may have a plus here or minus over there.

So, generally speaking we represent in the form of v n square, but you have to understand this is v n by 2, so you have to take square of that if you want to calculate the power, so power deliver to the load will be v n by 2 square divided by R load. So, this is given by four k T R n B; where R n is the noise resistance and for maximum noise power calculation R load should be equal to R n, which is true for maximum signal power transmission also.

So, now P comes out to be k T B. You can actually see here interesting thing that there is a no resistance value coming into picture. So, what is that mean that is noise power independent of the resistor, well actually speaking I would like to say that this particular expression has been derived assuming that this load is equivalent to R n. If R load is not equal to R n, in that particular case R will come into picture.

So, let us take the same example, but now we are going to calculate; what is the maximum noise power coming to the receiver end. So, for bandwidth of 10 megahertz temperature of 30 degree centigrade, we can calculate the value of P which is k T B. So, k is this, T is 273 plus 30, this is the bandwidth and this comes out to be 4.18 into 10 to the power minus 14 watt. So, you can see that this is a very very small number. Let us take the d B of this; so that is comes out to be minus 133 point 8 dB, which is equivalent to minus 103.8 dBm.

Now, I want to mention here that the receiver sensitivity of let us say mobile phone can be from minus 70 dBm to minus 100 dBm. In fact, there are many mobile phones which even have a sensitivity up to minus 110 dBm. So, this noise power is not negligible in comparison to minus 100 dBm or even lower value which can be received by the mobile phone ok.

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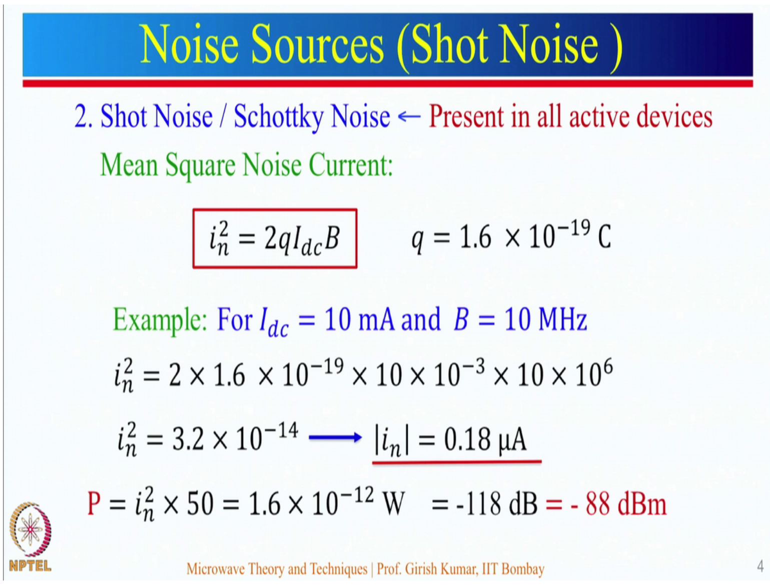


Now, let us just look at the another source of the noise which is a shot noise also known as Schottky noise, but basically I want to tell you this results from pn junction. So, any diode or transistor would have a pn junction. So, there will be current flowing through that pn junction and let us say that dc value of that current is i dc. Then mean square noise current is given by this particular expression, where q is the charge which is given by 1.6 into 10 to the power minus 19 Coulomb, and B is the bandwidth over which a receiver is going to operate or you can say you have to design amplifier to operate over that particular bandwidth.

So, let us say I dc is equal to 10 milliampere, and B is equal to 10 megahertz. So, let us see what is the value of i n square. So, i n square is given by two times q 1.6 into 10 to the power minus 19; I dc we have taken as 10 milliampere; B is bandwidth which is 10 megahertz. So, from here if we calculate the value of i n, that comes out to be 0.18 microampere. So, let us calculate; what is the noise power due to this particular noise current to do the power calculation; we have to assume the value of resistor. So, for most of the microwave circuit resistance value is typically taken as 50 ohm. So, P will be i n square into 50 and that comes out to be minus 118 dB which is minus 88 dBm.

So, you can see that this is fairly large value; if you think about a mobile phone receiver sensitivity, so that is why majority of the time first stage of the low noise amplifier does not used by biasing current of 10 milliampere, it may be of the order of 1 to 2 milliampere. And if you take this as let us say 1 milliampere. So, we will see that the value will decrease correspondingly, so i n square will decrease correspondingly by 10 times and that means, noise power will also decreased by about 10 dB which will be about minus 98 dBm. So, please remember when we are designing a low noise amplifier, we have to really be very very sure that the noise power generated by the amplifier should be as small as possible.

