Microwave Theory and Techniques
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Module - 7
Lecture - 33
Microwave Amplifiers - II: Stability and Constant Gain Circles

Hello, in the last lecture we had started talking about Microwave Amplifier. So, we will continue we started with the simple inverting amplifier using op amp.

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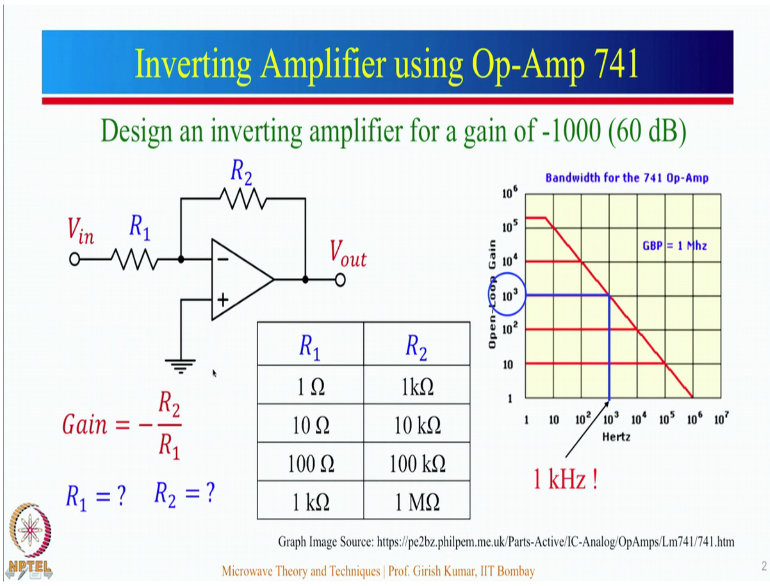

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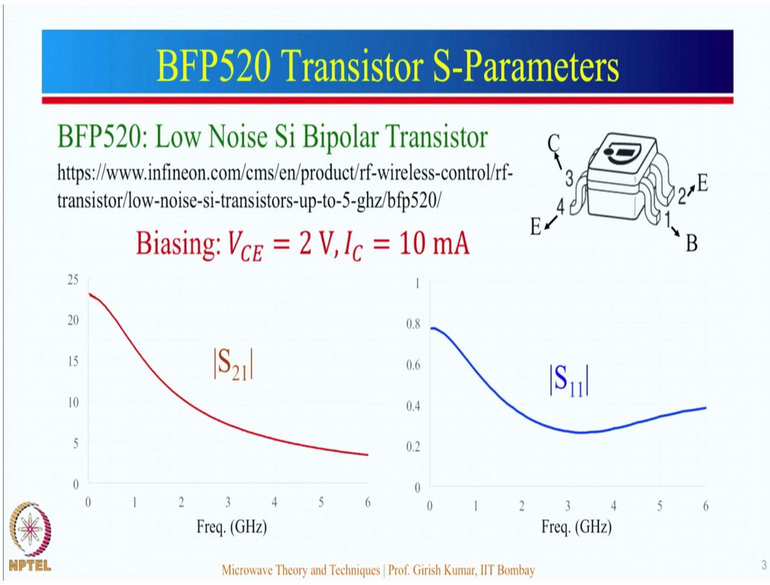

Then I mentioned about the transistor bfp 520, then I had mentioned about the transistor S parameters for given biasing conditions and we had seen that S 2 1 decreases as frequency increases. And this is the plot for the S 11, where also noticed that S 11 has a non zero value over the frequency band, hence it is important to design impedance matching network in the input side as well as at the output side.

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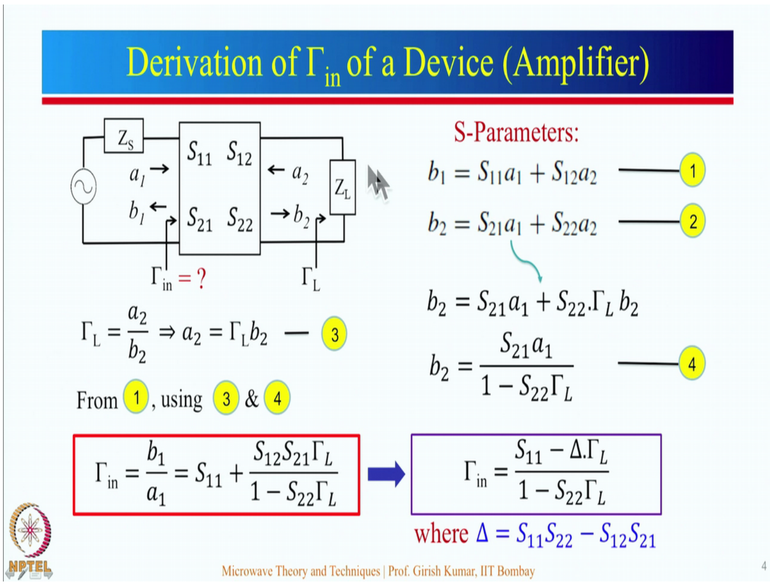

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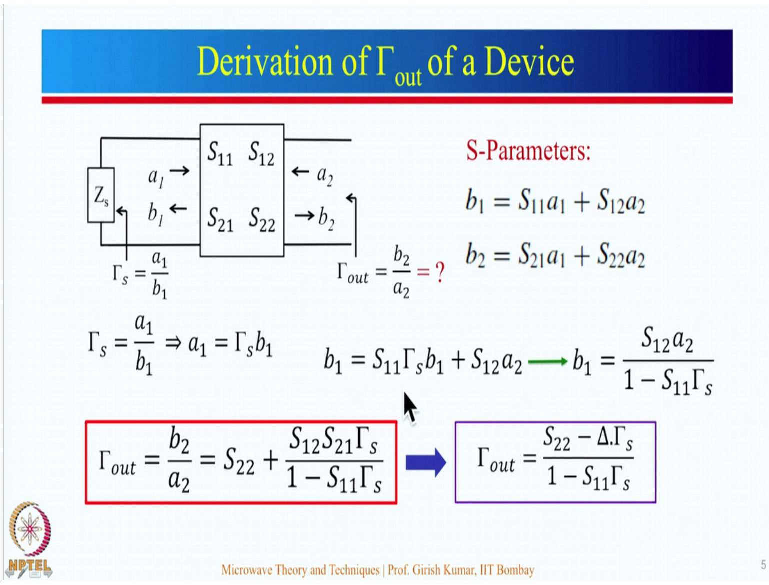

Hence we looked at the derivation for gamma in of the device, which is given by the this particular expression over here, then we looked at the derivation of gamma out of the device, which is given by this particular expression here.

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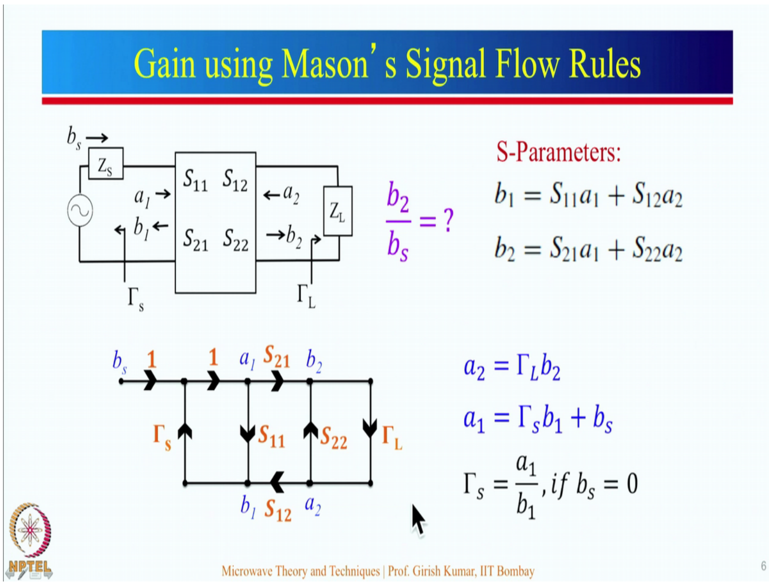

After that, we use masons signal flow rule to find out the gain of the amplifier, where the objective was to find b 2 divided by b s and we had used this signal flow.

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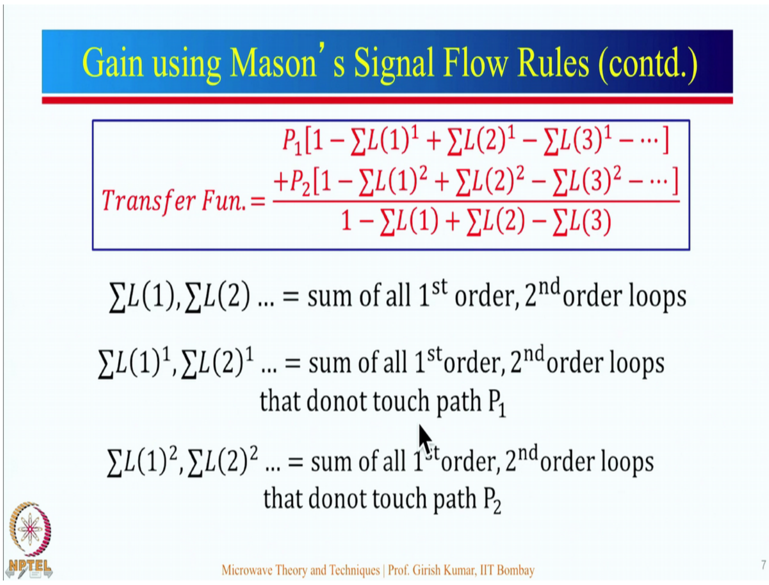

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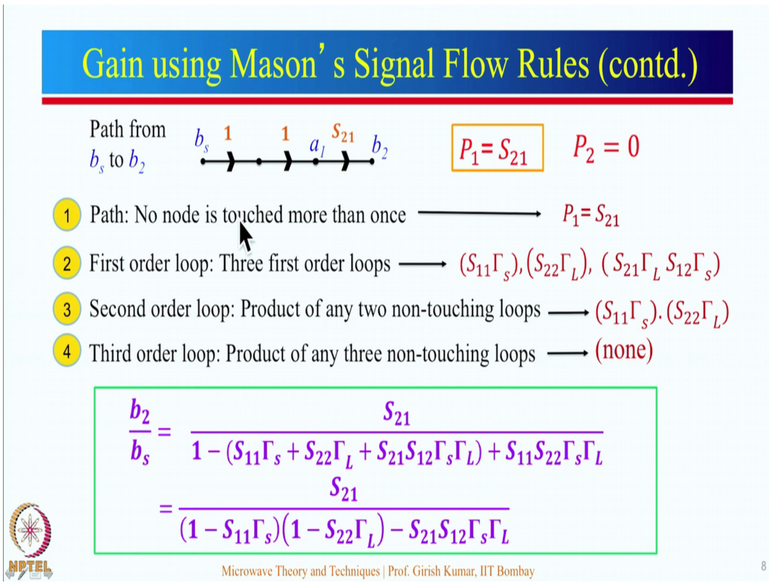

And then applied the masons signal flow transfer function formula and after that we calculated path from point b s to b 2 and then we found out what are the different loops and then we found out overall b 2 by b s.