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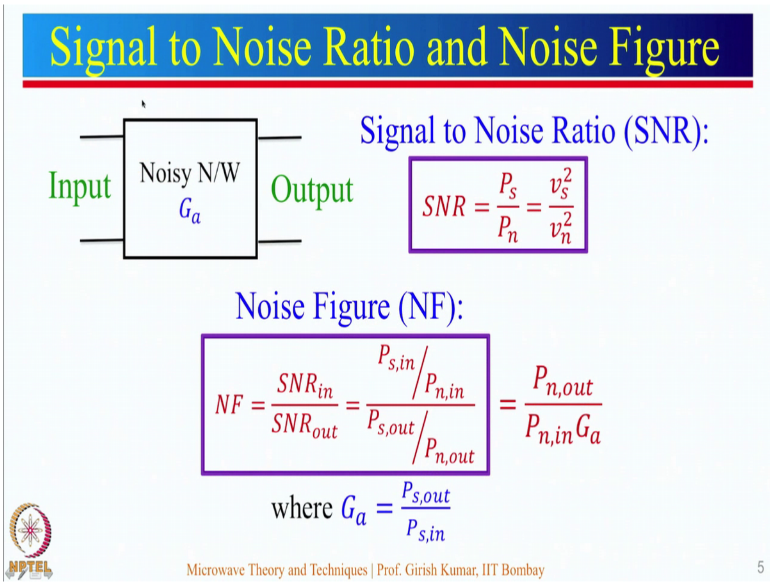


So, now let us define noise figure. So, first we will define signal to noise ratio, I am sure most of you are familiar with signal to noise ratio. Signal to noise ratio is defined as P s divided by P n which is nothing but signal power divided by noise power. And of course, these things will be related to voltage, so this will be v s square by R, this will also be v n square by R, so R, R, R will get cancel. So, this is the signal noise ratio in terms of the signal voltage in terms of the noise voltage.

Now, let us define noise figure symbol is NF. So, noise figure is defined as signal to noise ratio at the input side, divided by signal to noise ratio at the output side. So, you can see that this is the input of a noisy network, which has a gain of G a and this is the output. So, from where this noisy network is coming into picture think about any amplifier. Let us start with the transistor, a transistor amplifier would have several resistors, it will have a few pn junction. So, those pn junctions can be then represented by noise current.

So, the entire amplifier circuit can be represented in terms of its Thevenin equivalent or Norton equivalent. So, what is a Norton equivalent that will be a current in parallel with resistor. So, we can say that any amplifier can be ultimately simplified to a noise current and a resistor. So, we have already seen what are the noise current and resistor. I just want to mention here, that inductors and capacitors do not give rise to any noise that is true only for ideal inductor and capacitor.

A real inductor will be represented by inductor and series resistance, that series resistance will give rise to noise power. Similarly and lossy capacitor can be represented in terms of ideal capacitor in parallel with lossy resistors. So, for that noisy resistor we have to calculate; what is the noise power, but just to say in general any noisy network can be represented in terms of a current source and resistance for which we have already given you the expression; one can find out; what is the equivalent noise coming out of that particular circuit.

So, let us just define now, the gain of this particular amplifier is G a; so let us find out the expression for noise figure now, so that is signal to noise ratio at input by output. Now, of course some noise will be generated by the network, so this particular expression will be always greater than 1. So, let us see now signal to noise ratio at input is given by P s input divided by P n input you can see from here, and signal to noise ratio at output will be given by this particular expression, simplify this further that comes out to be P n out divided by P n in multiplied by G a. What is G a? G a is nothing but gain of this particular amplifier and that is equal to P s out divided by P s in.

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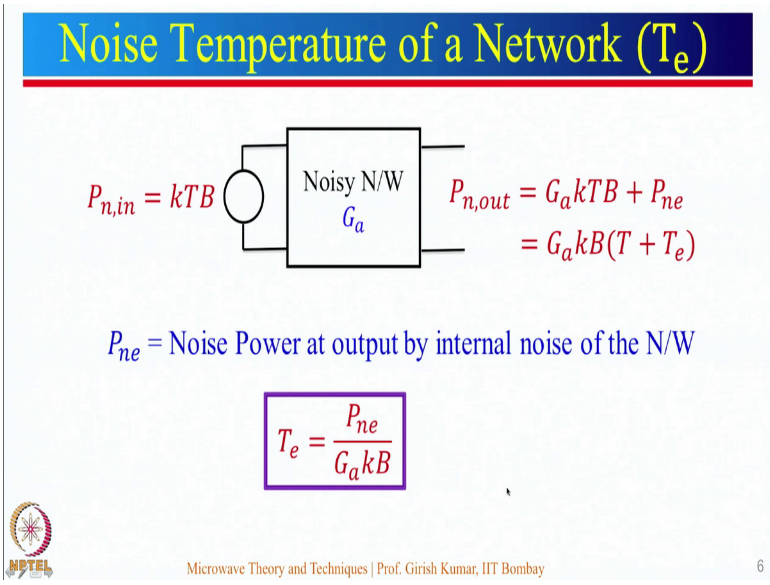