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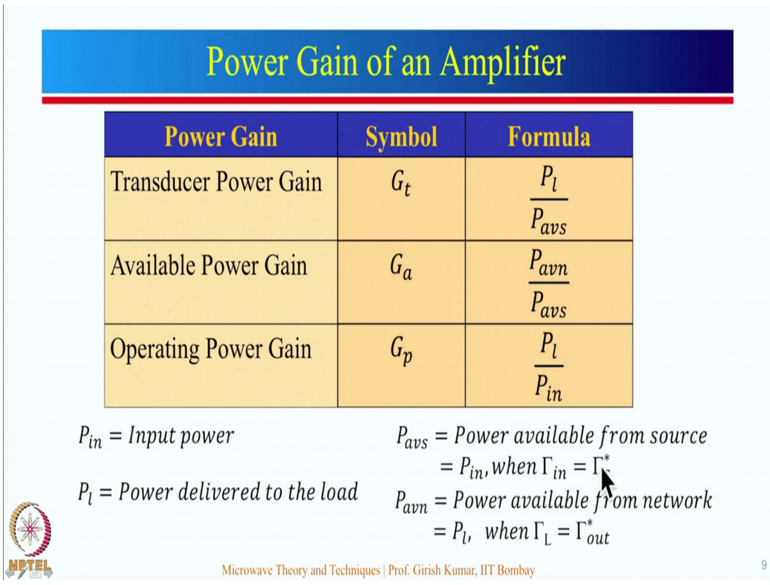

After that we looked at three different gain expressions; operating power gain, then transducer power gain, then we talked about available power gain and this happens when both gamma in is equal to gamma s conjugate and gamma L equal to gamma out conjugate.

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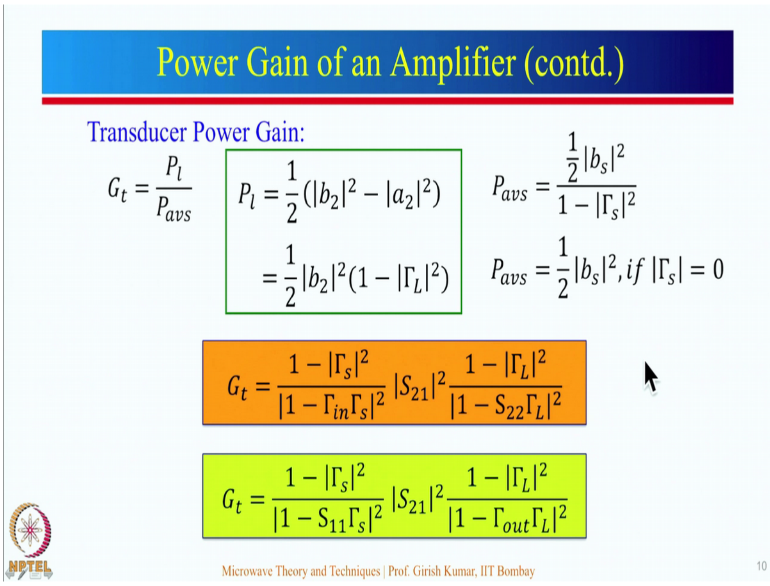

After that we looked at the expression for transducer power gain and we had seen this particular expression over here. So, this is the point where we left in the last lecture, let us continue from here now. So, as I mentioned this term corresponds to the input side, which can be maximized by using proper input impedance matching network. This is the gain because of that device S 2 1 and this term can be optimized by properly designing output matching network, even though as I mentioned earlier this is this input side, but gamma n depends upon gamma L.

So, somehow dependence of output is coming over here similarly, in this particular expression you can see there is a gamma out here, but gamma out depends upon gamma s. So, this output really speaking depends on again the input side. So, to make things simple we are going to take a few cases so, let us take these cases one by one.

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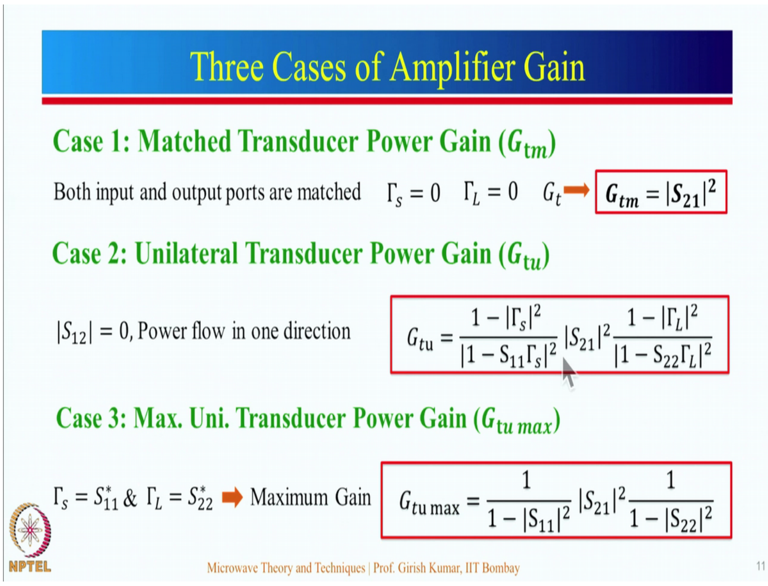

So, the first case which we are going to take is a case 1; matched transducer power gain, what this really means that if gamma s is equal to 0 and gamma L is equal to 0, then the previous expression reduces to this particular term only over here. So, this is kind of obvious because that is how S parameters of the device is measured because, what you do to measure S parameter, which terminate the output and the input side with 50 ohm.

So, if it is 50 ohm, then gamma s will be equal to 0 gamma L will be equal to 0 so, this is the matched gain. Now let us take a special case of unilateral transducer power gain, this is defined when S 1 2 is equal to 0. That means, power flow is only in one direction and in the other direction there is a no power flow that is why it is known as unilateral case, in case of bilateral power flow will be in both the direction.

So, when S 1 2 is equal to 0, you can see here that there is a no gamma in or gamma out term right now because, gamma in is now equal to S 1 1 and gamma out is equal to S 2 2. When S 1 2 is equals to 0. Now from G tu we can find out what is the maximum unilateral transducer power gain which is defined by G t u max and this situation will happen when gamma s is equal to S 11 conjugate a.

Please remember here gamma s should have been gamma in conjugate, but gamma in is now equal to S 1 1 gamma L should be equal to gamma out conjugate, but because S 1 2 is equal to 0 gamma out is equal to S 2 2 hence gamma l is S 2 2 conjugate now, we substitute this value over here. So, then this expression simplifies to this particular expression here let us see how? Gamma s is equal to S 1 1 conjugate. So, magnitude of that will be S 1 1 square.

Over here this is S 1 1 gamma s is S 1 1 conjugate. So, S 1 1 multiplied by S 1 1 conjugate will become S 1 1 square. So, you can see that, there is a 1 minus S 1 1 square here this one here is 1 minus S 1 1 square and whole square of that. So, one of these terms will get cancelled so, we are left with 1 over 1 minus S 1 1 square similarly, this term simplifies over here. Now you can see interesting thing over here G tm is equal to S 2 1 square when gamma s is 0 gamma L is 0.

But let us see now what this expression gives us S 2 1 square is there as it is, but now if S 1 1 is not equal to 0, let us assume say S 1 1 is equal to 0.5, then what will happen S 1 1 0.5 square of that will be 0.25 1 minus 0.25 is 0.75 this term will be now equal to 4 by 3.

So, that means, gain can be increased by a factor of 1.33, if we do proper impedance matching, let us look at this over here let take another example suppose S 2 2 is 0.6. So, 0.6 square will be 0.36 so, 1 minus 0.36 will be equal to 0.64. So, 1 divided by 0.64 you can say this factor will multiply the whole gain by approximately 1.6 time. So, one can get much larger gain by properly designing impedance matching network at both input side as well as the output side. However, at microwave frequency, whenever you want to design an amplifier it is very very important that you check the stability of the amplifier.

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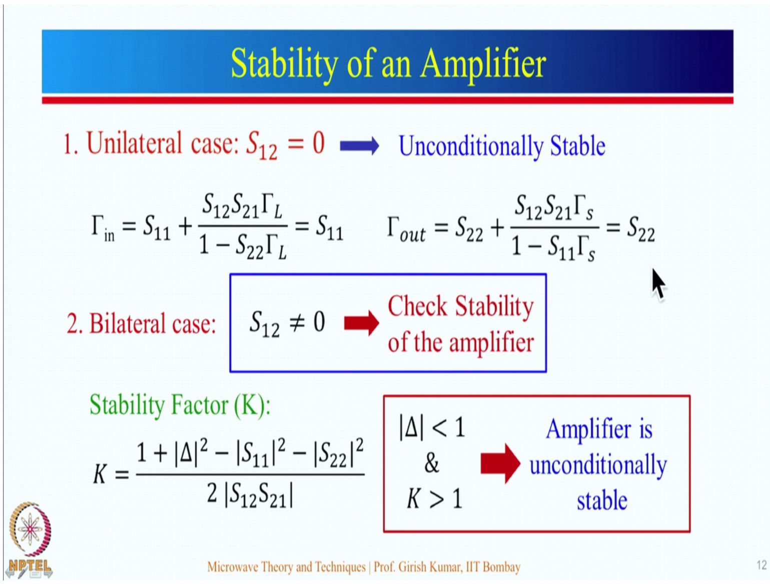

So, let us see how we define stability of an amplifier. Let us take of unilateral case, my in this situation S 1 2 is equal to 0, that is how unilateral is defined that signal is only going in one direction there is a no signal going in the opposite direction that means, S 1 2 is equal to 0. And if there is a no feedback going to the input side, this particular thing will remain always stable that means, it is unconditionally stable. And in this particular case now gamma in will be equal to S 1 1 you can just substitute S 1 2 equal to 0 and gamma out will be equal to S 2 2. So, it will always remain stable if it is a unilateral case.

Now, for bilateral case S 1 2 is not equal to 0, then we have to check stability of the amplifier. So, to check the stability of the amplifier we need to do two things; one is we have to see what is the stability factor K value. So, K is given by this particular expression, later on I will tell you how to drive this particular expression.