Now, we are going to define another term this is known as noise temperature of a network. I just want to mention that there is nothing like a noise temperature of a network, it does not mean that network will have a this much temperature or some other temperature. This is just a mathematical representation of noise figure, only to find out overall noise figure of a larger circuit. So, please remember noise temperature is not a physical quantity it is just a mathematical way of representation of noise figure.

So, let us just represent now. So, we had seen that P n is nothing but equal to k T B and that is given to the noisy network, so let us see what will be P n out. So, p n out will be nothing but gain multiplied by this particular power; so that will be G a k T B plus P n e which is nothing but noise generated by this particular network. So, this particular thing is now further written in this particular form, where P n e is represented in the form of T e multiplied by these terms over here.

So, we can say that P ne is noise power at output by internal noise of the network, and this internal noise can be due to all the resistors, due to all the current sources, due to all the voltage sources within the network. So from here we can say that T e is nothing but P ne divided by G a k B, you can actually think this whole thing in a slightly simpler manner as shown in the next slide.

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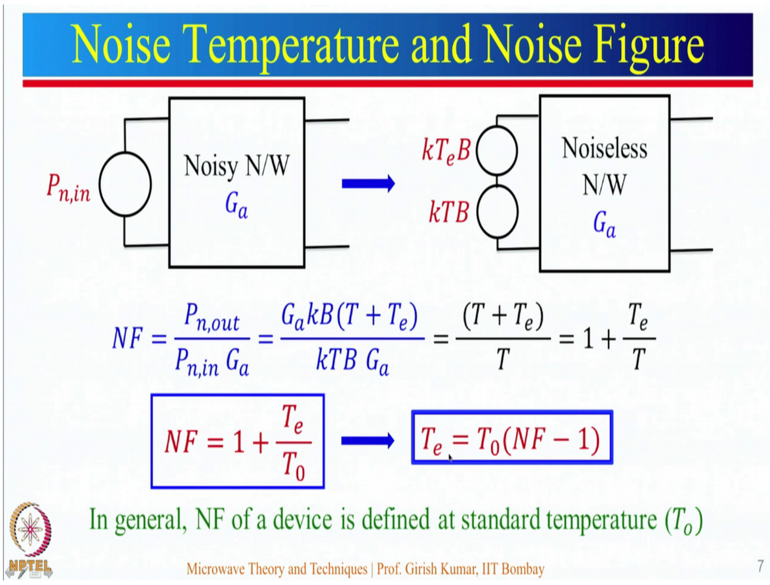


So, the same thing which we had shown in the previous slide; there was a noise input noisy network, but now this can be presented in this particular form where this network is noiseless network. So, how we have taken care of this noisy part? So, we had seen in the previous slide that this particular network generates noise of P n e, and that P n e is presented in the form of k T e B multiplied by G a. So, this term has been taken in the input side, and when finally we look at the output, output will have this term multiplied by this particular term over here.

So, we can now say what will be noise figure. So, noise figure is given by this particular expression. So now, let us substitute the values. So, P n out is nothing but G a k B T plus T e and what is this term P n in is k T B G a comes as it is. If we now simplify this, you can see most of the terms will get cancel G a k B, G a k B will cancel, so we are left with T plus T e divided by T which is equal to 1 plus T e divided by T; I just want to mention this is a general term for given T.

However, a manufacturer has to specify the noise figure of a network, they cannot define noise figure for any arbitrary temperature. So, what they do? they actually define noise figure for a standard temperature and I just want to tell this value of T 0 is equal to 290 degree k, so 290 is equal to 273 plus 17 degree centigrade. So, they have taken temperature as 17 degree centigrade of course in India, generally speaking we put have taken as 27 degree. In fact, that would have given a nicer number 273 plus 27 would have been 300 degree, but however since all these developments had been done in the western world they took 17 degree. So, we have to take T 0 as 270 plus 70 which is 290 degree Kelvin. So, the manufactures defined noise figure for this particular standard temperature.