So, right now let us take this as it is so, what are the conditions for the amplifier to be stable first thing is delta should be less than 1, what is delta? It is determinant of S matrix S 1 1 S 2 2 minus S 1 2 S 2 1 and when delta is less than 1 and K is greater than 1, then amplifier is unconditionally stable. So, what will happen if delta is greater than 1? I just want to tell you for majority of the devices delta will not be greater than 1, for majority of the devices delta is in general less than 1.

But; however, if delta is greater than 1, in that particular situation K should be less than 1. So, please remember, if delta is less than 1, then K should be greater than 1, if delta is greater than 1, then K should be less than 1 for the amplifier to be unconditionally stable.

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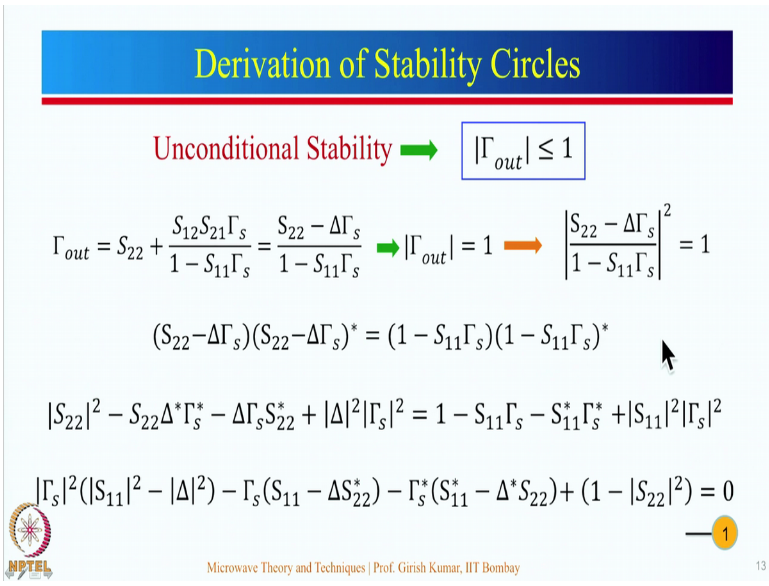

Now, let us look at now the derivation of stability circles; it will be obvious by the next slide what do I mean by stability circles? So, let us look at the condition for unconditional stability gamma out should be less than or equal to 1. You have to actually think about a smith chart, smith chart represents gamma equal to one circle and within that circle it actually represents all the real value of the impedance to be positive, imaginary part can be positive or negative. But what if the real part of the impedance is negative? In that particular situation gamma out will be greater than 1.

We can just a take a quick example here just imagine now; let us take an example of z out is equal to minus 10, then what is the expression for gamma out? This is equal to z out minus z 0 divided by z out plus z 0. If we take z out as minus 10 then gamma out will be minus 10 minus 50 which will be minus 60 divided by minus 10 plus 50 which will be plus 40.

So, minus 60 by 40 will be minus 1.5, in that particular case gamma out magnitude is greater than 1. For all positive value of real impedances gamma out will always be less than or equal to 1, in that particular case, we say it is unconditionally stable for all values of the load impedances. So, let us start with an expression for gamma out so, this is the expression for gamma out, we put the limiting case when this is equal to 1 and now substituting the value of gamma out over here so, this is the term.

Now, we have to simplify this particular term over here. So, let us see now the numerator part S 2 2 minus delta gamma s, since we have magnitude of that it is a complex number so, that is to be multiplied by it is complex conjugate. The denominator goes over here, so, which is right over here. So, now, we have to multiply these terms so, S 2 2 multiplied by this term here. So, you can see that this is complex conjugate so, that term will come here so, that complex conjugate will come over here it will pick up S 2 2 conjugate, minus delta conjugate and gamma s conjugate.

So, now we have to multiply these two so, S 2 2 into S 2 2 conjugate will be S 2 2 square, then minus S 2 2 then delta star, gamma s star. Then from here minus delta gamma s S 2 2 star or you can say S 2 2 conjugate and this multiplied by this will be delta square gamma s square. Similarly, we can write the right hand side so, this will be 1 1 minus S 1 1 gamma s, then minus S 11 star gamma s star because this will come inside and then, then magnitude term corresponding to the product of these two terms.

So, now what we have done over here? We are separated out the gamma s terms. So, gamma s square the corresponding coefficient is given here, then gamma s term which you can see from here put things together, then gamma s conjugate terms and then whatever is left out. So, now, this particular thing can be represented in the form of the circle so, let us see what is the form of the circle.

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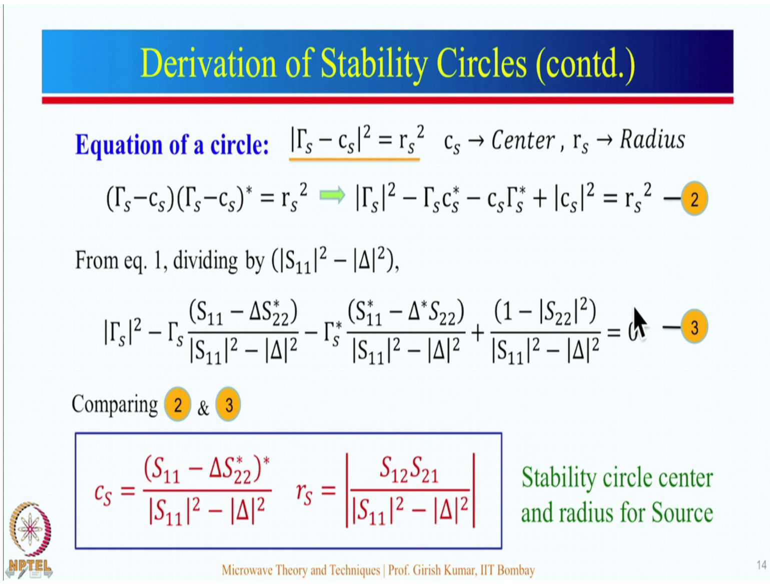

So, this is the equation of a circle in the polar coordinate, you might be familiar with the equation of circle as x square plus y square is equal to a square that is in rectangular coordinate or we say Cartesian coordinate. This is the equation in the polar coordinates so, what is this equation here? Let us see gamma s is the source reflection coefficient, this is the centre of the stability circle and this is the radius of the stability circle.

So, let us expand this term so this would be gamma s minus c s multiplied by it is conjugate value r s is a real value because, radius has to be a real value. So, that remains as it is, now we open this particular thing so, we have gamma s square minus the other terms are obtained multiplying these terms. So, gamma s c s conjugate minus c s gamma s conjugate plus c s square which is equal to r s square.

So, now, from the previous equation dividing that by this particular term we can write equation one in this particular form. Now comparing equation 2 and 3 let us see what we have. So, gamma s square is there as it is, this is gamma s c s conjugate so, this would be c s conjugate, this is minus c s gamma s conjugate so, this term will c s.

So, c s is given by this particular expression over here, how to find the value of r s? So, this is the one term which does not contain gamma s. So, corresponding to this here you can see that this will be now, c s square minus r s square. So, now, we already know what is the expression for c s? So, substituting the value of c s over here and simplifying for r s we can get the expression for the radius of the stability circle. In the similar fashion one can find out the centre and radius of the stability circle for the load.

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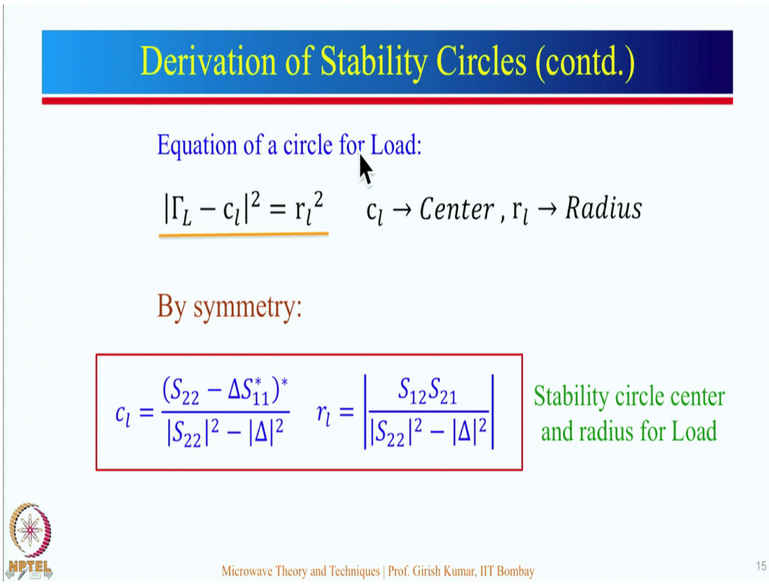

So, equation of a circle for load would be in a very similar fashion gamma L minus c l square is equal to r l square and by using this symmetry we can find out the expression for c l and r l.

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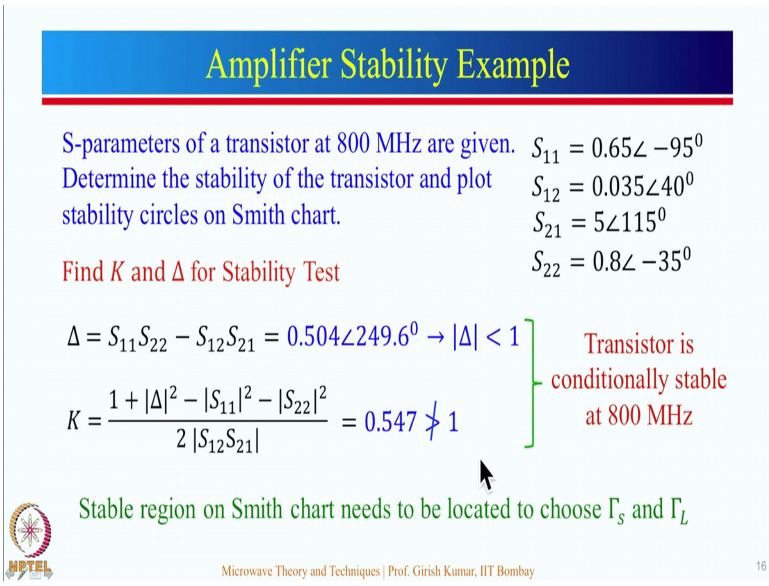

So, let us take an example; so, that you understand how to calculate these different stability circles and how these are represented on the smith chart? So, this is an example

S parameters of a transistor at 800 mega hertz are given. So, these are the S parameters so, we want to find out what is the stability of the transistor and plot stability cycles on smith chart.

So, for the given values of S parameters what we need to do we need to find the value of delta and K. So, delta is given by this particular expression all these S parameter values are given so, we can substitute these values of S parameters and then two complex calculation. This is the value which is obtained for delta; we can see that the magnitude of delta is less than 1.