Now, this can be simplified further to T e equal to T 0 times NF minus 1. Again I want to repeat that all these T e are only there to do the simplification after network, as we will see in the next slide.

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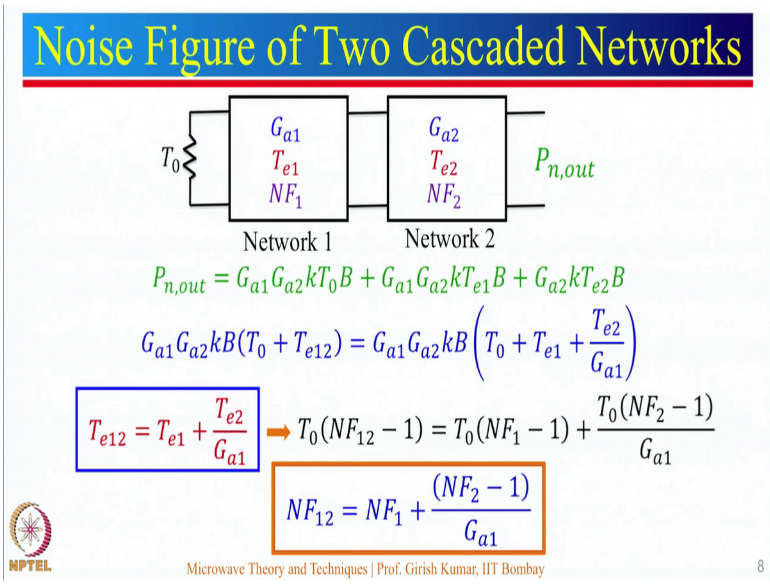


So, now we want to find out noise figure of two cascaded network. So, let us say we have a network one, which is defined by gain G a 1, T e 1, NF 1. In fact, most of the time manufacturer will only give G a 1 and NF 1. So we have to calculate T e 1, this is network to defined by G a 2 noise figure 2. So, now we want to find out what is P n out. So, I just want to mention here, so here it shows a resistor here, but it is represented by T 0. So, this is nothing but a standard temperature which is equal to 290 degree.

So, let us see how we can find out the value of P n out. So, P n out is given by this particular term which will correspond to k T 0 B multiplied by gain G a 1, G a 2. So, you can see that this particular term corresponds to this temperature. Then T e 1 is to be taken in the input side, so for this also k T e 1 B multiplied by G a 1 and G a 2, for T e 2 this will go to the input side of this one here. So, this will be only multiplied by G a 2, so you can see that G a 2 times k T e 2 B. Now, we want to find out over all noise temperature of this particular network, again I want to mention our actual objective is not to find the T e 1 2, our actual objective is to find out combined noise figure for these two network, this is again a mathematical representation.

So, we can say P n out will be actually represented by one single network. So, for that single network what will be the gain G a 1 multiplied by G a 2 k B and this will be the total thing which is seen at the input side, the right hand side terms can be now simplified in this particular fashion we can just take G a 1, G a 2 k p outside. So, we are left with T 0 plus T e 1 plus T e 2 divided by G a 1. So, from here we can calculate T e 1 2 as T e 1 plus T e 2 divided by G a 1 as I mentioned earlier, our objective is not to find T e 1 2 our objective is to find NF 1 2.

So, now we substitute the value of T e in terms of noise figure. So, we can now write to T e 1 2 as T 0 minus NF 1 2 minus 1, similarly we can write T e 1 and T e 2. So, by simplifying this particular expression, we can now write NF 1 2 as NF 1 plus NF 2 minus 1 divided by G a 1. So, how do we find overall noise figure, so let us just see that overall noise figure will be NF 1 2 is equal to NF 1 plus NF 2 minus 1 divided by G a 1. So, NF 1 comes out as it is corresponding to network 1, whereas NF 2 is now coming as NF 2 minus 1 divided by gain of the first stage. So, it is very important that first stage should also have decent gain also.

Suppose, if the gain is equal to 1, then what will happen overall noise figure will be combination of these two, but if the first stage has a gain of let say 10, then this term will be divided by 10; but if the gain of the first stage is 100 then this term will be divided by 100; that means, the second stage noise figure contribution will be very very small. Now, this particular thing can be extended to three cascaded networks also. So, let us see how this particular expression can be expanded to three networks.