Now, let us find out the value of K in the expression of K there are no complex numbers all are magnitude so, the calculation is relatively simple. So, we substitute these values and find out that K is equal to 0.547; that means it is not greater than 1. Hence, this particular transistor is not unconditionally stable, but instead of saying amplifier is unstable, we generally use a little different term we call it transistor is conditionally stable.

What it really means is that for certain cases, this particular transistor may be stable and if you satisfy those condition, then this transistor can be used as an amplifier, but when I discuss about the oscillator, the same thing I am going to say that this particular transistor is unstable.

So, please do not mix up here, since we are designing this transistor as an amplifier we want amplifier to be stable. So, we are going to find out what are those conditions, for which conditions the transistor is stable and for a oscillator design; we will actually see for which condition transistor is unstable and then we will use the unstable region to design oscillator. This will be more clear from the plot of the stability circles.

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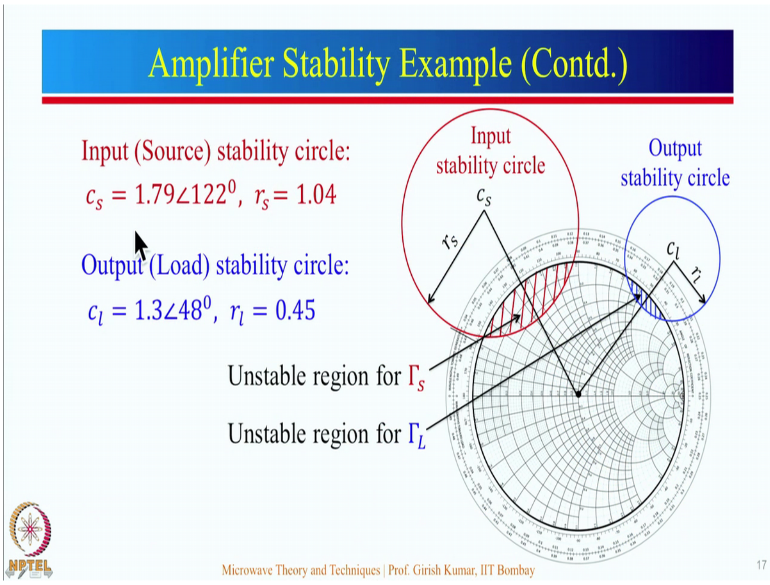

So, by using the expression of c s and r s we can calculate the values of c s and r s for given values of S parameter. So, this is the value c s and this is the value of r s and corresponding to the output stability circle these are the values of c l and r l. You can actually see over here that c s is not equal to 1.79; 1.79 will not be within the smith chart, please remember that smith chart only represents gamma less than or equal to 1.

Here this is the centre and radius of the gamma s which is input stability circle so, how do we locate that? So, what you need to do? You measure this particular distance here call this particular thing as one so, now, we need 1.7. So, let us assume that this particular distance is 10 centimeter. So, if this is 10 centimeter then 1.79 will be equal to 17.9 centimeter. So, you draw this particular line at an angle of 122 degree so, this will be angle 122 degree. So, this will locate the centre c s, now you can see r s is equal to 1.04, since we have assumed this to be 10 centimeter, this will be equal to now 10.4 centimeter.

So, with the radius of 10.4 you draw this particular circle, on paper you may not be able to draw the circle as it is going out of the smith chart what you can actually do is just calculate the difference of this number here. So, 1.79 minus 1.04 which is equal to 0.75, so, again assuming if this is 10 centimeter 0.75 will be 7.5 centimeters. So, locate this particular point here and then approximately draw this particular arc.

So, even if you cannot draw it is, now let us just look at the output stability circles, we have to locate the centre. So, again this one here is at an angle of 48 degree so, draw a line at an angle of 48 degree, then 1.3 minus 0.45. If you see the difference of this particular thing that difference is about 0.85.

So, if this is one; note down what will be 0.85 locate this particular point and approximately draw this particular arc or circle. Now this is the unstable region for gamma s, this is the unstable region for gamma L. So, you need to avoid this particular region over here so, do not choose the value of gamma s in this particular region, do not choose the value of gamma L in this particular region here.

So, if you choose the values of gamma s and gamma L in this particular region that amplifier will become unstable. However, however, when we want to design oscillator, we have to choose the values of gamma s in this particular region and for gamma L in this particular region. So, just to introduce the concept of the oscillator so, what do we generally do for oscillator? We choose a point in this particular region which is most unstable region.

So, let me just ask you a question you can think about it so, let us say point a, point b, point c, point d which is the most unstable point among a b c d? So, the actual answer is d because, this is deep inside the unstable region here. You can actually say that well, a is also unstable, b is also unstable, c is also unstable, why we do not take these points? The reason for that is because of external factors the biasing condition may change, the external components which you have used that they may have a tolerance value and also many of these devices parameter also may change from batch to batch and lot to lot.

So, it is always better for design of the oscillator you take a point which is most unstable. This will be more clear when I discuss about the oscillator design, but for amplifier please avoid this particular region and take gamma s value which is as far as possible. Similarly, for gamma L take the values as far from this particular place over here.

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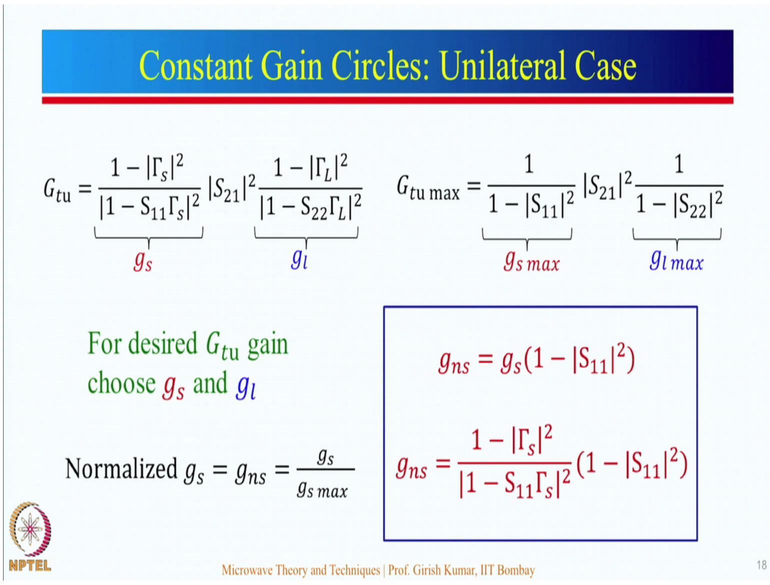

Now we are going to look at constant gain circle, so, will first start with the unilateral case. So, for unilateral case we had seen that, this is the expression for G tu and this is the expression for G tu max, generally speaking it will be specified that we have to design an amplifier for a given gain.

So, let us say for the desired G tu gain what we do? We choose the value of g s and g l, why we separately choose? Because, this represents the input side this represents the output side. So, we can design impedance matching network for input side as well as the output side separately, but before you proceed for this first of all you must find out what is g s max and g l max and what is the total G tu max? So, that desired G tu gain has to be less than G tu max. Suppose, if G tu max is 12 t p and if the problem says you have to design for a gain of 13 t p well that is not at all possible.

So, it is important to calculate the value of G tu max. So, now, for desired gain once you know that maximum gain is more than the desired gain, choose the value of g s and g l. So, the next step is you find out the normalized g s which is g n s this will be equal to g s divided by g s max, similarly, g n l will be g l divided by g l max.

So, in this expression let us substitute the value of g s max so, we actually we can find out g n s equal to g s, since g s max is 1 divided by this term here that will go in the numerator. So, now, g n s equal to this particular term, which is coming from here and this term corresponds to the g s max term. In this expression the only unknown is gamma s other values are known S 1 1 is known and g n s is known. So, we solve this particular equation for gamma s and that will lead to the constant gain circle.

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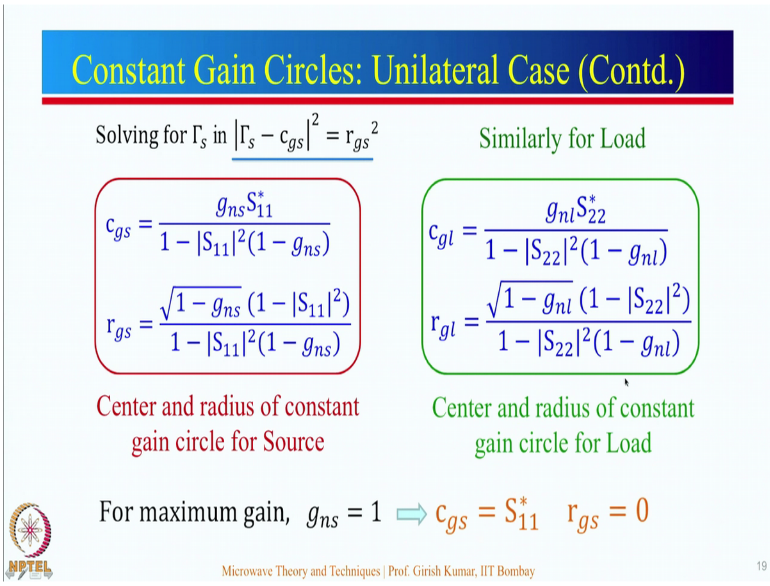

So, solving for gamma s in the form of gamma s minus c gs square is equal to r gs square by solving the previous equation we get c gs and r gs expression similarly, for load we get expressions for c g l and r g l. So, these are the expressions for centre and radius of constant gain for source, these are the expressions for centre and radius of constant gain circle for load. Let just look at the special case, for maximum gain g ns is equal to 1; that means, normalize value is equal to 1 and if you put g ns equal to 1 here let us see what we get. So, c gs will be equal to 1 over here and this term is 1 minus 1 so, that will become 0 so, this term becomes 0 so, c gs becomes equal to S 1 1 conjugate. Let us see what happens to r gs so, when g s is equal to 1. So, this becomes 1 minus 1.

So, that term will be 0, this will be 1 minus 1 this term is 0, but the denominator will be 1 minus 0. So, this particular thing becomes equal to 0, this is kind of obvious that for maximum gain we know that the gamma s should be equal to S 1 1 conjugate and that will be a single point. So, for maximum gain this expression reduces to this simple form where as for other values of g ns we have to calculate c gs and r gs.

In the same way one can derive the expression for the load side. So, these are the expressions for the centre and radius of the constant gain circle for load. In the next lecture, we will see what happens if S 1 2 is not equal to 0, what kind of a error is obtained if S 1 2 is not equal to 0 and then we will take a complete example of an amplifier design.

Thank you very much, see you next time, bye.