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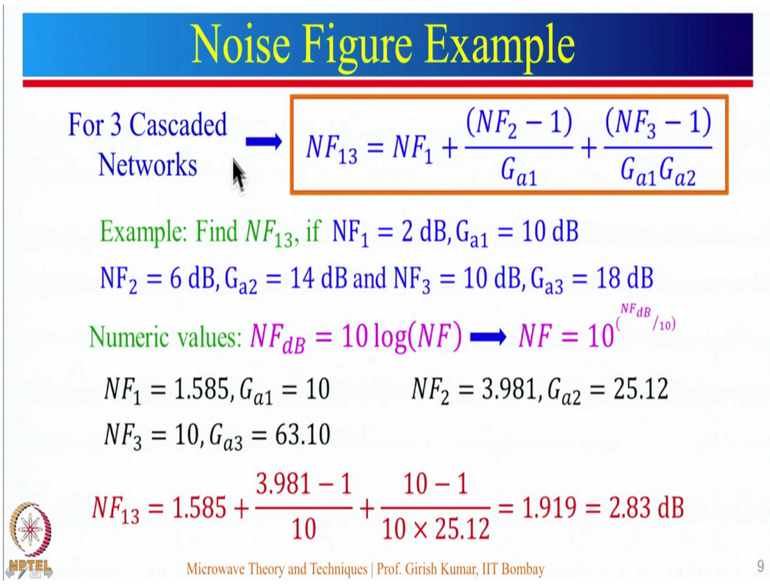


So, we have three cascaded networks. So, we want to find out the overall noise figure of these three networks. So, just recall the previous thing; so first stage comes as it is so NF 1, comes as it is NF 2 minus 1 divided by gain of the first stage, but for the third stage it is NF 3 minus 1 divided by gain of the first two stages. So, which is G a 1, G a 2 it will be more clear if you take an example. So, let us say we want to find out noise figure for three cascaded networks. So, the first stage has a noise figure of 2 dB gain of 10 dB, second stage has noise figure of 6 dB and gain of 14 dB, third stage is actually quiet bad as were as the noise figure is concerned it is 10 dB and gain is 18 dB. So, now we have to find out NF 1 3, so please do not keep all these dB values in this particular expression, you must find out the corresponding numeric value.

So, to find the numeric value we know that NF dB is given by 10 log NF. So, from here we can find the expression for NF which is 10 to the power NF dB divided by 10. So, for all these cases; now let us find out the numeric values. So, NF 1 comes out to be 1.585 corresponding to 2 dB, G a 1 comes out to be 10 for NF 2 6 dB, NF 2 is approximately 4, but the real value is 3.981. G a 2 14 dB comes out to be 25.12 and similarly NF 3 and G a 3 are given by these value.

So, now we can find out overall noise figure NF 1 3. So, you can see that NF 1 comes as it is, NF 2 minus 1 divided by gain of the first stage then NF 3 minus 1 divided by gain of first stage and second stage. So, overall noise figure is 1.919 which comes out to be

2.83 dB. So, if you look at this number here, you can see that this one here is 2 dB, so this has to be more than that, but you can see here this is 6 dB and 10 dB it is not sum of these 2 plus 6 plus 10, the reason for that is we had some gain for the first stage.

So, noise figure 2 gets divided by this particular gain, this noise figure 3 gets divided by these two gain values hence the contribution because of this is very very small. You can actually see that even though this noise figure is very high, contribution from this to the overall noise figure is very very small. So, just see that this is 10 minus 1, let us say 9 divided by 10 multiplied by 25.12 this really comes out to be only 0.04. So, you can see that the contribution is relatively small in this particular number.

So, just to summarize today we talked about low noise amplifier. We looked at two main sources of the noise; one is the thermal noise due to resistor and then we calculated; what is the noise power, available due to the resistor. Then we looked at the schottky noise, which is represented by i n square and that is mainly because of the pn junction within diodes or transistors, they are may be several pn junctions within a transistor.

So, basically an amplifier can be represented in terms of it is not an equivalent which is nothing but a current source in parallel with resistor. So, any amplifier circuit can be represented in these two simple current and resistance values; and from that we can find out what are the noise power available from them.

Then we talked about, signal to noise ratio and noise figure; we defined the term noise temperature, as I mentioned it is only a mathematical representation and we have to not worry too much about noise temperature, what we are interested in is to find out overall noise figure. So, we did find out the overall noise figure expression for three cascaded stages. And we took an example and we saw that the first stage is very very important. So, we must design the first stage which should have a very low noise figure. In the next lecture, we will actually look into how to design low noise amplifier.

Thank you very much. We will see you next time. Bye.